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Horizontal and Vertical Mathematization Processes of Junior High School Students in Solving Open-Ended Problems

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Mathematization is converting information from problems into mathematical models. The mathematization process is divided into horizontal and vertical mathematization. This descriptive qualitative research aimed to describe junior high school students' horizontal and vertical mathematization process in solving open-ended problems. The subjects are three students with good, medium, and poor mathematical problem-solving abilities. The instruments used were interview guidelines, mathematical problem-solving ability tests, and open-ended problem tests with topics area and perimeter of rectangles and circles. This research shows the horizontal and vertical mathematization process in solving open-ended problems. The horizontal mathematization process was; identifying the information and topics area and perimeter from the situation; representing the problem into some rectangle and circle figures and expressing the problem in the subject's own words; writing the mathematics language; finding the regularity of the relations to find the possible solutions; and making mathematical models. The vertical mathematization process was; using mathematical representations with symbols and formulas related to the area and perimeter of rectangles and circles; using formal algorithms; customizing and combining some models to get the correct answers; making logical arguments to support the solution and other possible solutions that suit the problem; and generalizing the solution using the concepts of area and perimeter of rectangles and circles to solve similar problems. Every student may have different strategies and solutions when solving open-ended problems. The results of this research can be used as input for future research related to the mathematization process in solving open-ended problems.

INTRODUCTION

Mathematics is related to human activities and things in the world. Freudenthal (1991) stated that learners should experience mathematics as human activity and must relate mathematics to reality. Therefore, learning mathematics is not only about calculating numbers but also must be related to real life. In Indonesia, mathematics is one of the compulsory subjects in formal schools, from Elementary School (*Sekolah Dasar*), Junior High School (*Sekolah Menengah Pertama*), until Senior High School (*Sekolah Menengah Atas*). The Regulation of the Ministry of Education and Culture of the Republic of Indonesia (*Peraturan Kementerian Pendidikan dan Kebudayaan Republik Indonesia*), number 35 in 2018, stated that learning mathematics aims to develop students' competencies as their basic ability in real-life. That statement indicates that learning mathematics must be connected to real-life situations. Therefore, students can understand mathematical concepts and relate them to real situations and vice versa.

Linking real-life phenomena to mathematical concepts is challenging for students. According to de Lange (2006), there are several steps that students need to take to solve contextual problems from real life using mathematical concepts, starting from understanding problems, making mathematical models, solving them with mathematical concepts, then interpreting them. Converting information from problems into mathematical models is a process of mathematization or mathematizing. According to Jupri & Drijvers (2016:2483), mathematization organizes and studies real-life phenomena with mathematics concepts. Based on that opinion, it can be explained that mathematization is organizing and studying mathematical ideas from real-life problems, such as transferring a realistic problem into a mathematical problem using mathematical symbols and vice versa. Real-life problems in mathematization can refer to phenomena from reality, imagination, or to mathematical situations that were meaningful and imaginable to the student because they already experienced or understood those situations before (Van den Heuvel-Panhuizen & Drijvers, 2013). Students need to transfer the information from the problem into mathematics symbols and then make mathematical models and solve it using mathematics. In line with Sunardi et al. (2014:3) opinion, a mathematical model is a translation of problems/questions into a mathematical language (mathematical form) to make it simpler and easier to understand.

Treffers and Goffree (1987) divided mathematization into horizontal and vertical components. Based on the opinion of Treffers and Goffree, there are two kinds of mathematization, and those are horizontal mathematization and vertical mathematization. Horizontal mathematization proceeds from everyday life to mathematical symbols by finding the regularities and relationships needed to identify information in everyday problems to mathematical symbols through schematization and visualization. De Lange (1987: 43) argues that horizontal mathematization transfers the problem to a mathematically stated problem to identify the specific mathematics in a general context by discovering regularities and relations, such as schematizing and visualizing. While vertical mathematization is a process that occurs within the mathematical system, for example: finding strategies to solve problems or applying formulas, or finding formulas. Vertical mathematization can be done after going through horizontal mathematization. De Lange (1987:43) also said that vertical mathematization is the mathematical processing and refurbishing of the mathematical models from real-world problems.

