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PROFILE OF STUDENTS' ANALOGICAL REASONING IN SOLVING MATHEMATICS PROBLEMS: A STUDY BASED ON SELF-REGULATED LEARNING

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Abstract

Analogical reasoning is an important ability for students related to reasoning activities by thinking carefully about the regularity of patterns found in mathematics. One of the factors that influence analogical reasoning is self-regulated learning, which shows the ability of the student to analyze, take strategies, and the ability to control their learning environment. This research aims to describe students' analogical reasoning in solving mathematics problems based on self-regulated learning. The subjects were one student with low self-regulated learning and one student with high self-regulated learning. The research method used is descriptive qualitative, and data were obtained through analogical problem-solving tests and interviews. Subjects were chosen based on the questionnaire instruments' categorization regarding the student's self-regulated learning and the mathematical ability test instruments. The results of this study were (1) Analogical reasoning of a student with low self-regulated learning: passed through the stages of structuring, mapping, and applying. At the structuring, the student identified all the information completely, but did not understand the problem correctly and could found the relationship between the source problem and the target problem. At the mapping stage, the student visualizing on her mind the forms of trigonometric equations that had similarities to previous problems then used to factoring the equation. At the applying stage, not completely mentioned all the possible solutions to the given problems. Meanwhile, (2) Analogical reasoning of a student with high self-regulated learning passed all the analogical reasoning stages of structuring, mapping, applying, and verifying. At the structuring, the student identified all the information completely and understand the problem. At the mapping stage, the student built a new mathematical model for the target problem. At the applying, the student mentioned all the possible solutions correctly. And student did the verification.

Keywords: analogical reasoning, self-regulated learning, problem-solving

INTRODUCTION

Reasoning is commonly used in teaching mathematics to understand a problem, build a mathematical model, and solve a problem. Reasoning prepares students with critical, creative, systematical, logical, and analytical thinking skills under the competency standards outlined in the *Peraturan Menteri Pendidikan Nasional* (2006). One of them is to use reasoning on patterns and properties and finding the connections between mathematical concepts. Thus, reasoning is an ability that must be possessed by individuals in learning mathematics.

Inductive reasoning is one aspect of reasoning, a process of generalizing or drawing conclusions from something specific to something general (Athanassopoulos & Voskoglou, 2020). English (2004) stated that analogical reasoning has been considered a part of inductive reasoning to detecting relational patterns and identifying repetitive patterns in dealing with variations in problems. Voskoglou (2012) stated that analogical reasoning is the activity of comparing information between the new concepts and concepts that have been learned in the past, then using the information to gain a deep understanding. Based on the description above, analogical reasoning is the process of reasoning by searching for patterns to find similarities between two or more particular problems.

According to Dienes (Susanto, 2015) and Hasratuddin (2013), stated that mathematics is the science of patterns, relationships, discipline, and hierarchical structure. The upper concepts have a relationship depending on the prior level. This statement implies that understanding previous concepts played a vital role in bridging the previous concepts to other concepts that will be studied. Analogical reasoning links a similarity between existing knowledge and the problem at hand. Hence, doing analogical reasoning can help in solving a mathematical problem.

Analogical reasoning becomes a crucial component in teaching mathematics because it helps students enhance creativity and make teaching and learning more effective (Lailiyah et al., 2018). Besides, the role of analogical reasoning in problem-solving, according to Mofidi et al. (2012) and Kaymakci (2016), refers to a medium for access students' prior knowledge and used to help foster in understanding concepts in-depth and increases the effectiveness of problem-solving performance with the ability to build access of the known problem to target problems by implementing a solution step that has identical structures with the new problem to be solved. It makes analogical reasoning an essential study in mathematics learning because analogical reasoning helps students more critically and carefully observe and analyze the patterns found in mathematics and other life problems outside of school.

In their research, Richland & Begolli (2016) found that analogical reasoning can effectively encourage students' higher-order thinking skills, and the same result by Podomi (2015) that analogical reasoning has a positive influence on student achievement in school. Furthermore, Ningrum & Mustikasari (2016) stated that analogical reasoning skills correspond to students' reasoning abilities, and students who have good analogical reasoning skills will have high reasoning abilities. Based on those opinions, analogical reasoning has a considerable influence on learning mathematics because it can improve problem-solving abilities and affect student learning outcomes. However, Widdiarto (Zaini & Retnawati, 2019) found that in the mathematics learning process, teachers frequently play an active role in instructional activities, use less varied methods, and do not allow students to practice reasoning skills. Meanwhile, Supratman et al. (2019) argued that many students made mistakes in solving problems using analogical reasoning.

