

## HIGH SCHOOL STUDENTS' GENERALIZATION VIEWED FROM LOGICAL-MATHEMATICAL INTELLIGENCE

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### Abstract

Generalization is an important element in understanding, recognizing, and examining mathematical situations. Students' generalization processes can be analyzed according to Mason's four stages of the generalization process, including the perception, expression, symbolic expression, and manipulation of generality. One of the things that may cause differences in students' generalization processes is logical-mathematical intelligence. This research aims to describe high school students' generalization processes in terms of logical-mathematical intelligence. This research is using a descriptive qualitative method. The subjects of this study consisted of 3 high school students in Surabaya with high, moderate, and low logical-mathematical intelligence. The data is obtained from the Logical-Mathematical Intelligence Questionnaire and Generalization Task. The material used in the Generalization Task is curved-face solid. The results of this study indicate that the student with low logical-mathematical intelligence has reached the stage of identifying patterns or regularity in the given information, but the student doesn't use those patterns to determine the next term. In other words, the student with low logical-mathematical intelligence has only reached the stage of perception of generality. The student with moderate logical-mathematical intelligence has reached the stage of identifying patterns or regularity in the given information and using those patterns to determine the next term but the student doesn't reach the stage of formulating the general formula of the pattern that has been found. In other words, the student with moderate logical-mathematical intelligence has only reached the stage of perception and expression of generality. The student with high logical-mathematical intelligence has reached the stage of identifying patterns or regularity and used those patterns to determine the next term. In addition, the student with high logical-mathematical intelligence also reaches the stage of formulating general formulas of patterns that have been found and using the general formula to solve the given problem. In other words, the student with high logical-mathematical intelligence has reached all the stages which is perception, expression, symbolic expression, and manipulation of generality.

**Keywords:** generalization process, logical-mathematical intelligence, high school students.

### INTRODUCTION

Generalization is an important element in learning mathematics. When students can make generalizations, it will help them in understanding mathematics. This is following the statement in NCTM (2000, p. 15) that "many concepts and processes, such as symmetry generalization, can help students gain insights into the nature and beauty of mathematics", in other words, generalization will affect students' mathematical understanding. Therefore, generalization is one of the main things and is needed in learning mathematics (Chua & Hoyles, 2014; Hashemi, Abu, Kashefi, & Rahimi, 2013; Yildiz, 2016). Not only in learning, but generalization is also important in students' thinking processes such as the statement by Hashemi et al.

(2019, p. 78) that generalization is "an important element of mathematical thinking process".

Besides being needed in mathematics learning and students' thinking processes, generalization is also needed to understand decomposition when examining mathematical situations (Dumitrascu, 2017). These situations can be in the form of situations found in math problems and everyday life. Without generalizations, students will find it difficult to understand various information as stated by Alexander and Duehl (2004, p. 36) "without generalization, there is no way for students to cope with an unlimited amount of information in both the abstract and the real world".

Generalization can be obtained from the results of noting the regularities found in the data being observed. The activity of noting regularities and formulating the

conjecture is stated as a generalization process (National Council of Teachers of Mathematics, 2000, p. 234). There are several studies related to the generalization process carried out by other researchers. Each researcher uses a different stage of the generalization process. In general, the last stage of the generalization process which is stated to be the same, that is the student states the regularity/pattern symbolically ((Radford & Puig, 2007; Aprilita, Mirza, & Nursangaji, 2016; Fadiana, Amin, & Lukito, A., 2017). Different from previous researchers, Swafford and Langrall (2000); Rivera and Becker (2007a); and Mason (2010) states that the last stage of the generalization process is when students use general equations to solve a given problem. Researchers are interested in reviewing the generalization process of students until students use the results of their generalizations to solve the problems given. Therefore, the researcher chose the generalization process indicator based on one of the three opinions, that is perception of generality, expression of generality, symbolic expression of generality, and manipulation of generality (Mason, Stacey, & Burton, 2010). Confirmation or description of the relationship between data according to the opinion of two other researchers will be represented by the expression of generality stage so that Mason's (2010) opinion about the generalization process stage will be used in this study.

