

**EXPLORING STUDENTS' COGNITIVE PROCESSES IN SOLVING NUMERACY TASK OF GEOMETRY AND MEASUREMENT REVIEWED BY MATHEMATICAL ABILITY****Citra Cahyaning Pertiwi**

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[rooselynaekawati@unesa.ac.id](mailto:rooselynaekawati@unesa.ac.id)**Abstrak**

Penelitian ini bertujuan untuk menganalisis bagaimana proses kognitif dari siswa SMA yang ditinjau dari kemampuan matematisnya dalam menyelesaikan masalah Asesmen Kompetensi Minimum (AKM) Numerasi terkait Geometri dan Pengukuran. Permasalahan diberikan kepada tiga siswa kelas X IPA SMA dengan kategori kemampuan matematis tinggi, sedang, dan rendah. Data hasil pekerjaan subjek dianalisis berdasarkan tahap proses kognitif yaitu tahap memahami informasi linguistik dan numerik dalam masalah; tahap menerjemahkan dan mengubah informasi tersebut menjadi notasi matematika, algoritma, dan persamaan; tahap mengamati hubungan antar elemen masalah; tahap merumuskan rencana untuk memecahkan masalah; tahap memprediksi hasil; tahap mengatur jalur solusi saat dijalankan; tahap mendeteksi dan mengoreksi kesalahan selama solusi masalah yang kemudian dilanjutkan dengan wawancara mendalam. Hasil menunjukkan bahwa siswa dengan kemampuan matematis tinggi melalui semua tahap pada proses kognitif dengan cukup runtut dan lengkap. Sedangkan siswa dengan kemampuan sedang dapat melalui tahap memahami informasi hingga tahap perhitungan dari perencanaan, namun ada beberapa kesalahan dan koreksi dalam menemukan solusi. Sedangkan siswa dengan kemampuan rendah belum mampu menunjukkan tahap proses kognitif secara runtut sehingga banyak tahap yang terputus dan harus kembali pada tahap memahami permasalahan. Berdasarkan hasil tersebut, diharapkan pada pembelajaran untuk persiapan Asesmen Kompetensi Minimum materi Geometri dan Pengukuran dapat lebih ditekankan dan dilatihkan pada pemahaman informasi pada permasalahan sehingga siswa dapat melanjutkan tahap penyelesaian permasalahan sesuai dengan rencana solusi yang dibuatnya.

**Kata kunci:** Proses Kognitif, Numerasi, Geometri dan Pengukuran.

**Abstract**

This study aims to analyze how the cognitive processes of high school students in terms of their mathematical abilities in solving the Minimum Competency Assessment Numeracy problems related to Geometry and Measurement. Problems were given to three 10<sup>th</sup> science students with categories of high, medium, and low mathematical ability. Data on the results of the subject's work were analyzed based on the cognitive processes stage, namely the stage of Comprehending linguistic and numerical information in the problem; 2) Translating and transforming that information into mathematical notations, algorithms, and equations; 3) Observing relationships among the elements of the problem; 4) Formulating a plan to solve the problem; 5) Predicting outcome; 6) Regulating the solution path as it is executed; 7) Detecting and correcting errors during problem solution which is then followed by in-depth interviews. The results showed that students with high mathematical abilities went through all stages of the cognitive process quite coherently and completely. While students with moderate abilities went through the stage of understanding information to the calculation stage of planning, but there were some errors and corrections in finding solutions. Meanwhile, students with low abilities had not been able to show the stages of cognitive processes in a coherent manner so that many stages were interrupted and had to return to the stage of understanding the problem. Based on these results, it is hoped that learning for the preparation of the Minimum Competency Assessment for Geometry and Measurement material can be emphasized more and so that the understanding of information on students' problems can continue the completion stage according to the solution plan that they have made.

**Keywords:** Cognitive Processes, Numeracy, Geometry and Measurement.

## INTRODUCTION

National assessment as a way to photograph and map the quality of schools and the education system as a whole (Ministry of Education and Culture, 2020). So only certain students need to be participants in the National Assessment. What is needed is information from a sample that represents the student population in each education unit at the grade level that is the target of the National Assessment. National assessments need to be carried out to improve the quality of education so that through this assessment / assessment can produce accurate information in improving the quality of learning and teaching which will later be oriented towards improving student learning outcomes (Ministry of Education and Culture, 2020). According to Kunandar (2013: 62) learning outcomes are certain competencies or abilities both cognitive, affective and psychomotor that are achieved or mastered by students after participating in the teaching and learning processes.

Regarding the competence of learning outcomes, based on the 2020 Ministry of Education and Culture of Minimum Competency Assessment pocket book, one of the assessment competencies in the national assessment is numeracy competence. This numeracy competence is contained in one of the national assessment instruments, namely the Minimum Competency Assessment instrument. Numeracy is the ability, confidence and willingness to engage with quantitative or spatial information to make informed decisions in all aspects of daily life (Alberta, 2018). The learning level at Minimum Competency Assessment Numeracy is divided into five levels. At the level of learning 5, it involves senior high school 10<sup>th</sup> grade students. The numeracy content related to this level has indicators that are adapted for 10<sup>th</sup> grade students. The contents are Geometry and Measurement, Aljabr, Uncertainty and Opportunity.

Geometry and Measurement is one of the main topics in mathematics. According to Dandyal (2015), geometry can be a stimulus for students' thinking skills in various aspects such as visual, verbal, drawing, logistical, and applied skills, so that it becomes a predictor in improving mathematical abilities at a later stage. As with measurement, measurement is an essential competency that is built on scientific knowledge for all fields and careers (John et. al. 2011). Therefore, the researcher chose geometry and measurement as a numeracy task in exploring students' cognitive processes.

For students in preparation for Minimum Competency Assessment and improving student learning outcomes can be done by focusing on efforts to train students to use their thinking potential. In the processes of thinking, it always involves and is related to learning activities, this is in line with what Panjaitan (2016) states

that learning is an activity related to cognitive processes. Cognitive processes have links and benefits in the problem solving processes, this is supported by the opinion of Schunk (2012: 299), one of the most important types of cognitive processes that occurs often during learning is problem solving. Therefore, the link between problem solving and cognitive processes is that problem solving can be understood as a cognitive processes that requires effort and concentration of thought, because in solving problems a person collects relevant information, identifies information, analyzes information and makes decisions.

In this study it can be concluded that the cognitive processes is a logical, concrete to abstract thought processes that involves processing information so that it raises ideas or knowledge, especially in solving a problem that can be traced from how students process their thoughts based on the type of reasoning that is mainly demanded by tasks, namely non- reasoning-measurement or measurement reasoning. Cognitive processes can be related to solving a problem, this is in accordance with what was expressed by Montague (2002) at the University of Miami regarding the stages of the cognitive processes in problem solving, namely: 1) Comprehending linguistic and numerical information in the problem; 2) Translating and transforming that information into mathematical notations, algorithms, and equations; 3) Observing relationships among the elements of the problem; 4) Formulating a plan to solve the problem; 5) Predicting outcome; 6) Regulating the solution path as it is executed; 7) Detecting and correcting errors during problem solution.

One of the factors that influence the problem-solving ability is the mathematical ability of students. Mathematical Ability is the ability, whether learnt or perceived as natural capability to process numerical data and conclude a mathematical calculation based on that data (Nam, 2013). Nurman's (2008) research results found that a student's mathematical ability has an effect on his mathematical problem-solving ability. Students with high mathematical abilities have high ability in problem solving, students with moderate mathematical abilities have fairly good problem solving abilities, and students who have low mathematical abilities have poor problem solving abilities.

