SOCIAL STUDENTS’ ADAPTIVE REASONING IN SOLVING MATHEMATICAL PROBLEMS

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Abstract
Adaptive reasoning is a component of basic mathematical skills that needs to be developed for students so that they can use mathematical procedures effectively. This research is a qualitative research that aims to describe the adaptive reasoning profile of secondary students in the Social Sciences department in solving mathematical problems. Research subjects were three students that solving mathematical problems correctly, solving mathematical problems less correctly, and solving mathematical problems incorrectly. The method used to collect data was to provide mathematical problem-solving tests and interviews. Data were analyzed based on students' adaptive reasoning activities in their activities to solve mathematical problems seen from three main aspects of adaptive reasoning, namely reflecting, explaining, and justifying. The results show that student who solved mathematical problems correctly indicated adaptive reasoning abilities in every aspect; student who solved mathematical problems less incorrectly demonstrated adaptive reasoning abilities that almost met all indicator aspects, and student who solved mathematical problems incorrectly did not demonstrate adaptive reasoning abilities in every aspect.

Keywords: adaptive reasoning, problem solving, social secondary students.

INTRODUCTION
Mathematics is a science that plays an important role in various disciplines. In accordance with the competency standards created by the National Education Standards Agency (2006), it shows that the main purpose of learning mathematics is to develop and utilize the students’ reasoning abilities. Mulyasa (2008) argues that reasoning ability is the ability to think systematically, logically, and critically in communicating ideas or problem solving. According to Suherman (2005), reasoning is a thought process that can be improved through direct and intensive exercises. The exercise is a series of tasks working on problems that are done repeatedly, so that a person becomes skilled in drawing conclusions. Hence, a reasoning ability is really essential when someone is solving a problem or making a decision.

Problems related to mathematics are often found in the daily life of all people, including students from elementary to tertiary levels. Basically, a problem is something that cannot be solved easily. Thus, it requires effort or deeper thought. Mathematics becomes a problem for students when they cannot apply formulas to solve a mathematical problem (Shadiq, 2004). To solve mathematical problems, logical considerations are needed to determine the strategy...
The National Research Council (NRC) (in Kilpatrick et al., 2001) introduces adaptive reasoning as a term. Adaptive reasoning has a connection to problem solving including mathematical problems. It is in line with the opinion of Siswono (2019), who said that adaptive reasoning is general reasoning in mathematics which refers to a capacity to think logically about the relation between concepts and situations. Thus, adaptive reasoning is closely related to mathematics. Kilpatrick et al. (2001) states that adaptive reasoning is one component of basic mathematical skills. Adaptive reasoning is the capacity to think logically, estimate answers, provide explanations about the concepts and procedures of the answers used, and assess its truth mathematically. This indicates that adaptive reasoning should be developed for students so they can adopt mathematical procedures effectively. From this explanation, it can be concluded that adaptive reasoning has a fairly close connection with a mathematical problem solving.

Academically, a secondary school level implements a major system which includes a major in Natural Sciences, Social Sciences, and Linguistics. Rufaidah's research (2015) shows that factors like intelligence and student’s interest influence the selection of a major. Thus, it can be inferred that students who are in the same particular department have academic abilities that are not much different one another. This is strengthened by the results of research by Hayati and Sujadi (2018) which show that the learning skills of students majoring in Natural Sciences are higher than the learning skills of students majoring in Social Sciences.

Therefore, it can also be stated that the ability of students in mathematics among departments is different. Moreover, students majoring in Natural Sciences get diverse subjects that hone their mathematical abilities more compared to other majors including Social Sciences. This is in line with the results of research by Akbar (2015) which show that students majoring in natural science must have mathematical abilities that are superior to students who major in social studies. In addition, Akbar (2018) also denotes that mathematical reasoning ability of students majoring in social studies is low. Although the social students’ mathematics reasoning ability is low, we do not know how is their adaptive reasoning, it is a different case. Yet, the researcher can not find a reaserch that discuss about social students’ adaptive reasoning.

In more details, students, with their adaptive reasoning ability, are able to explain and justify each of the steps that they use in solving problems. Consequently, it will be easier to explore students' adaptive reasoning abilities. As a result, it is principal to know how the adaptive reasoning of students majoring in social sciences so that we could develop improved adaptive reasoning for them.

Based on the description above, the researchers aim to conduct research on the adaptive reasoning profile of secondary students majoring in social sciences in solving mathematical problems.