As previously discussed, generalization is indeed considered an important element in learning mathematics, but in fact, students in Indonesia tend to have a moderate ability in making generalizations. This is following the results of a study conducted by Aprilita, Mirza, and Nursangaji (2016) in Pontianak which stated that as many as 70% of students were still classified as moderately capable. The results of the study by Situmorang and Zulkardi (2019) in Palembang also stated that 24 of the 32 students who were the subjects had moderate abilities. The initial stage in making generalizations is not difficult for students, but making generalizations in the form of algebra and words is a challenge for them (Rivera & Becker, 2007b; Chua & Hoyles, 2014). One of the things that might cause the difference in ability levels is intelligence.

Every student has intelligence with different levels, as stated by Armstrong (2009, p. 32) that "most students have strengths in several areas, so you should avoid pigeonholing a child in only one intelligence". Armstrong's statement is reinforced by Gardner's opinion (2011, p. xxix) which says that there are at least seven bits of intelligence in the human brain, that is linguistic intelligence, logical-mathematical intelligence, spatial intelligence, kinesthetic intelligence, musical intelligence, interpersonal intelligence, and intrapersonal intelligence. One of the intelligence related to mathematics is logical-mathematical intelligence, the individual's ability to think conceptually

and abstractly; operate mathematics; distinguish between logical and numerical patterns; and reason (Ilmiah, 2010; Armstrong, 2009, p. 6; Tirri & Komulainen, 2002, p. 158).

According to Arum, Kusmayadi, and Pramudya (2018), students' logical-mathematical intelligence contains several abilities such as the ability to classify, solve problems, make logical reasoning, and think analytically. When the level of logical-mathematical intelligence of students is different, the level of ability of students in classifying, solving problems, reasoning, and analyzing will also be different. These differences may lead to differences in the thinking process of each individual, in other words, the generalization process of each individual will also be different.

This study uses curved-face solid material. According to the 2013 curriculum, the sub material of curved-face solid consists of tubes, cones, and spheres. The sub materials used in this research are tubes and spheres. The use of sub-materials on the instrument is combined with pattern materials so that it is expected to be able to achieve this study aims.

There are several other studies related to the generalization that discuss ways to improve students' algebraic thinking (Carraher, Martinez, & Schliemann, 2008; Kaput, Carraher, & Blanton, 2008; Guner, Ersoy, & Temiz, 2013; Rivera, 2010), the level of students' ability to generalize (Aprilita, Mirza, & Nursangaji, 2016; Fitriani, 2016; Situmorang & Zulkardi, 2019), or ways to improve generalization skills (Wulandari, 2012). In addition, other studies describe the generalization process of students such as that conducted by Rinawati and Sarjoko (2020) with the subject of junior high school students and Sholichah (2017) with the subject of high school students viewed from gender. Based on some of these studies, there has been no research on the generalization process of students in terms of logical-mathematical intelligence. In addition, just like reasoning that needs to be developed in high school mathematics learning (Wulandari, 2012), generalizations also need to be developed in high school mathematics learning because it is part of reasoning. This opinion is also supported by Hashemi et al (2013) which state that the use of generalizations is necessary for learning effectiveness at all levels. One of these levels is High School. Based on the description above, this study aims to describe the generalization process of high school students with logical-mathematical intelligence.