Every student has their mathematization ability. Many factors can cause it, including the student's mathematical problem-solving ability. Problem-solving is used to help students to find the relevance between mathematics and other subjects and reality. According to Branca (1980), problem-solving is a basic ability in learning mathematics, so teachers should give, train, and accustom problem-solving to students in learning. According to National Professional Certification Agency or *Badan Nasional Sertifikasi Profesi*, problem-solving is the focus of learning mathematics. Learning mathematics in schools requires prioritizing the problem-solving approach to learning (BSNP, 2006). There are some types of problems; closed problems with a single solution, open problems with several solutions, and problems with various ways to find the solutions (BSNP, 2006). Therefore, learning mathematics also need to discuss open problems or open-ended problems. A closed problem is a problem that has been properly and wholly structured so that it has only one correct answer and is carried out by routine procedure. In contrast, an open-ended problem is a problem that is incompletely structured so that there are no fixed steps to ensure that the answer is correct (Foong, 2009).

Open-ended problems can improve High Order Thinking Skills because they can train creative thinking skills. However, Shadiq (2004) said that most mathematics learning nowadays focuses on procedural abilities, one-way communication, low-order thinking skills, routine questions, and low-level problems. Only several teachers have already used 'high-level ability' questions that required problem-solving or critical thinking. O'Neil & Brown (1998) and Shepard (1995) have stated the importance of open-ended mathematics problems. One is to refuse the stereotype that mathematics problems have only one solution. Problems of this type also encourage students to work on the same problem at different levels; some may have one solution, some will find several solutions, and others may find all possible solutions. However, most students still have difficulties solving open-ended mathematical problems. Purnamasari's research (2015) states that in solving an open-ended question, students can only reach the stage of understanding the problem, but students leave the following steps. This is also supported by the study of Darojat and Kartono (2016: 1) that in solving open-ended problems for junior high school students, students with poor problem-solving ability can only solve problems up to the stage of understanding the problem, students with medium problem-solving ability can solve problems up to the re-examining stage but are less thorough in carrying out the solving plan, and for high-ability students can solve problems up to the re-examining stage and are more detailed in implementing the problem-solving plan. Based on this, it can be seen that the ability to solve mathematical problems in solving open-ended questions for junior high school students is still low.

Some research examined the mathematization process carried out by students. The study by Jupri and Drijvers (2016), which examines the students' difficulties in the mathematization process in solving word problems in algebra, shows that formulating a mathematical model is the main difficulty. This highlights the importance of mathematization as an essential process in learning mathematics. Another research by Putri (2018) examines the horizontal and vertical mathematizing carried out by students in solving linear program questions. All of those research brings about the question of how are horizontal and vertical mathematizing carried out by students in solving open-ended problems because there is still not much research that examines about mathematization process in solving open-ended problems. According to Amalina (2020) and Nabila (2020), one of the levels that can be used as research targets to solve open-ended problems is the junior high school (SMP) level. Based on Piaget's theory of cognitive development, Junior High School students are in the formal operational stage (11 years of age), so they can think abstractly. That is, they can solve problems even though there are no real objects and think of several alternative solutions to problems.

Based on the description above, this research aimed to describe Junior High School students' horizontal and vertical mathematization processes in solving an open-ended problem. The importance of this research is to highlight the use of open-ended contextual problems and the mathematization process in solving them because there is still no previous research that examines the process of mathematization in solving open-ended problems.

METHOD

This research was descriptive research with a qualitative approach. This research was conducted in SMPN 28 Surabaya. The researcher chose the subjects using purposive sampling; the result of VII-J students' mathematical problem-solving ability test in SMPN 28 Surabaya was used to select the subject. The subject would be categorized into three categories of mathematical problem-solving ability; good, medium, and poor. The researcher chose one subject with good communication skills from every category.