Thinking in the process of solving a problem involves thinking to link the previous knowledge that has been experienced with the problem to be solved (Polya, 2004). Followed by Voskoglou's (2012) opinion, when facing a new situation and trying to solve new problems, maybe reminded of similar problems that have been resolved in the past and may be able to use the similarities of the problem-solving procedure to solve new problems. This situation occurs since we realize that the new problems are structurally similar to the familiar experienced in the past. Even the features of the two problems are quite different. In this case, Voskoglou said that the process of solving such problems is called analogical problem-solving. In line with this, Wardhani et al. (2016) stated that analogical problem-solving helps solve a problem by linking the similarities between a problem with one another.

Using analogy requires correspondence between a known problem (the source problem) and the target problem. English (2004) stated that the source problem has the same goal structure as the target problem, classified as an easy or moderate problem, can help solve the target problem or initial knowledge in solving the target problem. Meanwhile, the target problem has characteristics such as the source problem, which is made more modified, and the structure of the target problem is related to the structure of the source problem. In this case, the source problem and the target problem used to pay more attention to procedural similarities.

Isoda & Katagiri (2012) defined that analogical established perspectives in order to discovering solutions by recall an already-known problem that has been resolved and the most suitable. The thinking is to find an easily problem which is the same, then applied the rule in the entire problem. Analogical reasoning in classroom have the source problem and the target problem. The used of the source problem in analogical reasoning is to make students think that they have already learned something similar to target problem and treat the target problem in the same way as the source problem.

Ruppert (2013) describe analogical reasoning in solving problems through four stages:

a. *Structuring*
   Process of identifying any information on the source problem and the target problem by looking for the two problems' structural characteristics.
b. *Mapping*
   Process of finding and building conclusions by the similarities between the source problem and the target problem.
c. *Applying*
   Process of applying similarity of the source problem's relational problem-solving structure and solve the target problem by selecting the appropriate solution.
d. *Verifying*
   Process of verifying the source problem's solution and the target problem by checking the solution's suitability between the two problems.

Novick & Holyoak (1991) said that students tend to used analogical reasoning in solving mathematical problems if 1) students can identify whether a relationship between the target problem and the source problem or not, 2) students can identify the structure of the target problem according to the source problem, 3) students can find out how to use the procedural similarities of the source problem to solve the target problem.

In conditions that are completely limited due to the impact of the Covid-19 pandemic, it requires students to learn and complete a study independently and faced with an abundance of learning resources, so it requires the ability to analyze, take appropriate and relevant strategies also require good learning management. The learning method is related to students' ability to organize themselves or require as student self-regulated learning. According to Johnston-wilder & Lee (2010), self-regulated learning is the ability to build self-confidence, be diligent, reflect, and be positive. Self-regulated learning is a person's ability to understand and control the learning environment, which includes: learning objectives, time management, motivation, and how they use their knowledge in solving problems.

Hidayati & Listyani (2010) declared several characteristics of self-regulated learning there are: 1) No longer relying on others, defined as the ability not to expect guidance and direction from others. 2) Having a sense of self-confidence. 3) Discipline. 4) Responsibility in designing and monitoring learning activities. 5) Become pro-active and take the initiative. And 6) having self-control.

Students have different levels of self-regulated learning. Differences in students' self-regulated learning abilities indeed lead to differences in learning and solving problems, which determinants of students' analogical reasoning process. As research by (Haryati, 2015), concludes that individuals with high self-regulated learning tend to be able to learn better, have the ability to self-evaluate, organize effective learning, and have high knowledge. Based on that description, not to mention differences in students' self-regulated learning will trigger students' analogical reasoning to solve mathematical problems. Therefore, this study aims to describe students' analogical reasoning in solving mathematics problems based on students' self-regulated learning.

Arum (2017) described that students with high self-regulated learning have the ability to analyze problems properly, evaluate mistakes, gave arguments correctly, and drawing correct conclusions. Sulistyani et al. (2020) stated that differences in students' self-regulated learning influence problem-solving capabilities. Meanwhile, Fajriah et al. (2019) stated that self-regulated learning has a positive and significant impact on students' mathematical reasoning ability. Ningrum & Mustikasari (2016) stated that if there is a close relationship between analogical reasoning and students' mathematical reasoning, there is an increased student's reasoning ability through the analogical reasoning ability. This strengthens if there is a connection between student's analogical reasoning in solving mathematics problem that is also influenced by an individual's self-regulated learning. Hence, this study focuses on student's analogical reasoning in solving mathematics problem based on student self-regulated learning.

METHODS

This research is descriptive with a qualitative approach that aims to describe students' analogical reasoning in solving mathematics problems based on students self-regulated learning, which includes students behavior in the aspect of observing patterns, determining the relationships between the source problem and target problem, and building conclusions from similarities both of problems. The subjects were students who had studied trigonometric equation which is filtered based on the student self-regulated learning tests and mathematics ability tests, and then there were two subjects of the same sex, there are:

1. Subject with high mathematical ability and low self-regulated learning, and
2. Subject with high mathematical ability and high self-regulated learning.