## METHOD

Based on the purpose of this study, which is to describe the generalization process of high school students with high, moderate, and low logical-mathematical intelligence, this research is a descriptive study. The subjects of this study were 3 high school students who were selected based

on the results of a logical-mathematical intelligence questionnaire. These three subjects consist of 1 student with high logical-mathematical intelligence, 1 student with moderate logical-mathematical intelligence, and 1 student with low logical-mathematical intelligence. This study uses two supporting instruments, including Logical-Mathematical Intelligence Questionnaire and Generalization Task. The Logical-Mathematical Intelligence Questionnaire consists of 19 statement items adapted from the MI Inventory by Thomas Armstrong (2017) and the answers to this questionnaire use a Likert Scale with the criteria of Strongly Agree, Agree, Doubtful, Disagree, and Strongly Disagree. The Likert scale was chosen because it is easy to understand, making it easier for subjects to provide answers (Malhotra, Birks, & Wills, 2012; McDaniel Jr., Gates, Sivaramakrishnan, & Main, 2013). The results of this questionnaire were used in the selection of 3 research subjects.

The classification of logical-mathematical intelligence levels is obtained by comparing the scores of the Logical-Mathematical Intelligence Questionnaire using a reference adapted from the Range of Assessment Norms by Slameto (2010, p. 186). The references used are as follows:

**Table 1.** The Reference for Classification of Logical-Mathematical Intelligence Levels

Logical-Mathematical Intelligence Questionnaire Score	Level
$(\bar{x} + 0.5 SD) \leq N$	High
$(\bar{x} - 0.5 SD) \leq N < (\bar{x} + 0.5 SD)$	Moderate
$N < (\bar{x} - 0.5 SD)$	Low
$\bar{x}$ : average score of the questionnaire SD : Standard Deviation questionnaire score N : Logical-Mathematical Intelligence Questionnaire Score	

In the Generalization Task, the material used is curved-face solid with tube and sphere sub-materials. The Generalization Task consists of 1 essay question that has been validated by 2 expert validators, namely a lecturer in the Mathematics Department, Universitas Negeri Surabaya, and a high school teacher of Mathematics. The results of the Generalization of the subject will then be analyzed with the support of the results of observations during the work process. Following are the given problems in the Generalization Task.

**Figure 1.** Given problems in the Generalization Task.



**Table 2.** Generalization Process Indicator Code

Indicator	Activities	Indicator Code
<i>Perception of generality</i>	students find regularities or patterns in the given data	PoG
<i>Expression of generality</i>	students use regularities or patterns to determine the value of the next term or picture	EoG
<i>Symbolic expression of generality</i>	Students formulating general formula of patterns that have been found	SEoG
<i>Manipulation of generality</i>	students using the general formula to solve the given problem	MoG

Reference (Mason, Stacey, & Burton, 2010):

## RESULTS AND DISCUSSION

Based on the results of the Logical-Mathematical Intelligence Questionnaire scores conducted by 31 students, it was obtained

$$\bar{x}_{\text{skor angket}} = 65,35483871$$

$$SD = 14,717881$$

Based on these calculations, obtained 5 students with low logical-mathematical intelligence, 16 students with moderate logical-mathematical intelligence, and 10 students with high logical-mathematical intelligence. One subject was selected at each level of logical-mathematical intelligence of the same sex that is women. The selection was based on suggestions from the mathematics teacher

and homeroom teacher of the student concerned and also based on completeness of student answer. The following is the categorization of the logical-mathematical intelligence level of students using the reference in **Table 1** along with the description of the selected subject.

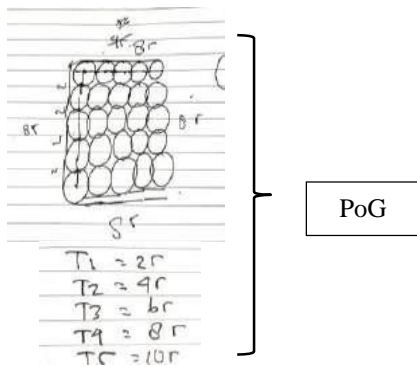
**Table 2.** Categorization of Mathematical Logical Intelligence Levels Based on Questionnaire Scores