The content of the problems in the Minimum Competency Assessment Numeracy that require problem solving skills and mathematical abilities is geometry and measurement, based on the Minimum Competency Assessment of Ministry of Education and Culture 2020 pocket book. The form of the problems geometry and measurement presented in this Minimum Competency Assessment Numeracy refers to aspects of problems in

everyday life or contextual problems. However, students' ability to solve and solve contextual problems is still lacking, this is in line with what was conveyed by Wijaya (2014) that less than one percent of Indonesian students are able to work on problems with complex situations that require mathematical modeling and reasoning skills. By knowing the cognitive processes of students in solving problems, especially in geometry and measurement of Minimum Competency Assessment Numeracy, it is hoped that there will be learning preparation for students to improve learning outcomes with interpretations of improving the quality of education. Therefore, this study aims to explore the Students' Cognitive Processes in Solving Numeracy Task of Geometry and Measurement reviewed by Mathematical Ability.

**METHOD**

**Sample of Research**

This research is a qualitative descriptive study. Students as subjects were selected using purposive sampling technique with certain criteria, namely based on mathematical abilities. In one class about 25-30 high school students of 10<sup>th</sup> grade science student carried out a math ability test under virtual supervision with a duration of about 45-60 minutes. The construction of the mathematical ability test in this study came from the Test of Mathematical Ability, which is one of the standardized tests that has been used by experts in the field of educational psychology and other educational sciences in research both to measure mathematical ability in pre-school age children and elementary school to school age. medium (Brown, Cronin, & Bryant, 214 J Nuerk, Moeller, & Willmes, 2015). In Siregar's (2020) research, Test of Mathematical Ability in third edition consists of several sub-tests, including: concepts and symbols in mathematics, arithmetic, mathematics in everyday life, story questions and attitudes in mathematics. In this study, a test of mathematical ability related to concepts and symbols was used on the absolute value inequality material. The mathematical ability was classified into three levels, namely high, moderate, and low math abilities. The third criterion for the level of mathematical ability according to Rofiki's opinion (Widarti, 2013: 4) makes the criteria for the level of student ability and the rating scale into 3 categories, namely high ability if  $80 \leq$  the value obtained is  $\leq 100$ , moderate ability if  $65 \leq$  the value obtained is  $<80$ , and low ability if  $0 \leq$  the value obtained is  $<65$ . From the results of the mathematics ability test, a sample of students were taken for each level by testing verbal communication and volition. The total sample obtained was 3 students.

**Instrument and Procedure**

Data collection was carried out through the written results of students' work on tests or problems that were different from the tests at the time of sampling and continued with interviews. Before students solve problems, researchers emphasized that student work results do not affect the assessment of subjects in their schools. First, students were given a Minimum Competency Assessment Numeracy problem about geometry and measurement that related to everyday life. Ministry of Education and Culture (2020) through the Minimum Competency Assessment pocket book and its implications in learning states that numeracy is the ability to think using mathematical concepts, procedures, facts, and mathematical tools in solving problems in everyday life with various contexts which is relevant for individuals as citizens of Indonesia and citizens of the world. Content on mathematical literacy or numeracy is used to measure the competencies needed in life and which is adjusted to the notion of numeracy. As for the content on the numeracy (Ministry of Education and Culture, 2020) that Measurement and Geometry Group, includes knowing a flat shape, the use of area and volume of a flat shape in everyday life, measurement of length, weight, time, volume and discharge, as well as using the area and standard units. Students solved geometry and measurement problems, and the researcher did the first analyze, and continued with interviews about their work.

The Numeracy task is a problem based on the Minimum Competency Assessment on Geometry and Measurement content. In first and second of Numeracy task is included in the personal context related to personal or individual self-interest. These two tasks were chosen because they represent Geometry content related to geometry figures and flat shapes, then in the measurement section it is related to sizes and calculation results to determine the sizes of these shapes.

**Numeracy Task 1**

*Read and understand the following passages to help answer question number 1*

Did you know that in 2025 as many as 321 million people in Indonesia are predicted to find it difficult to get clean water? (World Water Forum II/World Water Forum in The Hague, March 2000). The need for water is not only to maintain personal health, but some daily activities such as washing, cooking, cleaning themselves also require water in their use. It is unimaginable, if the water supply is running low while almost all of our daily activities require water. One way to save water from now on is by using an Aerator Faucet, because we can save up to 43 liters of water.

**Look at the picture!**



Paving      Printing

accepts orders to print paving in the form of a hexagon prism. To print the paving required a mold that has a side length of 11 cm and a height of 6 cm. Based on the size of the mold and the quality of the paving that the consumer wants, a mold material with the appropriate ratio is needed. The ratio of paving mold material is water:cement:sand is equal to 5:2:7. In the manufacture of this paving, the water used to print the pavings is as much as the water saved by using the aerator faucet. Using this water savings determine the maximum number of paving blocks that can be printed! Write down the solution steps and explanations!

**Numeracy Task 2**

**Look at the picture!**



Arif plays by the lake near his residence. Arif wanted to measure the width of the lake by observing the tree right in front of him which was right on the edge of the lake. Then Arif walked 175 meters and looked back at the tree he saw for the first time. He measured the tree's point of view which turned out to be  $60^\circ$ . Determine the width of the lake that Arif will measure! Write down the solution steps and explain!

Figure 1. Numeracy Task about Geometry and Measurement

Numeracy Task 1 is more complex than Numeracy Task 2, in Numeracy Task 1 it uses the application of geometry concepts to geometric figures associated with comparisons. Meanwhile, Numeracy Task 2 is application of trigonometry concepts to its measurements.

Cognitive processes can be related to solving a problem, this is in accordance with what was expressed by Montague's (2002) study at the University of Miami regarding the stages of the cognitive processes in problem solving, namely:

1. Comprehending linguistic and numerical information in the problem
2. Translating and transforming that information into mathematical notations, algorithms, and equations
3. Observing relationships among the elements of the problem
4. Formulating a plan to solve the problem
5. Predicting outcome
6. Regulating the solution path as it is executed
7. Detecting and correcting errors during problem solution

**Data Analysis**

The answer sheet is checked using stage of Cognitive processes was defined as follows Montague's (2002) study at the University of Miami and in the breakdown in several indicators.