The instruments used to facilitate researchers in collecting research data include non-test instruments in the form of a questionnaire on Student Self-regulated Learning (SSL), Mathematics Ability Test (MAT), Analogical Reasoning Test 1 (ART1), and Analogical Reasoning Test 2 (ART2) along with interview guidelines. Students were given a questionnaire of student self-regulated learning which results from the adoption of instrument development carried out by Hidayati & Listyani (2010). Based on the validity of the self-regulated learning instrument showed a validity value of 0.819 with a constructive validity test and the empirical validity, then it was said to be good. And 0.8797 which indicates high levels of category. Hence, the instruments are said to be valid and reliable for use. The self-regulated learning test is used to select a subject with low self-regulated learning and high self-regulated learning. The student self-regulated learning grouping is based on the category as shown at the following Table 1.

Table 1. Students Self-regulated Learning Category

Score	Category
$x \geq \bar{x} + SD$	High
$\bar{x} - SD \leq x < \bar{x} + SD$	Middle
$x < \bar{x} - SD$	Low

After the student self-regulated learning assessment was given, then the student is given a mathematical ability test (MAT) to select students with high mathematical abilities. The two selected subjects have high mathematical abilities because they have a good mastery of mathematical concepts to solve mathematical problems wholly and correctly (Yusrina & Masriyah, 2019). Then, the two subjects were given an analogical reasoning test. After the subjects were given an analogical reasoning test, they will be analyzed, and the students will be given an interview referring to the research indicators presented in Table 2.

Table 2.Indicator of Analogical Reasoning in Solving Mathematics Problems

Phase	Indicator of Analogical Reasoning in Solving Problems	Code
Structuring	Identify all the information on the source problem.	STC1
	Solve source problems.	STC2
	Identify all the information on the target problem.	STC3
	Declare have even solved similar problems.	STC4
	Find the source and the target problem's structural characteristics (in this case, finding similarities of the two problems).	STC5
Mapping	Develop a strategy considered the most effective one based on the relationship between two or more problems' characteristics to solve the target problem and state the reasons.	MPP
Applying	Apply the conclusions from the relationship obtained from the source problem and the target problem.	APL1
	Solve and explain the target problem-solving steps.	APL2
Verifying	Re-check the solution to the problem.	VRF

After all the data has been collected, the next step is to do time triangulation. The purpose of triangulation is to obtain valid and reliable research data. Triangulation was carried out by providing an analogical reasoning test 2 (ART2) and interviews at a different time before. Then the data were compared and checked for consistency. If the test results got different between the first data and the second data, then researchers take the data retrieval until obtaining the valid data. The data that has been collected then reduced and analyzed qualitatively. Furthermore, the data analysis results are presented, and conclusions are drawn.

RESULTS AND DISCUSSIONS

Following are the results of the analysis of students' analogical reasoning in solving mathematical problems based on tests that have been given previously. There will be code for interviews dialogue of the research data, such as PS for the transcript of the dialogue carried out by the interviewer, and for the subject will be coded as LS (student who had low self-regulated learning) and HS (student who had high self-regulated learning). For both, following by two-digit numbers located at the end, which means the number of questions or answers put forward.

Descriptions of the Analogical Reasoning Profiles of Student with Low Self-regulated Learning (LS) in Solving Mathematics Problems.

Based on the result of the analysis conducted on LS in solving the analogical problem can be revealed as follows.

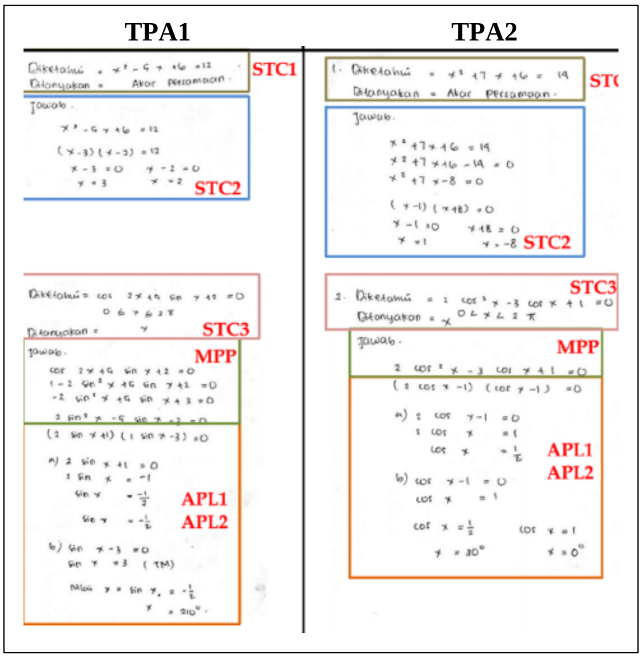

Figure 1.The Results of LS Problem-solving Task

Transcripts of LS interviews related to STC1 and STC2.