Questionnaire Score Range	Level	Subject	
		Score	Code
$19 \leq N \leq 58$	Low	30	SR
$58 < N < 73$	Moderate	66	SS
$73 \leq N \leq 95$	High	84	ST

**Result**

Based on the Generalization Task with problems as shown in **Figure 1**, the results obtained are as follows

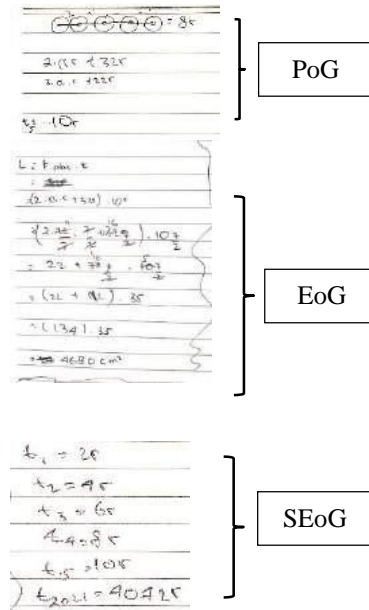
**Analysis Results of the Generalization Process of High School Students with Low Mathematical-Logical Intelligence (SR)**



**Figure 2.** SR's work

After reading the questions, SR looked at the picture of the container that had been given in the Generalization Task. SR found a pattern on the bottom of the container in the form of a change in the number of circles as shown in **Figure 2** above. The figure shows the shape of the base of the container in figure-5, SR found that the number of circles corresponds to the value of the square of the number in succession. In addition, SR also found a pattern on the height of the container which was directly expressed in the form of a formula

**Analysis Results of the Generalization Process of High School Students with Moderate Mathematical-Logical Intelligence (SS)**



**Figure 3.** SS's Work

After understanding the problem, SS ensures every information needed to solve the problem, one of which is calculating the sphere's radius. After that, look at the picture of the container that had been given to the question. SS found a change in the number of circles on the base of the container. The pattern is described simply as in PoG **Figure 3**. The number of circles drawn by SS represents the number of circles on the bottom of the container in Figure-5, the number of circles is then used to determine the formula for the circumference of the base of Figure-5. In addition to the pattern of the base of the container, SS also found a pattern on the height of the container. Then the formula for the circumference of the base of the container in Figure-5 is multiplied by the formula for the height of the container in Figure-5 to obtain the area of the wall for the container in Figure-5. The second question in the problem is the area of the wall of the Figure-2021 container. SS knows that it needs the circumference of the base and the height of the container Figure-2021. SS wrote down the pattern of the base circumference and the height of the container to determine the general equation as shown in SEoG **Figure 3**.

**Analysis Results of the Generalization Process of High School Students with Low Mathematical-Logical Intelligence (ST)**

The image shows two parts of a student's work. The top part, labeled 'PoG', lists the area of the container wall for different heights:
 
$$\begin{aligned} \text{luas selimut } 1 &= 2\pi r \cdot t = 2\pi r \cdot 2r \\ \text{luas selimut } 2 &= (2\pi r + 8r) \cdot t = (2\pi r + 8r) \cdot 4r \\ \text{luas selimut } 3 &= (2\pi r + 16r) \cdot t = (2\pi r + 16r) \cdot 6r \\ \text{luas selimut } 4 &= (2\pi r + 24r) \cdot t = (2\pi r + 24r) \cdot 8r \\ \text{luas selimut } 5 &= (2\pi r + 32r) \cdot t = (2\pi r + 32r) \cdot 10r \end{aligned}$$
 The bottom part, labeled 'EoG', shows the derivation of a general formula for the area of the container wall:
 
$$\begin{aligned} \text{luas selimut } 5 &= (2\pi r + 32r) \cdot 10r \\ &= \left(2 \cdot \frac{22}{7} \cdot \frac{7}{2} + 32 \cdot \frac{7}{2}\right) \cdot 10 \cdot \frac{7}{2} \\ &= (22 + 16 \cdot 7) \cdot 5 \cdot 7 \\ &= (22 + 112) \cdot 35 \\ &= 134 \cdot 35 \\ &= 4.690 \text{ cm}^2 \end{aligned}$$