Table 1. Indicators of Cognitive Processes

Stage of Cognitive Processes	Indicators	Code
Comprehending linguistic and numerical information in the problem	Able to relay information on existing problems using their own language	C1
	Convey how to understand information on the problem	C2
	Convey difficulties in understanding information and how to solve it (if any)	C3
Translating and transforming that information into mathematical notations, algorithms, and equations	Express information on problems in mathematical form, such as presenting data, equations, mathematical notation	T1
	Determine the mathematical concepts used in problem solving	T2
Observing relationships among the elements of the problem	Determine important information to solve problems	O1
	State the completeness or incompleteness of the problem information	O2

	Determine the relationship between the information used to solve problems	O3
Formulating a plan to solve the problem	Delivering a plan of steps / ways / flow of thought and work flow to find solutions to problems	F1
	Use a logical problem-solving plan	F2
Predicting outcome	Determine hypotheses / provisional estimates of solutions based on problem solving plans	P1
Regulating the solution path as it is executed	Use the steps in the problem-solving plan appropriately	R1
	Use clear liaison sentences between completion steps	R2
	Do the calculations correctly	R3
	Use mathematical notation appropriately and according to its meaning	R4
Detecting and correcting errors during problem solution	Convey error solving problems and convey the justification (if any)	D1
	Conveying his belief in solving the problems that have been done	D2
	Shows the thing / part of the answer that strengthens his belief in solving the problem	D3

This study used the flowchart model about Cognitive Processes by Montague was adapted from Ekawati's (2019) study about Students' Cognitive Processes in Solving Problem Related to the Concept of Area Conservation in terms of the dynamic processes that may occur during the student solution processes indicated by the step arrows is used as the analytical tool. The framework in this research had covered the stages of cognitive processes, became a flow for determining the direction of students' cognitive processes, and could direct the results of students' work and thought that show their cognitive processes because the model of using arrows in analysing the stages that might occur on student's cognitive process are proven as a helpful tool for keep

track student's behaviors (Yeo & Yeap, 2010), so using this framework of Cognitive Processes:

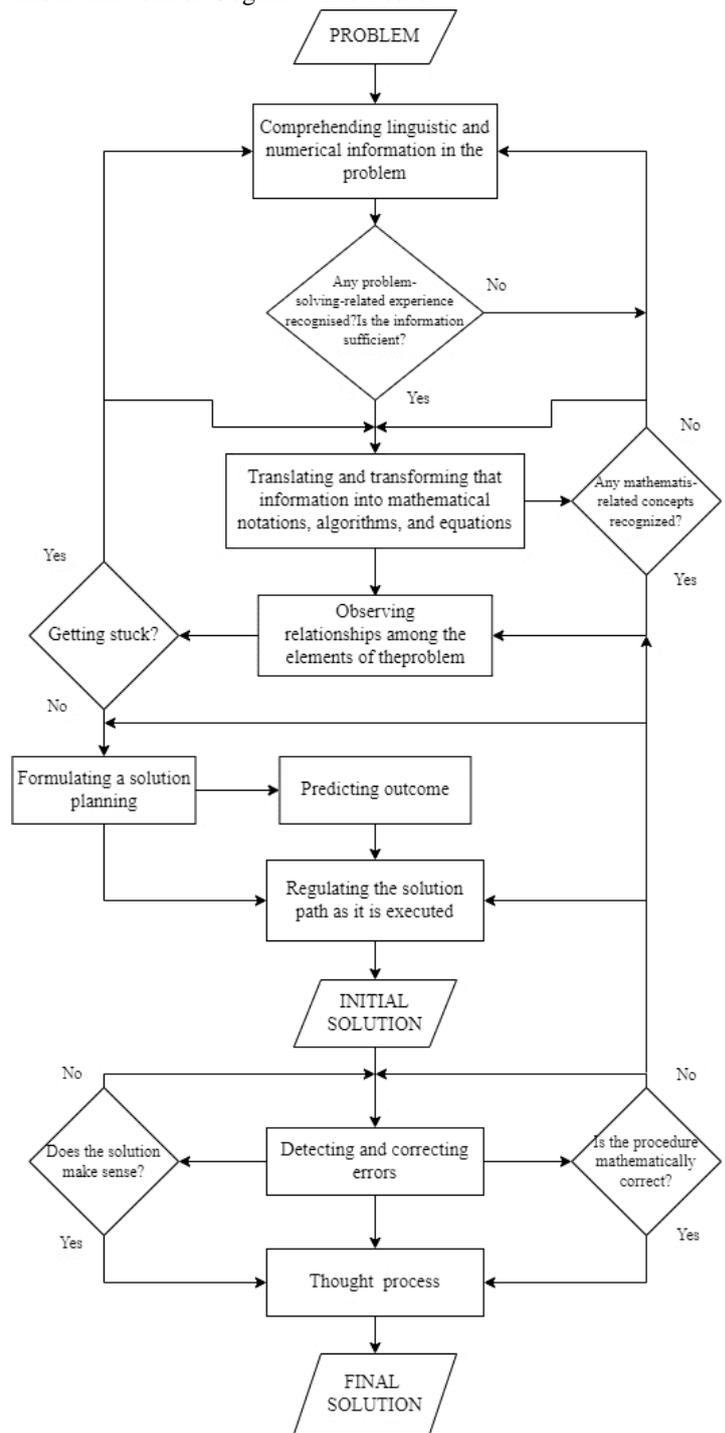


Figure 2. Framework of analysing students' cognitive processes

When the subject's responses did not indicate a particular cognitive processes occurred during the interview, the interviewer did not inquire about the processes further. Based on Cognitive Processes by Montague (2002), following interview guide:

Table 2. Interview Guide based on Cognitive Processes Stage

No	Cognitive Processes	Example of item questions
1.	Comprehending the linguistic and numerical information in the problem	<ul style="list-style-type: none"> <li>➤ Please read the question to me. If you don't understand certain words, say so.</li> <li>➤ Tell me what is asked in the question.</li> <li>➤ How do you understand the meaning of certain information from this question?</li> <li>➤ Which word/sentence/part of the graph makes it difficult for you to understand that you have not identified from this problem?</li> </ul>
2.	Translating and transforming that information into mathematical notations, algorithms, and equations	<ul style="list-style-type: none"> <li>➤ Have you come across a context like this before? In terms of what?</li> <li>➤ What mathematical concepts are used in this context?</li> <li>➤ How do you convert the information in the problem into mathematical forms and concepts?</li> </ul>
3.	Observing relationships among the elements of the problem	<ul style="list-style-type: none"> <li>➤ Which information from the problem do you think is important to use in the processes of finding answers?</li> <li>➤ Is there complete information provided?</li> <li>➤ Can you recognize the pattern/relationship of the information provided by the question?</li> </ul>
4.	Formulating a plan to solve the problem	<ul style="list-style-type: none"> <li>➤ Tell me how you are going to find the answer</li> </ul>
5.	Predicting outcome	What might happen if you carry out your plan?
6	Regulating the solution	Show me what to do to get the answer

	path as it is executed	
7	Detecting and correcting errors during problem solution	<ul style="list-style-type: none"> <li>➤ Tell me how do you convince yourself about your answer</li> <li>➤ Asking about the relationship between the mathematical results obtained by the questions on the question, such as the question: "Are you sure your answer makes sense to answer the question of the problem?"</li> <li>➤ Is there an image or the like that you made to strengthen your answer?</li> </ul>

Data analysis using descriptive techniques carried out through three stages, namely reducing data, presenting data and drawing conclusions (Miles & Huberman, 1994). Reducing data is the processes of selecting things or important information by the problem under study. After reducing the data, the data is presented and conclusions are drawn.

### RESULT AND DISCUSSION

Based on the results of the mathematical ability test of 25 10<sup>th</sup> grade science students and based on three categories of mathematical ability by Rofiki's (Widarti, 2013: 4) opinion, it was obtained each student representative from the high, moderate, and low categories.

Subject	Mathematical Ability	Code
Subject 1	High	S1
Subject 2	Moderate	S2
Subject 3	Low	S3

### The Cognitive Processes of High Mathematical Ability Student

The first subject was S1 in the category of high mathematical ability. The following was an analysis of the results of the work and interviews by S1 for the first and

second Minimum Competency Assessment Numeracy about Geometry and Measurement problems.