The mathematical problem-solving ability test consisted of problems with the topic of; the area of a circle, the perimeter of a circle, the area of a rectangle, and the perimeter of a rectangle. Then, the open-ended problem test would be given to the chosen subjects. The open-ended problem test consisted of contextual problems with the area and perimeter of circles and rectangles, used to describe the horizontal and vertical mathematization process in solving open-ended problems. After that, the subjects would be interviewed for information about the horizontal and vertical mathematization processes not written in the open-ended problem test results. Then, the open-ended problem test and interview results will be analyzed according to the mathematization indicators in the following table.

Table 1. The Mathematization Indicators

Activity	Indicators	Code
Horizontal Mathematization Process		
Identify mathematics concepts that relevant to the problem	Express the information relating to mathematical concepts relevant to the given problem	HM1
Represent problem in different ways	1) Make scheme, visualize problem into figures 2) Express the plan for solving the problem in their own words	HM2
Find the relations of problem language to mathematics symbols and language	Identify the sentences in the problem and the related mathematics symbols or words	HM3
Find the regularity of the relation and pattern related to the problem	Show the way to find the solution using figures or models as visualization and the explanation about the relations between the models and the information from the problem	HM4
Translate the problem into a mathematical model	Write the mathematical models from the problem	HM5
Vertical Mathematization Process		
Use different mathematics representations	1) Represent the problem in pre-formal representation with simple symbols and figures. 2) Represent the problem in formal representation, for example, with the area and perimeter of rectangles and circles	VM1
Use formal mathematics symbols, words, and processes	Solve the problem using mathematics symbols and words, and solve the algorithm	VM2

Customize, develop, and combine some mathematical models	1) Revise and customize the previous model in horizontal mathematization with the given problem. 2) Make other models for the problem	VM3
Make mathematical arguments	1) Give logical arguments to support the statement or reasons to show that the solution to the problem is correct. 2) Give logical arguments for other possible solutions to the problem	VM4
Generalize the solution	1) Use facts or ideas from the problem to make valid opinions in a different situation. 3) Make a general statement of the problem	VM5

These indicators were adapted from the steps that students must take in the horizontal and vertical activities of mathematization, according to de Lange (1987).

RESULT AND DISCUSSION

The mathematical problem-solving ability test was conducted in VII-J grade with 29 students. From the result of the mathematical problem-solving ability test and the teacher's consideration of their communication skills, the researcher chose one subject from every category of mathematical problem-solving ability. The code of every research subject showed in the table below.

Table 2. The Code of Research Subject

Subject	Score	Predicate	Code
FDBS	100	Good	S1
VCE	60	Medium	S2
EOP	14	Poor	S3

The chosen subjects would do the open-ended problem test and interview to examine their mathematization process. The horizontal and vertical mathematization processes carried out by research subjects are described based on de Lange's mathematization indicators in Table 1.

Result and Data Analysis

The following are the horizontal and vertical mathematization processes in solving open-ended problems based on the test data analysis and the interview.

1. **Subject with Good Mathematical Problem-Solving Ability**

   S1's interview and test results are shown in the following.

Table 3. Interview Transcript of S1

Transcript	Indicator
R: What is the information from the problem?	
S1: About the formula of perimeter and area of rectangles and circles. So, Mia wants to make a pond and a garden. She has land with a 4x5 area, the pond will be a square, and the garden will be a half circle, then the edge will be given by coral, but only 15 meters of coral, and the left area will be planted with grass that costs 40,000 /m^2.	HM1
R: How is your plan to solve this problem?	
S1: I make the design, then I guess the sizes. I choose 3 m and 2 m. I think that it will suit Mia's wishes.	HM2 HM4
R: Are there any possible solutions for Mia?	
S1: Yes, there must be.	HM4
R: Does your design suit Mia's wish? Or is there a revision?	