METHODS

The study was conducted during the even semester of the 2019/2020 academic year. Data collection was carried out at the Surabaya 16 Public High School. The subjects chosen in this study were three students of grade XI majoring in Social Sciences. Students selected as subjects are those with mathematical problem-solving test results that fit into three categories, namely; solving mathematical problems correctly, solving mathematical problems less correctly, and solving mathematical problems incorrectly.

This research implemented a qualitative approach. Qualitative research is a research procedure that produces descriptive data, namely descriptions of observed behaviors of subjects (Bogdan & Taylor in Siswono, 2019). The main instrument in this study was the author himself, and supporting instruments in the form of the Mathematical Problem-Solving Test (MPST) and interview guidelines.

Researchers studied and described the profile of students’ adaptive reasoning by giving MPST which was then followed by interviews. The researcher traced the data of how students connected the mathematical concepts that were appropriate to the mathematical problem situation during the reflecting, explaining, and justifying stages. Then, based on the data obtained, the researchers analyzed the adaptive reasoning profile of students in solving mathematical problems.

MPST was performed earlier to 33 students in the research class to see how students’ adaptive reasoning abilities were in solving mathematical problems. Based on the written results of MPST by those students, researchers analyzed each student's results. The analysis was carried out based on assessment guidelines and adjusted for indicators of adaptive reasoning in solving mathematical problems (see Table 1). The results of this analysis were then used to determine the final research subjects. From the results of the previous MPST analysis, three subjects were chosen. Subjects taken were those with three different categories, namely; student who solved mathematical problems correctly (has a perfect score, 12), student who solved mathematical problems less correctly (has a middle score, 8), and student who solve mathematical problems incorrectly (has a lowest score, 4). Then, MPST was given to the selected research subjects. The purpose of this MPST was to dig deeper into the adaptive reasoning of them in solving mathematical problems. Then, interviews were conducted to the research subjects regarding the answers.
they gave to MPST in order to dig deeper into the profile of students’ adaptive reasoning. The interview served to complete the students’ adaptive reasoning information from MPST’s results that were not yet in accordance with all indicators of adaptive reasoning.

MPST result data from the research subjects were analyzed by adjusting to the assessment guidelines and referring to the indicators of students’ adaptive reasoning in solving mathematical problems. Later, each student’s answer was connected to aspects of adaptive reasoning that had been determined in this study. Interview data were analyzed by researchers using qualitative analysis techniques. Interview data analysis was carried out through the stages of data reduction, data presentation, and drawing conclusions (Miles & Huberman, 2014).

The following is a task of MPST given to the research subjects: "Mr. Rudi has a plot of land with an area of 8 x 5 m² filled with weeds. On the land, there is a small building measuring 4 x 3 m² which is located side by side with two sides of the land at once. One day, Mr. Rudi tied a goat to a pole at the end of the building that was the closest to the middle of the land. If the rope that ties the goat is only two meters long, then determine the area of the land in which the weeds cannot be eaten by the goat!"

To facilitate the presentation of interview data, researchers used this indicators as in the following table:

### Table 1. Indicators

<table>
<thead>
<tr>
<th>Aspect of Adaptive Reasoning</th>
<th>Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflecting</td>
<td>Linking concepts that are relevant to the problems faced from their learning experiences</td>
</tr>
<tr>
<td></td>
<td>Applying appropriate concepts to the problems at hand</td>
</tr>
<tr>
<td></td>
<td>Re-checking whether the solution has been done in accordance with the chosen concept</td>
</tr>
<tr>
<td>Explaining</td>
<td>Explaining things that are known accompanied by strong arguments</td>
</tr>
<tr>
<td></td>
<td>Explaining the questions asked with strong arguments</td>
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<tr>
<td></td>
<td>Explaining concepts that are appropriate to the problem situation</td>
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<tr>
<td></td>
<td>Explaining the idea of appropriate solutions towards the problems based on learning experiences</td>
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<td></td>
<td>Explaining the ideas/strategies that have been used in solving problems</td>
</tr>
<tr>
<td>Justifying</td>
<td>Students justify the ideas/strategies they use in solving problems based on problem situation</td>
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</table>

RESULTS AND DISCUSSION

Adaptive Reasoning of Student who Solved Mathematical Problems Correctly (Subject 1/ S1)

**Figure 1. Mathematical Problem-Solving Test Results Subject S1**
From the results of the tests, interviews and researchers’ observations as well as an overview of Table 2, the researchers describes the adaptive reasoning of the subject who solved mathematical problems correctly (S1) in each aspect as follows:

**Adaptive Reasoning of Subject S1 on the Reflecting Aspect (R)**

On the reflecting aspect, S1 could represent the problem in the form of images that were in accordance with the given situation and do the right solution both in terms of the use of formulas and calculations. This indicates that S1 could determine the concept that suits the problem situation from its learning experience; in other words, S1 reflected by connecting concepts and situations. Consider the following interview quotes:

Q: Why do you think that these shapes are rectangular and circle?
A: Yes this is the land, a land must be rectangular. From its size, it can also be seen if the shape is rectangular. And buildings are usually rectangular everywhere. Then, as far as I know, something that was tied to a rope could only move in a circle.

S1 connected the situation of the problem with the situation in the real world when the subject gave the argument that the shape of the land and building in the problem were rectangle other than because of the size listed; in fact, the land and building are rectangular in reality. S1 also re-explained about the concept of coinciding that it used to make an appropriate picture. Although S1 also stated that it was not sure if it had remembered the concept well, S1 could use the concept of coinciding correctly on the picture. From the results of the written tests and interviews with S1, it can be concluded that student who solved mathematical problems correctly (S1) fulfills all indicators of adaptive reasoning in the reflecting aspect.

**Adaptive Reasoning of Subject S1 on the Explaining Aspect (E)**

On the explaining aspect, S1 explained with its own sentences about the things that were known and asked on the problem. S1 could express the argument that those have been stated in the problem. Once S1 understood the problem, it explained that the problem in the test was related to the concept of a flat area, an addition and a subtraction. S1 could also explain in detail with its own language how the situation in the problem was. Later, S1 explained the idea of how to solve the problem, i.e. S1 must find each area of known shapes, then use the principle of addition and subtraction to find the asked area.

S1 also explained in detail how the solution it had delivered in solving MPST. S1 explained that the first thing S1 did was to make a picture that suited the problem situation, then S1 could explain in detail about the picture along with logical reasons why S1 made such picture. Then, S1 explained in detail about the calculation/solution and the reasons why it did so. From the interview result, it can be concluded that student who solved mathematical problems correctly (S1) fulfills all indicators of adaptive reasoning in explaining aspect.

**Adaptive Reasoning of Subject S1 on the Justifying Aspect (J)**

On the aspect of justifying, S1 provided logical and appropriate arguments for each step of its solution and explanation. Consider the following interview quotes:

Q: What makes you believe that your answer is correct?
A: From the picture itself. In my opinion, the problem is is just right, the coinciding also seems right. Then the formula, the formula for the area of a rectangle and area of a circle are like these, the numbers I entered are also correct, the calculation is also, in my opinion, correct.

S1 always gave logical reasons based on its knowledge. S1 believed that the solution that he did was right because it was in accordance with the concept that he thought was appropriate and correct, as well as when S1 checked the solution. From the result of interviews, it can be concluded that student who solved mathematical problems correctly (S1) fulfills all indicators of adaptive reasoning on the justifying aspect.

From all available aspects and indicators of adaptive reasoning, it can be concluded that student who solved mathematical problems correctly (S1) can fulfill all indicators of adaptive reasoning in each aspect. In solving mathematical problems, S1 did reflecting activities to connect concepts and situations faced, and could explain what the problem situation and each step of the solution. On explaining aspect, S1 could justify every step of the solution it did. This is in accordance with the statement of Siswono (2017) that someone who has the ability of adaptive reasoning is able to connect between existing concepts and situations. Moreover, one can explain and justify everything he has done. In addition, it is reinforced by the words of Ostler (2011) that someone who has adaptive reasoning ability is able to know that the solution is correct, and provide the right reasons.

**Adaptive Reasoning of Student who Solved Mathematical Problems Less Correctly (Subject 2/ S2)**
From the results of the tests, interviews and researchers’ observations as well as an overview of Table 3, the researchers describe the adaptive reasoning of the subject who solved mathematical problems less correctly (S2) in each aspect as follows:

**Adaptive Reasoning of Subject S2 on the Reflecting Aspect (R)**

On the reflecting aspect, S2 could not solve the picture problems, but it could do the right choice of using formulas and calculation. From the explanation of S2, it seemed that it could answer well on understanding the questions and determining the concepts in accordance with the problem. However, S2 could not apply the concepts understood correctly. Thus, it did a re-examination, then, it could find the errors. Consider the following interview quotes:

Q: Which one do you think is wrong and how is it right?
A: The picture is wrongly located, it should be in the corner. If it is like this, it means it doesn’t suit the definition of “coinciding with two sides of land”

From this activity, although the subject made a mistake at the solution, S2 could do the reflection activity when checking the suitability of the solution to the problem situation and the corresponding concept. Also, the subject found that there was a mismatch between the solution and the problem situation. From the results of the written tests and interview with S2, researchers concluded that student who solved mathematical problems less correctly (S2) does not meet all the indicators of adaptive reasoning in the reflecting aspect; the student could not apply concepts that were in accordance with the problem.