In the first numeracy task, S1 is able to go through the comprehending linguistic and numerical information in the problem stage by rephrasing the problem and showing known information

$s = 11 \text{ cm}$	Perbandingan air : Semen : pasir = 5 : 2 : 7	<b>C1</b>
$t = 6 \text{ cm}$	Jumlah air = 43L	

Figure 3. The result work of High Mathematical Ability Student on first stage

In English

$s = 11 \text{ cm}$	Water:Cement:Sand Ratio = 5:2:7
$t = 6 \text{ cm}$	Amount of water = 43L

From the picture of the work, interviews were then conducted to clarify the writing of his work. The results of the interview with P is interviewer and S1 is participant.

**P:** Tell me what is asked in the question!

**S1:**  $s$  is the length of the side of the paving and  $t$  is the height of the paving, it is known that the ratio of water, cement, and sand to make paving is 5:2 :3 and the amount of water available is 43L from the savings using an aerator faucet, then how many paving can be printed? (C1)

**P:** What mathematical concepts are used in this context?

**S1:** The mathematical concept used is the volume of a prism, namely the area of the base times the height. Because the paving is in the form of a regular hexagon prism, we use the volume of a regular hexagon prism. Another concept used is a comparison to determine the volume of water in 1 paving mold. (T2)

**P:** Can you recognize the pattern/relationship of the information provided by the question?

**S1:** to find the volume of 1 paving then from the ratio of water, cement, sand it would be used to determine the volume of water for 1 paving. (O3)

**P:** why?

**S1:** because what is known is the amount of water used to print the paving, then this amount of water is divided by the volume of water in 1 paving so that the number of paving can be printed

From High Mathematical Ability Student's explanation, it was found that observing relationships among the elements of the problem and formulating a plan to solve the problem stage could be passed quite well, but S1 did not predict the outcome of the plan he made. For Regulating the solution path as it is executed stage, S1 shows the result of its work as follows

$L_{\text{alas}} = 3 \sqrt{3} \times 11^2$	<b>R1, R2, R3, R4</b>
$L_{\text{base}}^2 = 314,36722 \text{ cm}^2$	
$V_{\text{prisma}} = L_{\text{a}} \times t$	<b>R1, R2, R3, R4</b>
$V_{\text{prisma}} = 314,36722 \times 6$	
$= 1886,20332 \text{ cm}^3$	
$= 1,88620332 \text{ L}$	
$= 1,9 \text{ L}$	

Figure 4. The result work of High Mathematical Ability Student on Regulating and Thought Processes

The calculations carried out by S1 have met the correct procedure. After calculating the area of the regular hexagon base, determining the volume of paving and determining the volume of water in 1 paving, then he divided the amount of water which is 43L by the volume of water in 1 paving which is 0.7L so that the result is 61.42 paving. From the results of these calculations, S1 gave a conclusion to answer the first problem of the numeracy task

Jika air yang akan diisi adalah 0,7L, maka banyaknya jumlah paving block maksimum yang dapat dicetak adalah :	<b>R1, R2, R3, R4</b>
$43 : 0,7 = 61,42 \rightarrow 61 \text{ paving}$	
0,7	

Figure 5. The result work of High Mathematical Ability Student on Conclusion

In English:

If the water to be filled is 0.7L, then the maximum number of paving blocks that can be printed is
$\frac{43}{0.7} = 61.42 \Rightarrow 61 \text{ pavings}$

The researcher tried to carry out detecting and correcting errors during the problem solution stage by asking S1 about the final result,

**P:** why do you round up the number of paving to 61?

**S1:** I rounded 61.42 down because the decimal is less than 5

**P:** what if the excess was 0.52 paving, how did you round off?

**S1:** I will round up to 62 paving

**P:** if the excess of 0.52 paving would be worth selling?

**S1:** I don't think so, Miss, so the paving is still 61 paving so that the paving is intact if it is later sold.

From the cognitive processes of High Mathematical Ability Student in the first numeracy task, it can be shown in the flow diagram in figure 9.

In the second numeracy task, S1 re-explained the problem of determining the approximate size of the river width and S1 also wrote down some known information

Samping = 175 m  
Sudut elevasi = 60°

C1

Figure 6. The result work of High Mathematical Ability Student on first stage

In English:

Side = 175m  
Elevation angle = 60°

S1 explained that the meaning of the side was that from the illustration in the problem there was a right triangle shape, the word side meant that the side beside the angle was 60°, namely walking distance of Arif was 175 meters. The researcher asked S1 to illustrate a right triangle and side, hypotenuse, and front side positions from an angle of 60°

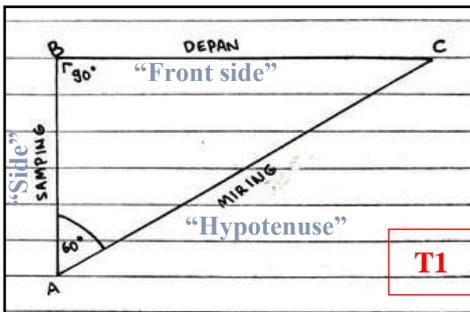


Figure 7. The illustration of Right Triangle

From the results of the illustration, researchers conducted in-depth interviews to find out the meaning of the illustrations made by S1.

P: What mathematical concepts are used in this context?

S1: the concept used is the basic trigonometry ratio, namely  $\sin \alpha = \frac{de}{mi} = \frac{\text{front side length}}{\text{hypotenuse length}}$ ,  $\cos \alpha = \frac{sa}{mi} = \frac{\text{side length}}{\text{hypotenuse length}}$ ,  $\tan \alpha = \frac{de}{sa} = \frac{\text{front side length}}{\text{side length}}$ . The information contained in the problem and the selection of the concepts used was quite relevant and could be used to solve problems. (T1)

This shows that S1 was able to pass the translating and observing stages quite well. In planning the solution of the problem, S1 revealed that he would use the concept of a trigonometry comparison of tan angles.

P: Why use trigonometry ratio of tan angles?

S1: because  $\tan = \frac{de}{sa} = \frac{\text{front side length}}{\text{side length}}$ , because the front angle of 60°, namely BC was the width of the river to be searched and the side angle of 60° was the Arif's distance walking from A to B which was already known and also known value of  $\tan 60^\circ$ . (O3)

S1 also skipped the prediction of the answer based on the plan she had made. In the execution of the S1 job was shown as follows

$\tan 60 = \frac{\text{depan}}{\text{samping}}$   
 $\sqrt{3} = \frac{\text{depan}}{175}$   
 $\text{depan} = 175\sqrt{3} \text{ m}$   
Jadi, lebar danau tersebut adalah  $175\sqrt{3} \text{ m}$

Figure 8. Calculation and conclusion of the second problem from High Mathematical Ability Student

In English:

$$\tan 60^\circ = \frac{\text{Front Length}}{\text{Side Length}}$$

$$\sqrt{3} = \frac{\text{Front Length}}{175}$$

$$\text{Front Length} = 175\sqrt{3}$$

So, the width of river is  $175\sqrt{3} \text{ m}$

The calculation processes had been done was correct and the use of notation was also correct, it's just that the conclusion section needed to be re-corrected so that the answer more made sense. The researcher tried to ask about the right conclusion.

P: what was the approximate width of the river if it is to be stated in general using  $\sqrt{3}$ ?

S1: I don't think so because people are generally unfamiliar with the number  $\sqrt{3}$

P: how to make the estimated width of this river acceptable to people in general?

S1: I needed to find the value of  $\sqrt{3}$  and multiply by 175. The result is 303.109, so the approximate width of the river is 303.109 meters.