Figure 4. ST's Work-1

After understanding the questions, ST wrote in detail the information obtained, needed, and asked. Then ST confirms every information needed to solve the problem, one of which is to calculate the sphere's radius. ST's next step is to observe the given image. ST finds a pattern on the base and height of the container. In contrast to SR and SS, ST does not state the pattern in the form of a picture but directly in the form of a formula. ST writes the formula for the area of the container wall sequentially and then substitutes the pattern for the circumference of the base and the height of the container in the wall formula, making it easier for ST to find the area of the wall of the container in Figure-5.

The image shows two parts of a student's work. The top part, labeled 'SEoG', shows the identification of arithmetic sequences for the base circumference and height:
 
$$\begin{aligned} & * 0, 8, 16, 24, 32, \dots & * 2, 4, 6, 8, 10, \dots \\ U_n &= a + (n-1) \cdot b & U_n &= a + (n-1) \cdot b \\ U_{2021} &= 0 + (2021-1) \cdot 8 & U_{2021} &= 2 + (2021-1) \cdot 2 \\ U_{2021} &= 0 + 2020 \cdot 8 & U_{2021} &= 2 + 2020 \cdot 2 \\ U_{2021} &= 16.160 & U_{2021} &= 4.042 \end{aligned}$$
 The bottom part, labeled 'MoG', shows the substitution of these values into the area formula:
 
$$\begin{aligned} \text{luas selimut } 2021 &= (2\pi r + 16 \cdot 160) \cdot 4.042 \\ &= \left(2 \cdot \frac{22}{7} \cdot \frac{7}{2} + 16 \cdot 160 \cdot \frac{7}{2}\right) \cdot 4.042 \cdot \frac{7}{2} \\ &= (22 + 16 \cdot 560 \cdot 7) \cdot 2021 \cdot 7 \\ &= (22 + 56 \cdot 560) \cdot 14 \cdot 147 \\ &= 56 \cdot 542 \cdot 14 \cdot 147 \\ &= 800.465.554 \text{ cm}^2 \end{aligned}$$

Figure 5. ST's Work-2

The second question in the problem is the area of the wall of the Figure-2021 container. ST knows that one way to get the wall area of the Figure-2021 container is to use the general equation of the pattern of the circumference of the base and the height of the container. ST realized that the pattern that ST found was an arithmetic sequence, so ST immediately used the general equation  $U_n$  of an arithmetic sequence to find  $U_{2021}$  which would be the coefficient of  $r$  in the formula for the circumference of the base of the Figure-2021 container. ST reuses the general equation  $U_n$  arithmetic sequence to find  $U_{2021}$  which will be the coefficient  $r$  in the formula for the height of the container Figure-2021. After that ST multiplied the formula for the circumference and height of the Figure-2021 container that ST had found to determine the wall area of the Figure-2021 container.

Discussion

Based on the analysis of the data obtained from the results of the Generalization Task carried out by three students with low, medium, and high logical-mathematical intelligence, there were differences in the generalization process of students with high, medium, and low levels of logical-mathematical intelligence. The following is a description of the generalization process.

Generalization Process of Students with Low Logical-Mathematical Intelligence

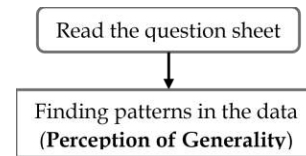


Figure 6. Flowchart of the generalization process of students with low logical-mathematical intelligence

Students with low logical-mathematical intelligence can understand the information provided well and are aware of the existence of patterns in the information provided. Students with low logical-mathematical intelligence can also determine the pattern formed from the image given to the problem, this can be seen when students can draw the shape of the base of the container in Figure-5. This is following the results of research by Arum, Kusmayadi, and Pramudya (2018) which states that students with low logical-mathematical intelligence can classify the information provided and understand patterns.