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S1: It suits Mia's wish already. Maybe there would be a revision because I think my design is too small for the pond and garden.	VM3
R: Is your design correct already?	
S1: Yes, because there are some corals left.	VM4
R: Can you solve other similar problems? How?	
S1: Yes. First, I guess the possible measures and try them through trial and error to find the correct solution.	VM5

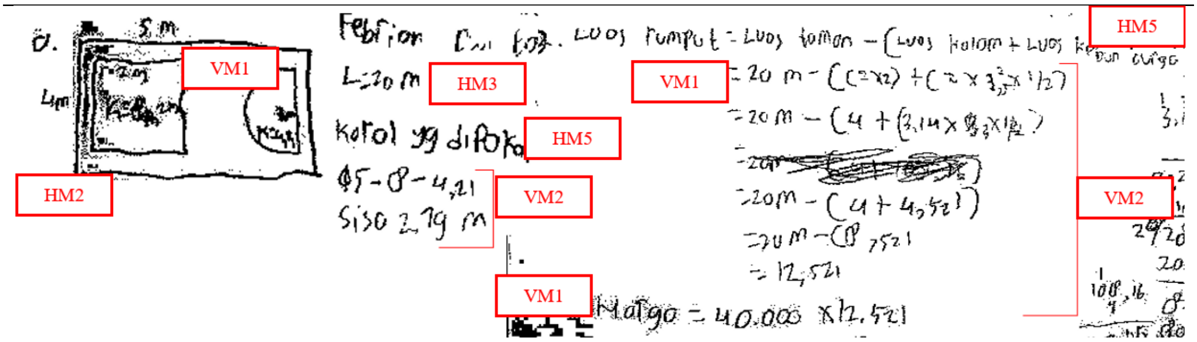

Figure 1. S1's Answer to Open-Ended Problem Test

From the interview transcript, S1 identified mathematics concepts relevant to the problem by identifying the information and the question asked in the problem about designing Mia's fish pond and flower garden so that her coral is enough to surround them. S1 can also define the mathematical concept related to the problem: the perimeter and area of rectangles and circles. In the next step, the problem was represented in different ways by visualizing the problem and expressing the plan for solving the problem. Then, S1 found the relation of problem language to mathematics symbols and language by symbolizing the area of Mia's land. S1 found the regularity of the connection and pattern by estimating the possible sizes to define the suitable design and also said there are some possible solutions for the design. The mathematical models S1 wrote are the design sketch with rectangles and semicircular shapes, the left coral, and the grass area. The illustration of the design already matched the information from the problem. In the answer, S1 used pre-formal representations of the situation through figures and symbols, such as K (*Keliling*/perimeter), L (*Luas*/area), r (radius), a semicircle shape for the flower garden, and two rectangle shapes for the land and the fish pond. S1 also used formal representation through the perimeter, area, and grass costs formula. S1 mostly did not write the formula of the area and perimeter using symbols but wrote it directly with the numbers and the result. The subject carried out the correct solving algorithm in the calculation but mostly did not write the symbols in the formula. Then, S1 customized and combined the design made in the horizontal process with the chosen size and the found perimeter, but S1 said that the design could be revised to make it better. Subject S1 did not change the design because it followed Mia's wishes. S1 could provide a logical argument to support the solution and give opinions for other possible answers. In the last step, S1 could solve a similar problem using ideas from the problem given before to make general statements.

2. Subject with Medium Mathematical Problem-Solving Ability

S2's interview and test results are shown in the following.

Table 4. Interview Transcript of S2

Transcript	Indicator
R: What is the information from the problem?	
S2: Mia wants to make a fish pond and flower garden, but she only has enough coral for 15 m. She wants a design with a rectangle for the pond and a semicircle for the garden.	HM1
R: How is your plan to solve this problem?	
S2: I guess the sizes are less than 15 meters for the coral; I will calculate the perimeter of the rectangle and semicircle.	HM2 HM4
R: Are there any possible solutions for Mia?	
S2: Yes, but this one is easier.	HM4
R: Does your design suit Mia's wish? Or is there a revision?	
S2: Yes, because already less than 15. So there is no revision.	VM3 VM4
R: Can you solve other similar problems? How?	
S2: Yes, I calculate the total perimeter and then the area.	VM5