Based on the explanation, the cognitive processes flow diagram for High Mathematical Ability Student in numeracy task 2 is obtained in figure 9.

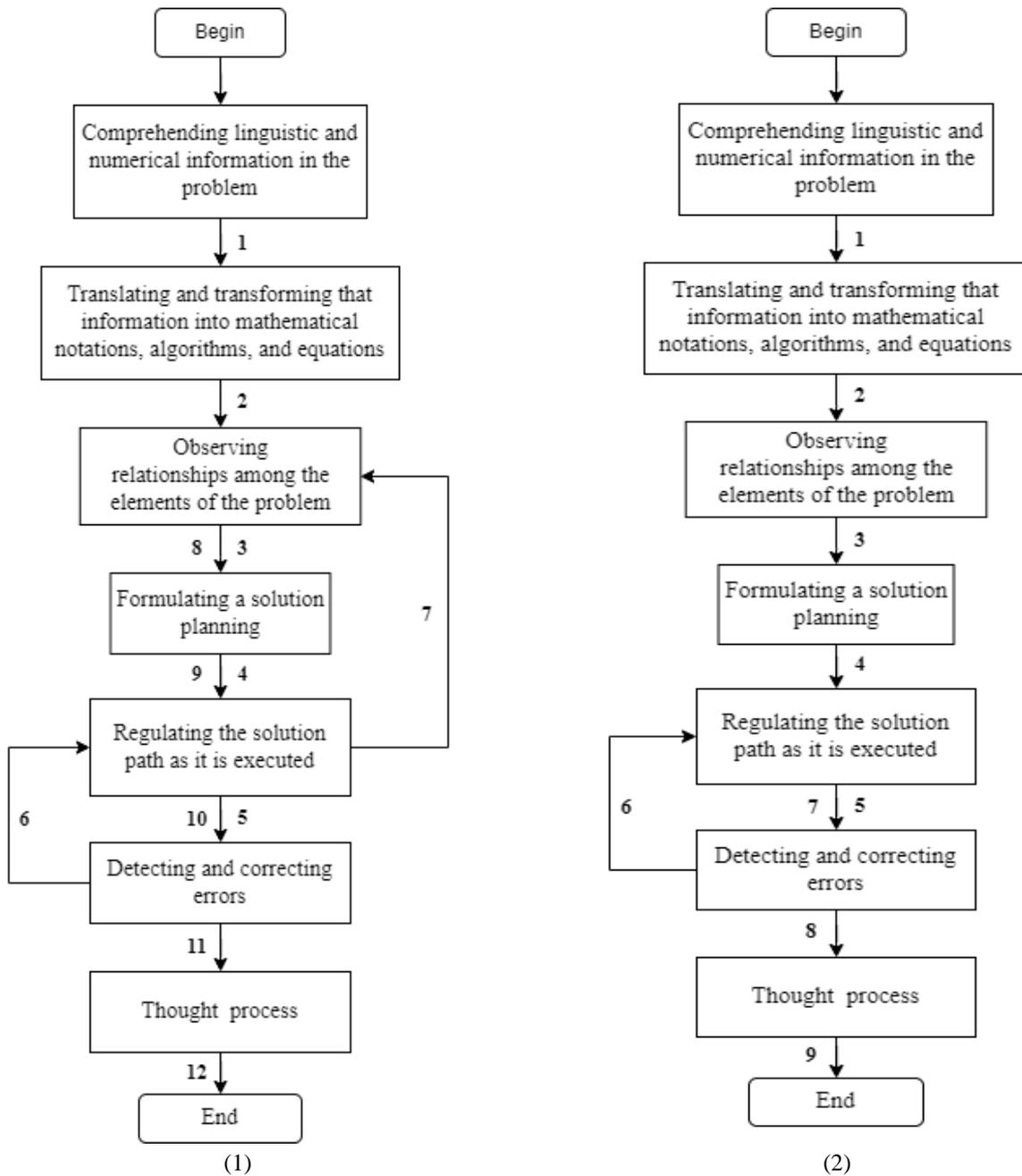


Figure 9. Cognitive Processes of High Mathematical Ability Student

**The Cognitive Processes of Moderate Mathematical Ability Student**

The second subject was S2 in the category of moderate mathematical ability. The following was an analysis of the results of the work and interviewed by S2 for the first and second Minimum Competency Assessment Numeracy about Geometry and Measurement problems.

In understanding the first numeracy task, S2 stated that he was quite confused and needed to repeat himself in understanding the problem. S2 tried to convey the contents of the problem in their own language that we were asked

to determine the number of paving that were printed and explained with appropriate steps. S2 also re-read the problem completely and then he also wrote down some known information

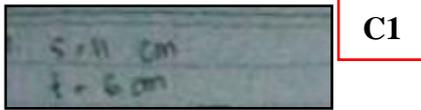


Figure 10. The result work of Moderate Mathematical Ability Student on first stage

Based on the picture above, S2 explained about the work in in-depth interview with P as a researcher and S2 as a participant.

- P:** Explain again about the problem in the first question!  
**S2:** the meaning of  $s$  is the length of the paving side, which is 11 cm and  $t$  is the height of the paving, which is 6 cm.  
**P:** How about the concept used to solve this problem?  
**S2:** there are several concepts to solve this problem, namely the area of the hexagon and the volume of the prism. (T2)  
**P:** How the relationship between the two concepts?  
**S2:** in making paving in the form of a hexagonal prism, it was necessary to have a 6-sided prism volume, namely the area of the base times the height. (O3)

This indicated that S2 was quite capable of comprehending linguistic and numerical information in the problem stage, capable to translating and observing stage the information in the problem. Then the researcher continued the question of what ratio of water he calculated, S2 was quite hesitant to answer that to determine the volume of water for all paving. In this case the researcher reaffirmed,

- P:** whether 0.7L was possible as the volume of water for all paving?  
**S2:** (re-read the information in the problem and found that) the water volume for all paving is 43L, so 0.7L is the volume for 1 paving.

After getting stuck on the several relationship between problem information, S2 could proceed to formulating a plan to solve the problem stage. S2 said that by finding the volume of water for 1 paving, the available water would be divided by that volume, so that the number of paving would be obtained. From this plan, S2 performed the following calculations

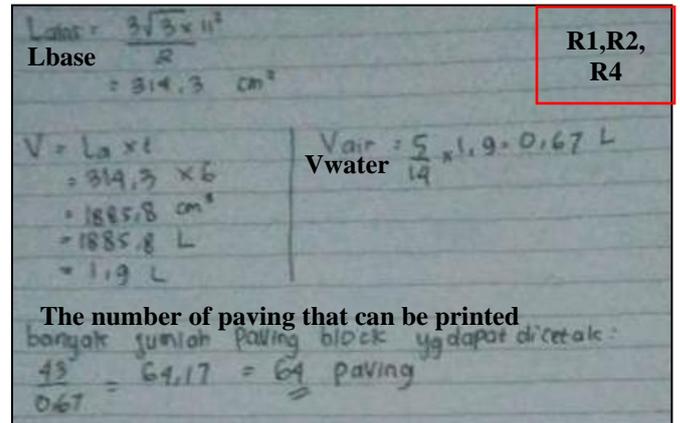


Figure 11. Calculation and conclusion of the first task from Moderate Mathematical Ability Student

S2 had performed the appropriate and correct calculations. However, there were some errors in the concept of rounding to determine the volume of water in 1 paving. The researcher asked S2 to recalculate the result from  $\frac{5}{14} \times 1.9$ .