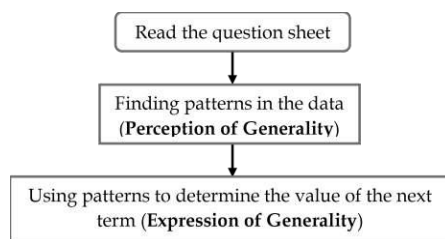
Students with low logical-mathematical intelligence tend to find patterns in the form of pictures easier, as was done when they found patterns in the shape of the base of the container. In addition, students with low logical-mathematical intelligence also tend to find it easier to find simple number patterns such as patterns formed at the height of the container. When students look for more complicated patterns such as patterns related to the circumference of a circle formula, students with low logical-mathematical intelligence tend to find it difficult. One of the characteristics of individuals who tend towards logical-mathematical intelligence is to show expertise with logic and use various skills in mathematics (Yusuf & Nurihsan, 2006), in other words, individuals who do not tend logical-mathematical intelligence tend not to show expertise with logic and do not use various skills in mathematics. As seen in the results of the Generalization Task of students with low logical-mathematical intelligence, students do not realize that the four curved



edges of the base of the container will form a complete circle when combined so that its length is equal to  $2\pi r$ .

Students with low logical-mathematical intelligence do not reach the stage of using regularities or patterns to determine the value of the next blanket area. This occurs because students with low logical-mathematical intelligence tend to avoid complex mathematical calculations or require long stages. One of the characteristics of individuals who tend towards logical-mathematical intelligence is to enjoy complex operations (Yusuf & Nurihsan, 2006) and able to process numbers well (Armstrong, 2009), in other words, individuals who do not tend logical-mathematical intelligence tend to show no interest in performing complex operations or calculations and do not show the ability to process numbers.

### Generalization Process of Students with Moderate Logical-Mathematical Intelligence



**Figure 7.** Flowchart of the generalization process of students with moderate logical-mathematical intelligence

Students with moderate logical-mathematical intelligence can understand the information provided well and are aware of the existence of patterns in the information provided. Students with moderate logical-mathematical intelligence can also determine the pattern on the shape of the base of the container and the height of the container. In addition, students can find that the bottom of the container in Figure-5 contains  $5 \times 5$  circles and find that the height of the container always increases by one sphere or increases by  $2r$ . This is following the results of research by Arum, Kusmayadi, and Pramudya (2018) which states that students with logical-mathematical intelligence are showing the ability to classify the information provided and understand patterns.

In the results of the Generalization Task of students with moderate logical-mathematical intelligence, it can be seen that students have no difficulty in determining the area of the wall of the Figure-5 container. Students with moderate logical-mathematical intelligence can easily determine the height of the Figure-5 container. Students also understand that the four curved edges of the base of

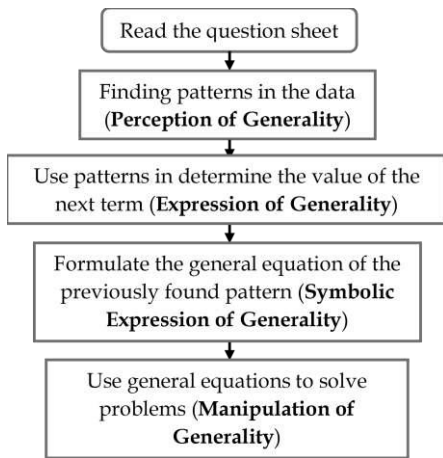
the container will form a complete circle when combined so that its length is equal to the circumference of a complete circle. In the next step, students with moderate logical-mathematical intelligence are adding  $2\pi r$  to the length of the other side that students have found to determine the formula for the circumference of the base of the container Figure-5. This is following the opinion of Yusuf and Nurihsan (2006) that one of the characteristics of individuals who have a tendency to logical-mathematical intelligence is demonstrating expertise with logic to solve problems and using various skills in mathematics (p. 231). Then, students with logical-mathematical intelligence are using the pattern of the circumference of the base and the height of the container to determine the area of the next container wall.