**Adaptive Reasoning of Subject S2 on the Explaining Aspect (E)**

On the explaining aspect, S2 explained about the things that were known and asked on the problem. It could show the details on the test. S2 explained that it could understand the problem by imagining how the situation in the problem was so that it could determine the concept contained in the problem, namely the concept of the area of rectangular and circle. Also, S2 confessed that it did not have a specific strategy planned to solve the problem. From the explanation of S2, it was known that the initial idea to solve the problem was to make a picture that matched with what was known in the problem so that it could determine what steps it should do next.

From S2’s explanation of its activity in solving problem, it really understood the problem when it could draw a picture, according to it, in accordance with the situation of the problem. Then, it could find the area that was asked on the problem. S2 explained in detail about its activities when solving problems, but the subject could not give a reason strong enough for each step it did. From the results of interview with S2, it can be concluded that student who solved mathematical problems less correctly (S2) meets all indicators of adaptive reasoning in explaining aspect.

**Adaptive Reasoning of Subject S2 on the Justifying Aspect (J)**

On the aspect of justifying, S2 could make a justification for the mistakes made at its solution. Although the results of the S2’s MPTS solution were not quite correct, S2 could find errors in making image representations that were less appropriate to the problem situation. S2 could explain precisely how the picture should be more suited to the problem situation. From the results of interview with S2, researchers summarized that student who solved mathematical problems less correctly (S2) meets all indicators of adaptive reasoning on the justifying aspect.

From all aspects and indicators of adaptive reasoning, it can be stated that student who solved mathematical problems less correctly (S2) can meet almost all indicators of adaptive reasoning in each aspect. Although S2 did not make a correct solution on its written test, it could make a justification for its mistakes. This is relevant to the study of Hidayati (2017) that students with mathematical abilities are not careful enough in the process of implementing problem solving. Even though the subject’s categories differ, student who solved mathematical problems less correctly in this study can be labeled to have a moderate problem-solving ability. In addition, the subject’s justifying activities was in line with Kilpatrick (2001) that students who have adaptive reasoning abilities are able to assess the correct works and justify it.

**Adaptive Reasoning of Student who Solved Mathematical Problems Incorrectly (Subject 3/3)**

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![Figure 2. Mathematical Problem-Solving Test Results Subject S2](image-url)
From the results of the tests, interviews and researchers’ observations as well as an overview of Table 4, the researchers describe the adaptive reasoning of the subject who solved mathematical problems incorrectly (S3) in each aspect as follows:

**Adaptive Reasoning of Subject S3 on the Reflecting Aspect (R)**

On the reflecting aspect, S3 could not represent the problem in the appropriate form to the given situation. S3 did not do the solution correctly even though it wrote the correct formula. S3 told that since the first start, it could not determine what concept appropriate to the problem situation. Apparently, S3 was confused with the word “coincide” in the problem. S3 stated that it forgot about the concept. Although writing the actual formula was indeed appropriate for the problem, S3 could not adjust the formula to what it was drawing. From written test results and interview with S3, it can be concluded that student who solved mathematical problems incorrectly (S3) does not meet the indicators for aspects of reflecting on adaptive reasoning.

**Adaptive Reasoning of Subject S3 on the Explaining Aspect (E)**

On the explaining aspect, S3 did not answer in detail almost all of the researchers’ questions related to this aspect. In other words, S3 did not meet the indicators in the explaining aspect. S3 did not explain in detail about the things that were known to the problem with its own words. In fact, S3 stated that it could not understand the problem. Although it could not understand the situation of the problem well, S3 understood what the question was about. It was proven when the subject could explain what was asked on the problem. In its words, S3 could not determine what concept in accordance with the problem was because the subject itself did not understand the test. S3 also explained that the solution provided was only because it just wrote what it had in mind. S3 also did not explain in detail every step of its solution. Consider the following interview quotes:

Q: Do you understand this problem? And why is that?
A: I don’t understand because I forgot about the concept of coincidence.