First Test

PS1-01 : "Mention the information contained in question1!"

LS1-01 : "It is known that the quadratic equation $x^2 - 5x + 6 = 1$, then asked to find the root of the equation."

PS1-02 : "Explain your steps of problem 1!"

LS1-02 : "First, we know that the problem was the quadratic equation then tried to factor it, it turns out that it can just be factored and find the result."

PS1-03 : "Did you have difficulty working on question 1?"

LS1-03 : "No."

Second test

PS2-01 : "For question number 1, mention the information contained"

LS2-01 : "Problem number 1, we know the equation of the quadratic, and asked for the root of the equation."

PS2-02 : "How did you solve it?"

LS2-02 : "First it was simplified, 14 were moved to the left become -14 . Then $6 - 14 = -8$, the equation becomes $x^2 + 7x - 8 = 0$. Then you get the root."

Based on the written answers in Figure 1 and the interview excerpt at STC1, LS in solving mathematics problems began by writing down all the information (what is known and asked). In solving the source problem (STC2), LS stated that she did not found difficulties and could solve the given source problem, but in the LS work results shown in Figure 1, there were incorrect steps for solving the quadratic equation and seem hustle. In ART1, LS worked on quadratic equations without equating 0 first, but in ART2, LS equated the equation with 0. After explored through interviews with LS above, she stated that in working on quadratic equations, an important step is to equate 0 equations first. And after being given other similar questions outside ART1 and ART2, LS did it by first equating the equation with 0. In line with Jumaisyaroh et al. (2015), students with low levels of self-regulated learning have a low sense of responsibility for the learning process and less focus on the questions.

To explore information related to the STC3 and STC4 stages, the following are the results of the interview with LS.

First Test

PS1-04	:	"Have you ever come across a question similar to question number 2?"
LS1-04	:	"Once, miss."
PS1-05	:	"Are they the same problem with problem number 2?"
LS1-05	:	"Actually it is almost the same, the question is looking for x."
PS1-06	:	"And what does x mean here?"
LS1-06	:	"Value for x miss, what is asked in both problem is the same question, right? You have to be asked to find the value of x."
PS1-07	:	"How did you get that meaning of x?"
LS1-07	:	"Hhhmmm, what is it…..? I think it just looking for the value of x, miss."
PS1-08	:	"Mention the information contained in question 2!"
LS-08	:	"The information was known, miss? A trigonometric equation."

Second test

PS2-03	:	"Have you ever come across a question similar to question number 2?"
LS2-03	:	"Yes, Once. A trigonometric equation, but in a different form and number."
PS2-04	:	"Mention the information contained in question 2!"
LS2-04	:	"Number 2, known a trigonometric equation. Then asked…….alike question number 2…the roots."
PS2-05	:	"Did you know roots was referred to?"
LS2-05	:	"Hmm…The roots is x, so the question is a quadratic and finds for the roots, then it means to find for x."

At the STC3 stage, LS mentioned the known information and what is being asked in the target problem ultimately and wrote it down. However, in the interview, LS did not mention the conditions for the roots in the trigonometric equation known in the problem. This is in the line with the statement of Sulistyani et al. (2020) which showed that students with low self-regulated learning mention the information incompletely. LS could not understand the meaning of roots in the problem given, so she didn't provide valid arguments to explain their statement's meaning. This corresponds with Arum (2017) and Ekananda et al. (2020) that students with low self-regulated learning did not explain and understand the main point of the problem and did not present a logical argument supporting the problem-solving step. In this case, LS did not understand the meaning of the value of x because she did not focus on her mind and only repeats the information she was previously received without processing his knowledge. In the interview, LS answered questions by the interviewer doubtfully, and there was a time lag when she was going to convey his opinion. At the STC4, LS stated that he had encountered a similar question previously revealed in the interview LS1-04 and LS2-03.

At the STC5 stage, LS processed information by linking previous knowledge about quadratic equations and found similarities in the characteristics of the source problem and target problem, which lies in what is asked by the problem, and the problem is in the form of a quadratic equation. The following is a dialogue interview with LS, which relates to the STC5.

First Test

PS1-09	:	"Is there a relationship between question number 1 and question number 2?"
LS1-09	:	"Yes, the point is, you are both looking for x, right?"
PS1-10	:	"Are there any other similarities between question number 1 and question number 2? "
LS1-10	:	"The question is the same and both can be factorized? So the similarity in the pattern are the same"
PS1-11	:	"How did you find the relationship?"
LS1-11	:	"Hhhmmm ... I don't know any other, but problem 1 and problem 2 were both squared miss, the one is in the form of cosine, and the other is x squared."