- P:** are you sure with your answer about  $\frac{5}{14}$  times 1.9? Please re-calculate!  
**S2:**  $\frac{5}{14}$  times 1.9, I get the result 0.67567. Oh, this is rounded up, right, Miss? Means 0.7 should be.  
**P:** whether the continuation of the calculation processes will affect the next calculation?  
**S2:** yes, Miss, it has an effect on the number of paving results, which means it should be  $\frac{43}{0.7}$  is equal to 61.42. So the number of paving are 64 pavings.

In this case, detecting and correcting errors during the problem solution stage could be passed by S2 properly through the guidance of questions that lead S2 to realize the error and correct it. Based on the explanation, it is obtained the flow diagram of the S2 cognitive processes in the figure 15.

In the second numeracy task, S2 is able to re-explained the problems to be solved and write down some known information.

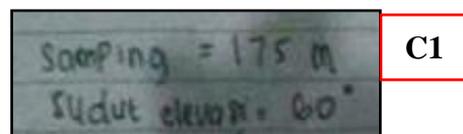
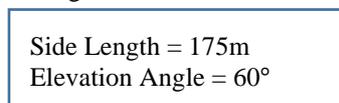


Figure 12. The result work of Moderate Mathematical Ability Student on first stage

In English:



S2 explain the meaning of the information explained through the following illustration

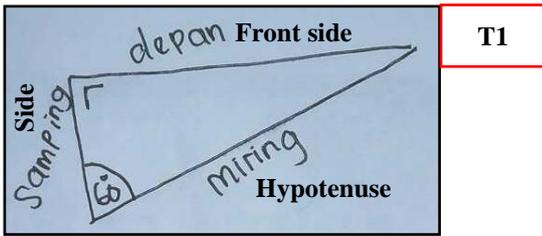


Figure 13. Illustration of Right Triangle

**P:** From this illustration, which part shows the distance Arif walked and the width of the river?

**S2:** I think the width of the river is the hypotenuse and the distance Arif walks is the side.

**P:** Why does the hypotenuse indicate the width of the river?

**S2:** because from the point Arif walked. Arif walks from the of angle 60°.

**P:** What are the conditions for determining the distance between two points?

**S2:** What's the closest distance, Miss? Must be upright. Means, Oh the width of the river on the front side of Miss.

**P:** OK, right. Now explain the meaning of the side, front, and hypotenuse side.

**S2:** the meaning of "side" is beside the angle 60; the meaning of "front" is the side in front of the angle 60; meanwhile the "hypotenuse" is the oblique of right triangle.

**P:** What concept did you use to solve this problem? And how is the relationship between the information?

**S2:** I use the concept of trigonometry comparison. Using  $\tan 60 = \frac{de}{sa} = \frac{\text{Front Length}}{\text{Side Length}}$ , because we already know the value of  $\tan 60$  and side length and we will look for the width of the river, namely Front Length.

From this, S2 had been able to full fill translating and transforming that information into mathematical notations, algorithms, and equations stage and observing relationships among the elements of the problem stage, although there had been some stuck in the explanation of the relationship between concepts and known information.

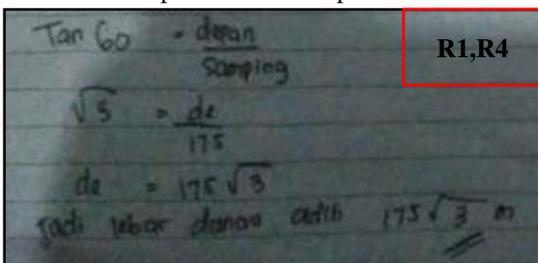
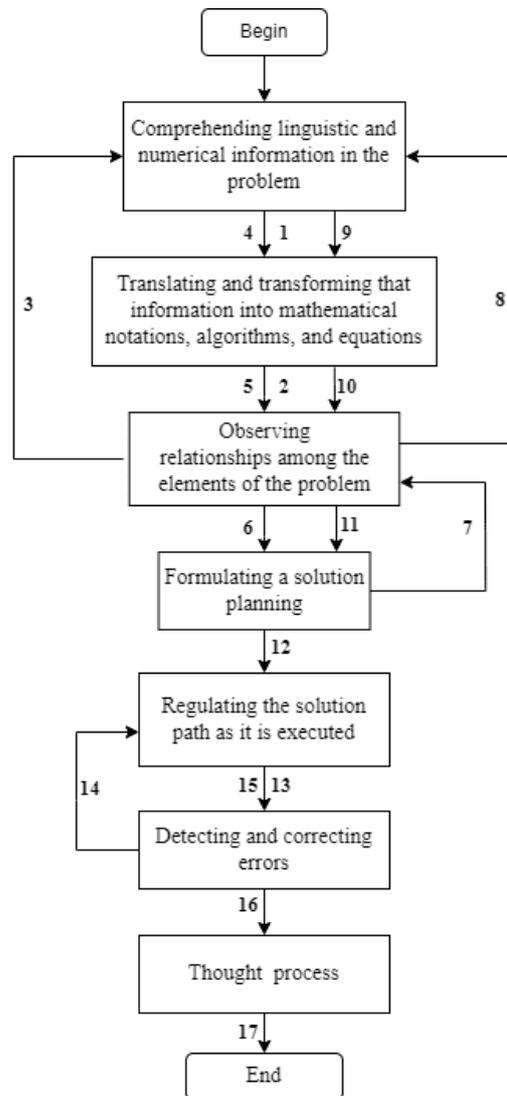


Figure 14. Calculation and conclusion in second task of Moderate Mathematical Ability Student

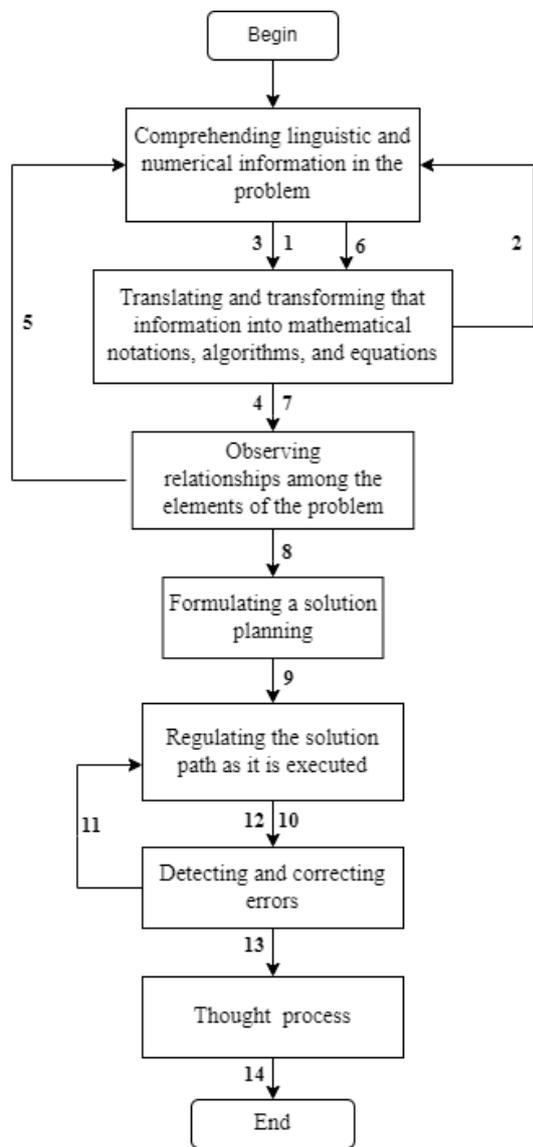
$$\begin{aligned} \tan 60^\circ &= \frac{\text{Front side}}{\text{Side}} \\ \sqrt{3} &= \frac{\text{Front side}}{175} \\ \text{Front side} &= 175\sqrt{3} \\ \text{So, the width of river is } &175\sqrt{3} \end{aligned}$$

The calculation processes carried out by S2 was correct and according to the procedure. However, in the conclusion section, there were results that didn't make sense if  $175\sqrt{3}$  was an estimate of river width because it was not commonly used in measurements. From this, S2 stated that the estimated river width of  $175\sqrt{3}$  needed to be converted into decimal so that it was 175 times 1.732 so that the size of the river width was approximately 303.1. From the explanation, a flow diagram of S2's cognitive processes was obtained in the second problem in the figure 15.