When they were about to answer the second question, students with moderate logical-mathematical intelligence were trying to formulate general equations for the pattern of the circumference of the base and the height of the container. Students with moderate mathematical logical intelligence demonstrate the ability to find numerical patterns when determining the general equation for the height of a container. Students find an abstract relationship in the index of the container image and the coefficient of the radius of the sphere in the formula for the height of the container. This is following the opinion of Gardner (2011) that logical-mathematical intelligence is the ability to distinguish numerical patterns. Students with moderate logical-mathematical intelligence have determined the general equation for the height of the container, but the students did not find the general equation for the circumference of the base. One of the characteristics of individuals with logical-mathematical intelligence is being able to think abstractly and conceptually (Gardner, 2011) and using various skills in mathematics (Yusuf & Nurihsan, 2006), in other words, individuals with moderate logical-mathematical intelligence will show less ability to think abstractly, such as determining abstract relationships in image indexes and differences in patterns and not showing expertise or understanding related to material related to number patterns. Students with moderate logical-mathematical intelligence did not find general equations for the pattern of the circumference of the base of the container so that students did not find general equations for the area of the container wall, in other words, students with moderate logical-mathematical intelligence were said to have not reached the stage of formulating general equations.

Students with moderate logical-mathematical intelligence have not reached the stage of using general equations to solve the given problem which is determining the wall area of the Figure-2021 container. One of the characteristics of individuals who tend towards logical-

mathematical intelligence is to like to experiment (Armstrong, 2009) and enjoy complex operations (Yusuf & Nurihsan, 2006), in other words, individuals with moderate logical-mathematical intelligence tend not to show interest in finding other solutions in solving problems and show no interest when performing complex operations or calculations. This is following the results of the Generalization Task of students with moderate logical-mathematical intelligence when finding a pattern around the base always increases by  $8r$  and using this method to find the circumference of the base of the Figure-2021 container, but students know that this method is not suitable to be applied in determining the circumference of the base of the Figure-2021 container because it takes quite a long time. Students with moderate logical-mathematical intelligence also do not show the desire to make other efforts which is finding a general equation for the circumference of a circle that can be used to determine the wall area of the Figure-2021 container. In addition, students with moderate logical-mathematical intelligence tend to avoid complicated or complex calculations when trying to solve the given problems.

**Generalization Process of Students with High Logical-Mathematical Intelligence**



**Figure 8.** Flowchart of the generalization process of students with high logical-mathematical intelligence

Students with high logical-mathematical intelligence can understand the information provided well and are aware of the existence of patterns in the information provided. Students with high logical-mathematical intelligence can also determine the pattern on the shape of the base of the container and the height of the container. Students can find that the circumference of the base of the container always increases by  $8r$ . In addition, students with high logical-mathematical intelligence also found that the height of the container always increased by  $2r$ .

This is following the results of a study by Arum, Kusmayadi, and Pramudya (2018) which states that students with high logical-mathematical intelligence can classify the information provided and understand the patterns in the information.

Based on the results of the Generalization Task, students with high logical-mathematical intelligence can easily determine the area of the wall in Figure-5. At first, students with high logical-mathematical intelligence focused on the pattern around the base of the container, then the students multiplied by the pattern formed from the height of the container. This method shows that students with high mathematical logical intelligence can solve problems with logic. This is following one of the characteristics of individuals who tend towards logical-mathematical intelligence which is showing expertise with logic to solve problems (Yusuf & Nurihsan, 2006). In addition, the method also shows that students with high logical-mathematical intelligence master mathematics well because students directly write the formula for the area of the wall and change the circumference of the base and height with the patterns found without describing or calculating the patterns one by one. Following the opinion of Yusuf and Nurihsan (2006) that individuals who tend towards logical-mathematical intelligence can use various skills in mathematics (p. 231).