Second Test

PS2-06	:	"Is there a relationship between question number 1 and question number 2?"
LS2-06	:	"Yes, the point is, you are both looking for the roots, then the equation was order 2 and both can be factorized."

After finding the similarities in both problems' structural characteristics, at the MPP stage, LS used the theory of trigonometric identities and took steps to convert the equation in the target problem into the same form, into the sine form or cosine form. LS took steps to equalize the equation on the target problem by converting the cosine into the sine form without mentioned the steps taken and provided parentheses to clarify the steps used. It shows that LS did not have the initiative to compose an expression of his ideas so other people will understand them (Hidayati & Listyani, 2010). Based on the more simplified form, LS plans to solve the target problem similar to the source problem's solution through factoring. LS choose not to provide unique codes or manipulations to build simpler mathematical models of $cos\ x$ and calculated the trigonometric equation form to get the factor result. The mapping from the target problem to the source problem occurs by visualizing forms of trigonometric equations that had similarities with the source problems.

In implementing the target problem-solving strategy (APL1), LS used the suitability of the target problem according to the source problem. Furthermore, at the APL2 stage, LS explained the process until she got the answers, as LS explained in the following interview.

First Test

PS1-12	:	"Are the steps you took to solve question number 2 similar to solving question number 1?"

LS1-12 : "Yes, miss, both are factored to get the roots."

PS1-13 : "Explain how you took steps to find the solution to question number 2!"

LS1-13 : "First, equate cosine or sine in the equation. Then, it is factored, then you get the value of x."

PS1-14 : "Why it is $2\sin x + 1 = 0$ and $\sin x - 3 = 0$?"

LS1-14 : "Because it is equal to 0, miss."

PS1-15 : "If sine is a negative sign, what does it means?"

LS1-15 : "To finding the sine value, remember about the quadrant, above the sine is positive (while drawing a quadrant)."

PS1-16 : "So below, the two quadrants are negative?"

LS1-16 : "Yes, miss."

PS1-17 : "Based on your answer sheet, is it correct to put the minus sign of sine on that quadrant?"

LS1-17 : "Yes, here (refers to quadrant III)."

PS1-18 : "That means there is only one value of negative sine on the quadrant?"

LS1-18 : "Isn't?"

Second Test

PS2-07 : "Are the steps you took to solve question number 2 similar to solving question number 1?"

LS2-07 : "Yes, miss, both are factored until getting the x."

PS2-08 : "Explain how you took steps to find the solution to question number 2!"

LS2-08 : "First, the equation is cosine. Then, it is factored in, and find the value of x. We get $x = 30$ and $x = 0$."

PS2-09 : "On your answer sheet, why you don't solve for $\cos x = \frac{1}{2}$?

LS2-09 : "For point A miss? …That is…Oh, it was the same as the result above."

In the completion process, LS ignored the condition for the roots and mentioned the solution incompletely. Following by the research of Jumaisyaroh et al. (2015), which stated that students with low self-regulated learning still relying on others, in this case to teachers, which causes the student to give up easily in solving problems, and when facing a mathematics problem, they did not produce optimal solutions. Besides, LS did not write conclusions on their answer sheets, it is similar to the research of Arum (2017), student with low self-regulated learning did not write a conclusion to the answers she has obtained and tended to be passive in answering questions from the interviewer and mentioning the answers incompletely. LS explained the solution's steps hesitantly and did not hesitate to ask the interviewer to confirm her correctness.

In the verification step, LS was sure of the answer she had, but she did not try to enter the value of x into the known equation based on the following interview excerpt.

First Test

PS1-19 : "Are you sure the all solution you got is correct?"

LS1-19 : "Yes, miss."

PS1-20 : "How can you be sure of your answer?"

LS1-20 : "InsyaAllah, it is correct, miss. Before, I solve it seriously, step by step."

PS1-21 : "After finding the answer, have you try to put the value of x into the known equation?"

LS1-21 : "No, miss."

Second Test

PS2-10 : "Are you sure the all solution you got is correct?"

LS2-10 : "InsyaAllah, miss."

PS2-11 : "After finding the answer, have you try to put the value of x into the known equation?"

LS2-11 : "Not yet, miss. I have no time"

It showed that LS didn't re-check all the steps taken from the beginning until she got the result. According to the characteristics of a student with low self-regulated learning by Hidayati & Listyani (2010) and Arum (2017), a student with low self-regulated learning does not verify due to a lack of sense of responsibility and act without thinking profoundly and did not evaluate her learning effectively. In this case, LS did not understand the meaning of the value of x because she did not focus on her learning and only repeated the information she has previously obtained without processing his knowledge, as stated by LS in dialogue LS-05 and LS-06. Likewise, what can be seen in Figure 1, the results of LS*'s* work on the target problem are less than perfect because they only wrote one solution at known intervals.