(1)

In English:



(2)

Figure 15. Cognitive Processes of Moderate Mathematical Ability Student

**The Cognitive Processes of Low Mathematical Ability Student**

The third subject was S3 in the category of low mathematical ability. The following was an analysis of the results of the work and interviews by S3 for the first and second Minimum Competency Assessment Numeracy about Geometry and Measurement problems.

The first numeracy task was not well understood by S3 so he needed to read over and over again the existing problems. From some understood information, S3 wrote down the known information based on what he understands. S3 explained that the meaning of  $s=11$  cm was the length of the paving side which was 11cm and  $t=6$ cm was the height of the paving which was 6cm. S3 also stated that the concept used in solving this problem was the 6-sided area formula and the new formula which

was less precise than his own estimation. Researchers were quite interested in the new formula expressed by S3.

**P:** what formula it is and how can you get that formula?

**S3:** it is a side formula. The formula for the area of the hexagon to find the number of paving by dividing the side formula by the formula for the area of the hexagon.

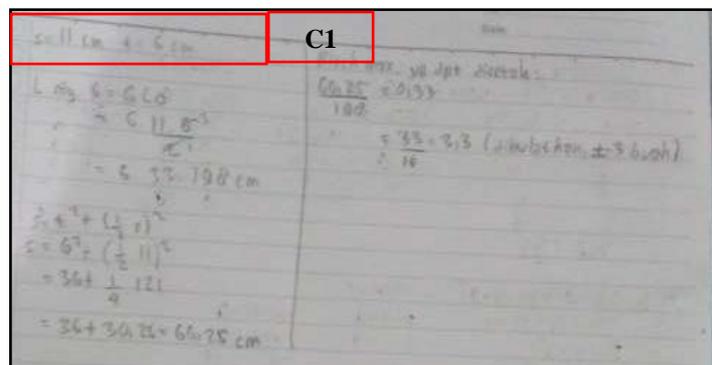


Figure 16. The result work of Low Mathematical Ability Student in first problem

From the results of planning and work by S3 there were many discrepancies and many inappropriate procedures that affect the calculations and results. Regarding this, there were many stages that could not be passed and resulted in S3 having to continue to return to the first stage, which was comprehending linguistic and numerical information in the problem stage.

In the second numeracy task that was solved by S3, he reiterated the contents of the problem quite well. S3 was also able to convey the concept that would be used to solve the problem, namely trigonometry comparison, but unfortunately S3 was not able to illustrate the problem in geometric form and S3 was not able to correctly execute the concept of trigonometry comparison.

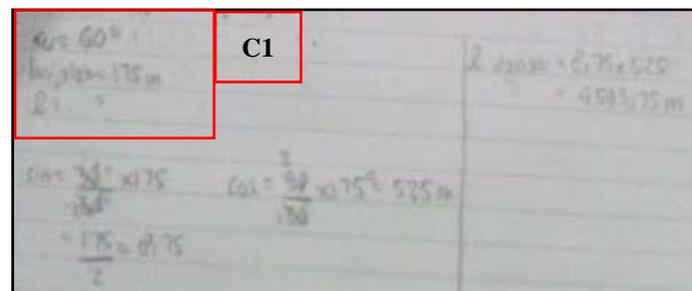


Figure 17. The result work of Low Mathematical Ability Student in second problem

From this S3 work, it shows that S3 had not been able to go through stages in the cognitive processes or S3 was stuck in many stages so S3 had to go back to comprehending linguistic and numerical information in the problem stage. Based on the explanation of the

cognitive processes of S3 in solving the first and second problems related to geometry and measurement, it could be shown in the flow chart in the figure 18.

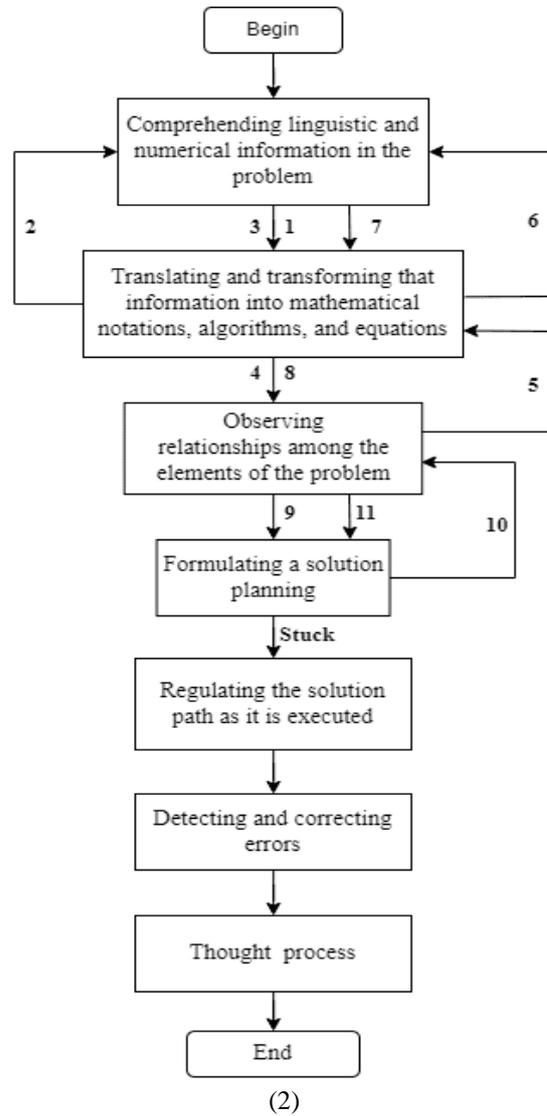
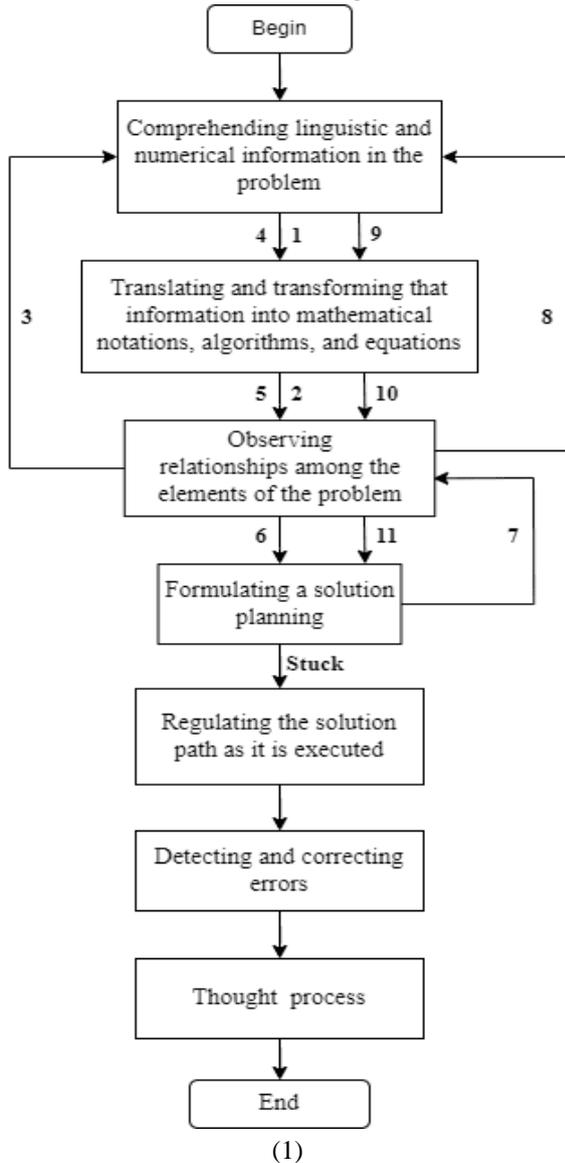