Q: How could you finish this problem?
A: Just try as much as I could
Q: Can you tell me things noticed?
A: There is a land. There are buildings and goats
Q: Explain again how you solved this problem!
A: Yes, I read the problem, then write first what is known, the land, the small building, the rope. At first, I was confused about the coincidence, but then I let myself just write what can be written first. Then, I wrote what was asked. Then I drew the land, the building, the rope, and I calculated the area of the land, the building, and the rope.

Q: Why did you do such a solution?
A: Because that is the way it should be done.

From the results of interview, it can be concluded that the adaptive reasoning of student who solved mathematical problems incorrectly (S3) does not meet the indicators for explaining aspect in adaptive reasoning.

**Adaptive Reasoning of Subject S3 on the Justifying Aspect (J)**

On the justifying aspect, S3 could not find its own mistakes when given the opportunity to re-examine its work. With the help of a little guidance from the researcher, S3 still did not think that it was making a wrong solution. According to the subject, it was all correct. Moreover, S3 could not provide a strong argument that could prove that the solution or explanation was correct. From the results of interview with S3, it can be concluded that student who solved mathematical problems incorrectly (S3) does not meet the indicators for the justifying aspect of adaptive reasoning.

From all aspects and indicators of adaptive reasoning available, it can be concluded that student who solved mathematical problems incorrectly (S3) does not meet the indicators of adaptive reasoning in each of its aspects. In its explanation, S3 did not provide a logical and detailed explanation. S3 was also unable to reflect and justify its work. The results of this study are relevant to the results of the research by Hidayati (2017) that students who have low mathematical abilities are not able to explain logical reasons to strengthen the answers they had. Even though this subject category is different, student who do not solve mathematical problems correctly can be labeled to have low mathematical, problem solving, abilities. Hence, it can be concluded that the reasoning ability of student who
CONCLUSIONS

Students’ Adaptive Reasoning Who Solved Mathematical Problems Correctly

On the reflecting aspect, student was able to make appropriate solutions both when making problem representations and calculating its solutions. Student was able to determine, connect, and apply concepts that were appropriate to the problem situation. On the explaining aspect, student explained clearly in describing its statements and provided logical reasons for each statement.

On the aspect of justifying, student could provide strong and logical arguments for each statement and completion step that was done, which reinforced that the results of the subject's work were correct and in accordance with what they should be. Overall, it means that student who solves mathematical problems correctly has good adaptive reasoning skills. It comes from one’s ability to connect situations and proper concepts, be able to explain in detail and justify every step of the solution.

Students’ Adaptive Reasoning Who Solved Mathematical Problems Less Correctly

On the aspect of reflecting, student was able to determine the concepts appropriate to the problem, but the subject was not able to make solutions correctly. On the explaining aspect, student explained clearly and in detail about the whole process of solving the problem. However, the subject did not provide a strong reason for each step of the solution. Nevertheless, each subject's explanation fulfilled all the indicators in the explaining aspect. On the aspect of justifying, student was able to find mistakes and make a justification on the more appropriate solution for the written results. Overall, it means that student who solve mathematical problems less correctly shows its adaptive reasoning abilities when the student is able to find errors and make their justification with strong and logical explanations.

Students’ Adaptive Reasoning Who Solved Mathematical Problems Incorrectly

On the reflecting aspect, student was not able to connect and apply concepts that were appropriate to the problem situation. On the explaining aspect, student was not able to explain in detail when answering almost all questions of researchers. Besides, the subject was not able to provide clear and logical reasons for each statement and solution’s step. On the aspect of justifying, student was not able to give strong and logical reasons for every solution one did. Overall, it means that student who solves mathematical problems incorrectly does not have adaptive reasoning skills because one is unable to connect concepts and situations accordingly; explain and justify each step of one’s solution.

RECOMMENDATIONS

The result of the study indicates that the adaptive reasoning profile of students in social studies major varies. For mathematics instructors, it is recommended to familiarize students with problems based on reasoning and require students to be able to explain and justify their solution steps with the aim of improving and equalizing students’ adaptive reasoning abilities in the social sciences department. And for other researchers who will conduct research in order to find out the profile of students’ adaptive reasoning, it is recommended that they review the students' adaptive reasoning from other various aspects, such as gender, learning styles, mathematical abilities, and so on.

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