Descriptions of the Analogical Reasoning Profiles of Student with High Self-regulated Learning (HS) in Solving Mathematics Problems.

Based on the results of the analysis conducted on HS in solving analogical problems can be revealed as follows.

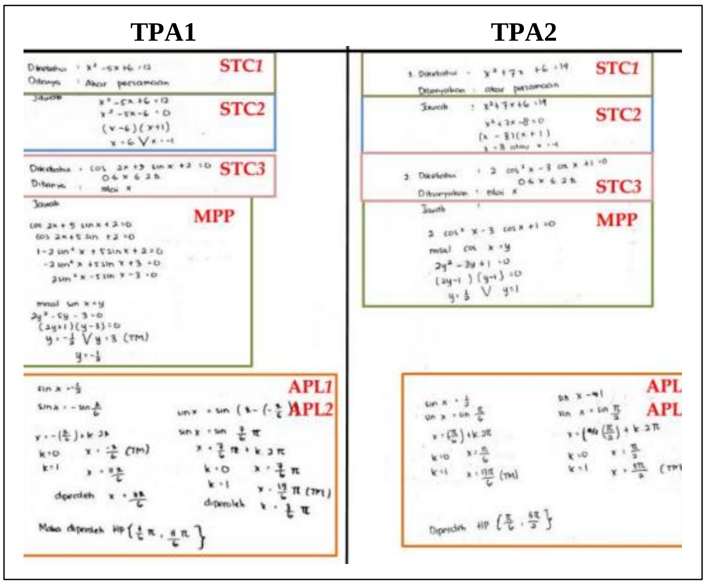

Figure 2. The Results of HS Problem-solving Task

Figure 2 on the STC1 stage (structuring 1) HS identified the problem by rewriting all the source problem information. HS has shown that she could understand the meaning of the questions well and convey it in her language, as in the following dialogue excerpt.

First Test

PS1-01 : "Mention all the information on problem 1!"

HS1-01 : "Problem one, known a quadratic equation $x^2 - 5x + 6 = 12$ and asked for the roots of that equation."

PS1-02 : "Explain your step for solving problem 1!"

HS1-02 : "The quadratic equation was not equal to 0, then we change the equation, we got $x^2 - 5x - 6 = 0$, then factorize it by $(x-6)(x+1)$ and we got the roots was $x=6$ and $x=-1$."

Second Test

PS2-01 : "Mention all the information on problem 1!"

HS2-01 : "Problem one, known a quadratic equation and asked for solving that equation."

PS2-02 : "And what solve?"

HS2-02 : "Oh…we find for roots that match with the equation given."

PS2-03 : "Explain your step for solving problem 1!"

HS2-03 : "First, we must equal the quadratic equation with 0, we subtract with 14, then we got $x^2+7x-8=0$, and then factorize it by $(x-8)(x+1)$ and we got the roots was $x=8$ and $x=-1$."

In the interview transcript above, HS explained the steps in solving the question well and clearly which indicated that she was able to understand the meaning of the problem regarding the root of the equation. Following Fajriah et al. (2019) and Arum (2017), higher self-regulated learning will have an impact on the success of their study. In this case, HS organized effective ways of learning to maximize its ability, to explain and understand the problem, so that its study becomes more optimal. At the STC2 stage, HS solved the source problem correctly, showed that it was appropriate with the opinions of Hidayati & Listyani (2010) and Johnston-wilder & Lee (2010), students with high self-regulated learning was able to form positively, persistently, and not easily give up in learning mathematics.

At the STC3 stage, HS identified the known and asked information in the target problem, then wrote it down on the answer sheet. HS said that she had previously encountered a similar problem with a slight difference in known trigonometric equations, but she could still understand the meaning of the problem, which is to find the root of the equation such that if the root substituted into the initial equation, it will produce a correct equation (STC4 stage). The following is a dialogue revealed by HS in the interview.

First Test

PS1-03 : "Question number 2, have you ever seen something similar?"

HS1-03 : "Once, miss, with a different form of the equation."

PS1-04 : "What kind of form?"

HS1-04 : "The equation is a difference in trigonometric functions and the numbers."

PS1-05 : "Is the problem the same?"

HS1-05 : "Yes, looking for the value of x or the roots."

PS1-06 : "In this problem, what is the meaning of the root of the trigonometric equation?"

HS1-06 : "If the value of x was obtained and then substituted into the equation, it will be correct, miss. In this problem, it means the left side became 0"

PS1-07 : "In question number 2, what information did you get?"

HS1-07 : "In problem number 2, I obtained the trigonometric equation mentioned with the x value must be greater than or equal to 0 but less than or equal to 2π."

PS1-08 : "Then what was asked in question number 2?"

HS1-08 : "We are finding a set of solutions that appropriate with the conditions given."