Figure 18. Cognitive Processes of Low Mathematical Ability Student

From the exposure of each stage of the cognitive processes of each subject, here was a comparison between the three subjects to the stage of the cognitive processes Table 3. Comparison between the cognitive processes of the subject

Stage of Cognitive Processes	Subject of Research		
	Subject 1	Subject 2	Subject 3
Comprehending linguistic and numerical information in the problem	Have abilities at this stage, able to understand and explain the contents of the problem	Sufficiently able to understand the problem even though there is some information that is	Not able to understand the problem completely and fully

		still not understood	
<i>Translating and transforming that information into mathematical notations, algorithms, and equations</i>	Able to transform known and understood information in mathematical form or in geometric illustrations. Able to analyze the right concepts that will be used in solving problems	Sufficiently able to translate information on the problem into mathematical forms, notations, and illustrations. There are still some inaccuracies in choosing mathematical concepts in solving problems	There is some information that can be changed in mathematical form, although not completely and in detail. The mathematical concepts that will be used are still not completely correct and there are wrong concepts.
<i>Observing relationships among the elements of the problem</i>	Able to link the information on the problem with the concept used	There is still some confusion in determining the existing information with the concept to be used	Not able in using information on problems and use of concepts so it is necessary to repeat in understanding the problems and concepts that will be used.
<i>Formulating a plan to solve the problem</i>	The planning and completion steps offered are appropriate and logical	There are still some planning steps that do not have a logical basis even though	The planning that will be carried out is not appropriate because the subject has not been

		they are appropriate	able to understand the problem and use the concept in drafting a solution
<i>Predicting outcome</i>	The subject stated that he did not have a predictable answer or solution to the problem	Subjects pass through this stage of the processes because they do not think about predicting the solution	Do not go through this processes because the subject is still not able to complete the previous stage
<i>Regulating the solution path as it is executed</i>	In the execution of planning and calculations carried out according to the procedure and is quite appropriate	Calculations are carried out according to the plan even though there are some calculations that are not correct	There are many errors and inaccuracies at this stage because they have not been able to optimally go through the previous stage
<i>Detecting and correcting errors during problem solution</i>	The subject realizes the illogical reason for the conclusion he came to and corrects it	Subject realizes and corrects inaccurate calculations	The subject still continues to repeat at the stage of understanding and using the right concepts in solving problems

From the results of the comparison of the cognitive processes of the subject's activities, in this study the students' cognitive processes were carried out using problems based on Minimum Competency Assessment

Numeracy on geometry and measurement material, resulting in students' cognitive stages that were quite optimal from high, medium, and low level students. The novelty that could be shown from previous research from Ekawati's (2019) study was the discussion of problems that were more directed at the preparation of national assessments and were related to personal contexts.

The novelty that can be shown from previous research from the Ekawati's (2019) study is the discussion of issues that are more directed to the preparation of national assessments and are related to personal contexts. In the numeracy task problem, students with high abilities are able to solve problems through the cognitive process stage starting with a fairly good understanding of the problem, this is reinforced by Stillman's (1996) statement which hypothesizes that the factors that influence the success of the solution are unsatisfactory understanding skills, lack of understanding of mathematical concepts and inhibited impulsive responses to problems. However, a good understanding of the problem is not enough to show that students with high abilities are able to give conclusions based on problems such as the rounding process in the first numeracy task conclusion that is still based on concepts and not on the answers to problems. In addition, in the conclusion section of the second numeracy task, it is about the form of root numbers that are less common for the community when used in measurements so that the conclusions drawn are still based on concepts and not based on answers to problems. Another novelty for students with moderate mathematical abilities, there are still many obstacles in finding the relationship between the concepts that he has planned and the execution of problems such as in the first numeracy task, the subject is still confused in using the concept of volume on a hexagon prism with a comparison of ingredients to determine volume. water on 1 piece of paving. Then in the second part of the numeracy task, the subject was still confused in using the illustration of a right triangle on which parts to interpret the width of the river and the distance Arif walked. Meanwhile, students with low abilities are very less able to understand information on problems which then cause a lot of being stuck on the stages of cognitive processes, such as in the first numeracy task, the subject is only able to write down some known information such as the side length and height of the paving size, but not have the appropriate idea to use the concept of volume of a regular hexagon prism. Likewise in numeracy task 2, the subject was only able to rewrite information about the distance Arif walked and the angle of elevation. The concept ideas used are also not in accordance with the concept of trigonometry.

From the results of the stages of work done by the three students, it was found that in the problem solving

process one can go through the stages of the cognitive process in a coherent manner but can also return to the previous stage to determine the right solution and calculation or even stop at a certain stage and roll back to the previous stage, this is in line with the statement that breakers may follow a cyclical process in which the breaker moves back and forth, may get stuck and must take steps back along the way (Mason, 2015).

## CONCLUSION AND SUGGESTION

The results of this study indicate that students with high mathematical abilities overall able to go through coherently and completely the stages of cognitive processes. Subjects with mathematical abilities in this category are able to go through the stages of understanding problems, translating problems in mathematical form, observing the relationship between information and mathematical concepts used, so that they can smoothly execute plans for problem solutions. Meanwhile, students with moderate mathematical abilities can go through the problem understanding stage, translate problem information, state the concepts, even though they are at the stage of linking information with concepts that are still not able so they need to repeat at the previous stage, but the subject can realize immediately at the stage of linkage between concepts and information so that the subject can continue at the planning execution stage, although there are still some calculations that need justification. For subjects with low category mathematical abilities, they are only able to go through the stage of understanding information and slightly able to translate information in mathematical form, but overall this subject is still not able to go through the stages of cognitive processes, there are still many errors and need guidance in the processes. From the results of this study, a learning processes and practice of non-routine problems related to geometry and measurement materials is needed which can later be used as initial preparation for national assessments. It is hoped that learning for the preparation of the Minimum Competency Assessment for Geometry and Measurement material can be emphasized more and so that the understanding of information on students' problems can continue the completion stage according to the solution plan that they have made.

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