Students with high logical-mathematical intelligence reach the stage of formulating general equations from the patterns found. The first step taken by students with high logical-mathematical intelligence is to focus on the  $r$  coefficient on the pattern around the base of the container then students use the formula  $U_n = a + (n - 1)b$  to determine the  $r$  coefficient in Figure-2021 after realizing that the pattern formed is an arithmetic sequence. This is also done on the container height pattern. The work shows that students with high logical-mathematical intelligence can find numerical patterns (Gardner, 1993), mastering mathematics and able to use expertise in mathematics (Yusuf & Nurihsan, 2006) namely an understanding of sequences and series material. In addition, the work of students with high logical-mathematical intelligence is also following the results of research by Arum, Kusmayadi, and Pramudya (2018) which states that students with high logical-mathematical intelligence can understand abstract patterns and relationships. It is proven that students with high logical-mathematical intelligence can find an abstract relationship between the Image index and the differences in the patterns that have been found.

After determining the general equation for the circumference of the base and the height of the container, students with high logical-mathematical intelligence use the results of the general equation to determine the circumference of the base and the height of the Figure-

2021 container. Then students with high logical-mathematical intelligence multiply the circumference of the base of Figure-2021 by the height of the container of Figure-2021 to determine the wall area of the Figure-2021 container. One of the characteristics of individuals who tend towards logical-mathematical intelligence is liking or enjoying the counting process (Armstrong, 2009). This can be seen from the student worksheets that appear to write down every calculation from the beginning to the end sequentially and neatly.

The discussion in this study is only guided by the subject's written answers so that it is not enough to reveal the generalization process carried out.

## CONCLUSION

### Conclusion

Based on the results of the Generalization Task, students with different levels of logical-mathematical intelligence will show different generalization processes so that the following conclusions are obtained.

Students with low logical-mathematical intelligence have reached the stage of finding the pattern of the circumference of the base and the height of the container in the information provided, but students do not use the pattern to determine the area for the next container wall. In addition, students with low logical-mathematical intelligence do not reach the stage of formulating general equations so that students with low logical-mathematical intelligence also do not reach the stage of solving the given problem, in other words, students with low logical-mathematical intelligence only reach the stage of perception of generality.

Students with moderate logical-mathematical intelligence have reached the stage of finding the pattern of the circumference of the base and the height of the container and using the pattern to determine the area of the next container wall, but students with moderate logical-mathematical intelligence have not reached the stage of formulating general equations from the pattern of the circumference of the base of the container so that students do not find the general equation for the area of the container wall. This makes students with moderate logical-mathematical intelligence not reach the stage of formulating general equations from the patterns found so that students also do not reach the stage of solving the given problem, in other words, students with moderate logical-mathematical intelligence only reach the stage of perception and expression of generality.

Students with high logical-mathematical intelligence have identified the pattern of the circumference of the base and the height of the container and used these patterns to determine the area of the next container wall. In addition, students with high logical-mathematical intelligence

understand that the pattern of the circumference of the base of the container and the pattern of the height of the container that is formed is an arithmetic sequence so students use  $U_n = a + (n - 1)b$  as a general equation. The general equation is then used to determine the formula for the wall area of the Figure-2021 container, in other words, students with high logical-mathematical intelligence reach all stages in the generalization process which is perception, expression, symbolic expression, and manipulation of generality.

### Suggestion

The generalization process found in this study was carried out sequentially without skipping or repeating the previous stage. Besides that, this study is still lacking in exploring the generalization process of the subject because it is only guided by written answers, it is suggested to other researchers who want to conduct research related to the generalization process to discuss the generalization process which is carried out in a non-sequential manner and conduct interviews with research subjects.

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