Second Test

PS2-04 : "Question number 2, have you ever seen something similar?"

HS2-04 : "Once, miss, with a different form of the trigonometric function and the number."

PS2-05 : "In question number 2, what information did you get?"

HS2-05 : "In problem number 2, I obtained the trigonometric equation $2\cos^2 x-3\cos x+1=0$ and the roots must be greater than or equal to 0 but less than or equal to 2π. And we are finding a set of solutions that appropriate with the conditions given."

HS also emphasized that the root of the equation looking for must be in a known interval range. In this case, she said it must be in the interval of $0\le x\le 2\pi$. In working on a given problem, HS showed that she could focus on her learning activities and try to solve them as best as possible, following students' characteristics with high self-regulated learning as stated by Hidayati & Listyani (2010).

In the STC5 stage, HS found the relationship between the two problems (source problem and target problem) in differences and similarities. First, she found the differences in both problems' equations, one is a quadratic equation, and the other is a trigonometric equation. The second thing she found was that the problem was asked to find the roots of the equation. The following is an interview dialogue that was related to the STC5 stage.

First Test

PS1-09 : "Do you think there is a relationship between question number 1 and question number 2?"

HS1-09 : "There is ... the connection."

PS1-10 : "What did you find?"

HS1-10 : "There are similarities that I have found, the form of problem one and problem two and what we are looking for is the roots of the equation, but, what the difference is that number 1 is the quadratic equation and number 2 is the trigonometric equation. However, we are both looking for the root of the equation from question number 1 and question number 2."

Second Test

PS2-06 : "Do you think there is a relationship between question number 1 and question number 2?"

HS2-06 : "Yes, I do."

PS2-07 : "What did you find?"

HS2-07 : "The similarities that I have found are we looking for the roots, the first equation is a quadratic equation, and the second is trigonometric equation order 2."

By pocketed both problems' differences and similarities, HS then used the pattern she was found to solve the target problem by made an analogy of quadratic equation to the trigonometric equation. This is the following interviews revealed to the MPP stage.

First Test

PS1-11 : "Then are the steps you took to solve question number 2 similar to the steps you took to solve question number 1?"

HS1-11 : "Yes, the step I used in number 2 is similar to step number 1."

PS1-12 : "Which part of the similar step?"

HS1-12 : "The factorized to find the value of x."

PS1-13 : "Then why do you suppose $y=\sin x$?"

HS1-13 : "I took the step to equal $\sin x = y$ to make it easier for me to find the roots. So we can suppose $\sin x = y$, and we can find the root, then we change y again to $\sin x$."

Second Test

PS2-08 : "Then are the steps you took to solve question number 2 similar to the steps you took to solve question number 1?"

HS2-08 : "Yes, the step when factorized until getting the value of x."

PS2-09 : "Then why do you suppose $y = \cos x$?"

HS2-09 : "I took the step $y = \cos x$ to make it easier for me to find the roots. So it more simple to factorized and then we get back to change $y = \cos x$."

Based on the results of the HS*'s* written work in Figure 2 MPP code, HS recalled the quadratic equation form in the source problem and mapped the target problem to the source problem by first carrying out the process of coding the characteristics of the problem or building a new mathematical model for the target problem that she had simplified (converted the equation into sine form). The new mathematical model of the given trigonometric equation was converted into a quadratic equation by using assumptions. After interviewed HS, she took this step because she thought this strategy was the easiest way to find the roots. After all, the equations became more simple and identic to the source problem. In this case, the subject planned her learning method, which is effective, and carried out the learning tasks well (Hidayati & Listyani, 2010). HS also explained that when the new mathematical model was built, it would later be returned to the initial form to determine the problem's actual value.

At the applying stage (APL1), HS used strategy by implemented the planned completion idea and wrote down the completion steps. At the APL2 stage, HS produce all possible solutions based on the known conditions in the problem and did not forget to conclude the final result of the target problem, this corresponds to the previous studied that students with high self-regulated learning wrote the conclusions of the problem (Ekananda et al., 2020). HS then described the steps in solving the target problem in a very firm, coherent, and detailed as in the following dialogue excerpt.

First Test

PS1-14 : "Explain how you took steps to find the solution to question number 2."

HS1-14 : "First, let us look at the problem, there is cosine and sine, then we change $\cos 2x$ to sine, which is $1 - 2\sin^2 2x$. Then we add up the numbers and find a simple result. Since there is a minus sign in front, we multiply the right side and the left side by -1, so the one in front has a positive variable. Then we suppose $\sin x = y$. From there, we can find the factors. There are two results, namely $y = -\frac{1}{2}$ or $y = 3$. However, y here means $\sin x$, so the value $x = 3$ does not satisfy. Then we use the trigonometric equation formula for sine. Namely $\sin x$equals… ..$\sin \theta$. Here, because the θ is minus, it becomes minus. The first formula $x = \theta + k \cdot 2\pi$. Here, as I said earlier, the x is a minus, so we follow the minus sign. Then we get if $k = 0$ then $x = -\frac{\pi}{6}$. If $k = 1$ then x is equal to $\frac{11\pi}{6}$. But $x = -\frac{\pi}{6}$ does not fulfill because …… must be more or equal to 0. So we got the solution is $\frac{11}{6}\ \pi$. Then the second formula is $x = (\pi - \theta) + k \cdot 2\pi$. Then you find that if $k = 0$ then $x = \frac{7}{6}\ \pi$ and if $k = 1$ then $x = \frac{19}{6}\ \pi$. However, $x = \frac{6}{19}\ \pi$ does not fill because it is more than 2π, while the limit is up to 2π. Then the set of solutions is $\frac{7}{6}\ \pi$ and $\frac{11}{6}\ \pi$."

Second Test

PS2-10 : "Explain how you took steps to find the solution to question number 2."

HS2-10 : "First, let us look at the problem, there are cosine, then we suppose $\cos x = y$. From there, we get $2y^2 - 3y + 1 = 0$. From the simple form, we factorized and there are two results, namely $y = \frac{1}{2}$ or $y = 1$. Then back to the assumption $y = \cos x$. From two values of x, we used *cosinus* rule $y = \cos \theta$. Then we got $x = \frac{\pi}{6}$, $x = \frac{13\pi}{6}$, and $x = \frac{\pi}{2}, x = \frac{5\pi}{2}$. But we eliminate based on the intervals, then $x = \frac{13\pi}{6}$ and $x = \frac{5\pi}{2}$ didn't correct because more than 2π. However, we got the values were $x = \frac{\pi}{6}$ and $\frac{\pi}{2}$."

HS explained the completion process that was carried out from the start until he answered with just one question from the interviewer. In solving, HS did not forget the interval of x asked by the problem and then choose the x value found and matches it with the interval of $0\ to\ 2\pi$. In the interview, HS seemed very confident in the answers to the problems given and was brave in expressing her opinion, showing that she overcame the problems faced in her learning activities.

At the verification stage, HS could correct both source and target problem, draw correct conclusions, and provide logical reasons for each step, this according to the previous research that students with high self-regulated learning able to conduct verification after getting an answer and wrote the conclusions of the problem given (Ekananda et al., 2020). HS evaluate the learning outcomes and explained how it took to verify the answers. The following is a transcript of the interview with HS in the verification step.

First Test

PS1-15 : "Are you sure the steps you used to solve question number 1 and question number 2 are correct?"

HS1-15 : "Yes, I am sure that my answer is correct because I have proven that when the x I entered into the equation, the equation is correct."

Second Test

PS2-11 : "Are you sure the steps you used to solve question number 1 and question number 2 are correct?"

HS2-11 : "Yes, I am sure with my answer. I have proven, when the x I entered into the equation, the equation is correct."

CLOSURE

Conclusion

Based on the results of the research and analysis that has been described in the previous discussion, which refers to research questions, it can be concluded that the analogical reasoning profile of students in solving mathematical problems based on self-regulated learning can be described through the tendency of actions taken by the subjects on each indicator as follows.

The analogical reasoning profile of a student with low self-regulated learning in solving mathematics problems through the stages of structuring, mapping, and applying, but not up to the verifying stage. At the structuring stage, the student wrote down all the information entirely but did not explain the meaning of the questions correctly. At the mapping stage, the student did not have the initiative to formulate an expression of their ideas so that others can understand them. At the applying stage, the student did not solve problems optimally, did not provide conclusions on their work, did not fluently put forward steps to completion, and did not provide valid arguments to explain their statements' meaning.

The analogical reasoning profile of a student with high self-regulated learning in solving mathematics problems through all the analogical reasoning stages, namely structuring, mapping, applying, and verifying. At the structuring stage, the student could understand the meaning of the problem and mentioned all the information. At the mapping stage, the student coded the problem's characteristics by constructing a new mathematical model of the target problem. At the applying stage, the student presents the problem-solving steps for completion carefully, firmly, and clearly, could build conclusions, and provided logical reasons for each work process carried out. At the verification stage, the student provided evidence of the solutions given.

Recommendation

Referring to the results of the discussion and the conclusions obtained, researchers provide several suggestions:

1. The results indicate that there are differences in the analogical reasoning of student low self-regulated learning and high self-regulated learning in solving mathematical problems. Teachers should using varied learning which helped to develop their analogical reasoning skills.
2. Other researchers who will carry out similar research related to analogical reasoning profiles, advised to use different problems or other reviews by connecting some materials such as algebra, geometry, and others and adapting other self-regulated learning instruments; thus, the data obtained is more entirely and in-depth.

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