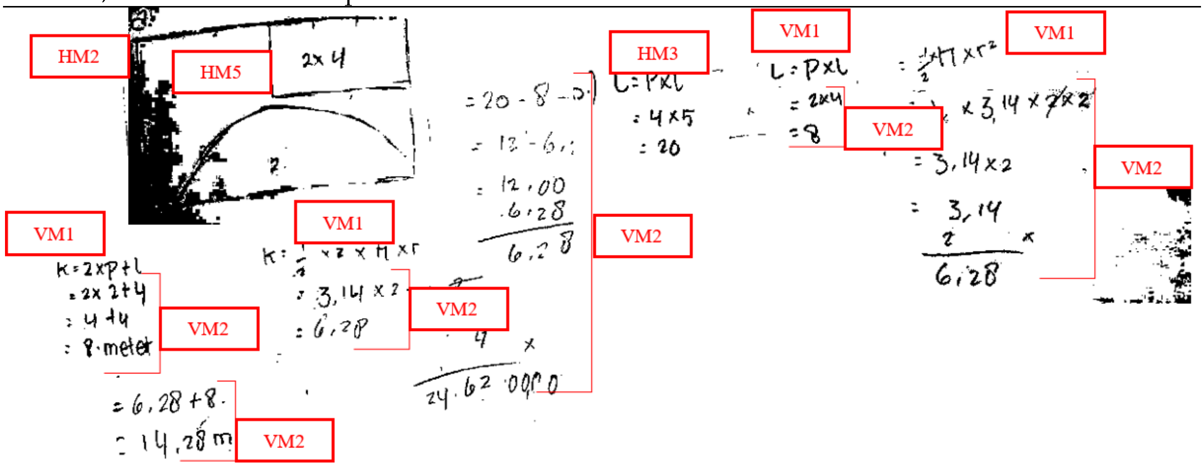

Figure 2. S2's Answer to Open-Ended Problem Test

From the interview transcript, S2 identified mathematics concepts relevant to the problem by identifying the information and the question asked in the problem about designing Mia's fish pond and flower garden with only 15 m corals. S2 can also define the mathematical concept related to the problem: the perimeter and area of rectangles and circles. In the next step, the problem was represented in different ways by visualizing the problem and expressing the plan for solving the problem. Then, S2 found the relation of problem language to mathematics symbols and language by symbolizing the area of Mia's land. S2 found the regularity of the connection and pattern by estimating the possible sizes to define the suitable design and also said there are some possible solutions for the design. S2 only wrote the model for the design with rectangles and semicircular shapes but did not write the left coral and the grass area. The sketch of the design already matched the information from the problem. In the answer, S2 used pre-formal representations of the situation through figures and symbols, such as K (*Keliling*/perimeter), L (*Luas*/area), r (radius), a semicircle shape for the flower garden, and two rectangle shapes for the land and the

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fish pond. S2 also used formal representation through the perimeter, area, and grass costs formula, but S2 wrote the wrong formula for the perimeter. The subject did some calculations incorrectly, so the solving algorithm was incomplete. Then, S2 customized and combined the design made in the horizontal process with the chosen size and the found perimeter. S2 did not revise even though the design has not followed Mia's wishes. S2 could provide a logical argument to support the solution and give opinions for other possible answers. The last step, S2, could solve a similar problem using ideas from the problem given before to make general statements.

3. Subject with Poor Mathematical Problem-Solving Ability

S3's interview and test results are shown in the following.

Table 5. Interview Transcript of S3

Transcript	Indicator
R: What is the information from the problem?	
S3: Mia wants to make a rectangular fish pond and a semicircular flower garden on her 4x5 land. She also wants to put coral on the edge, but only enough for 15 m.	HM1
R: How do you make your design?	
S3: Using a ruler.	
R: I mean, how is your plan to solve this problem?	
S3: I calculate.	HM2 HM4
R: Are there any possible solutions for Mia?	
S3: Yes….maybe… (*hestitated*)	HM4
R: Does your design suit Mia's wish? Or is there a revision?	
S3: Yes… yes, because it is correct already (*confused*)	VM3 VM4
R: Can you solve other similar problems? How?	
S3: Yes, by calculating the perimeters and then adding them.	VM5

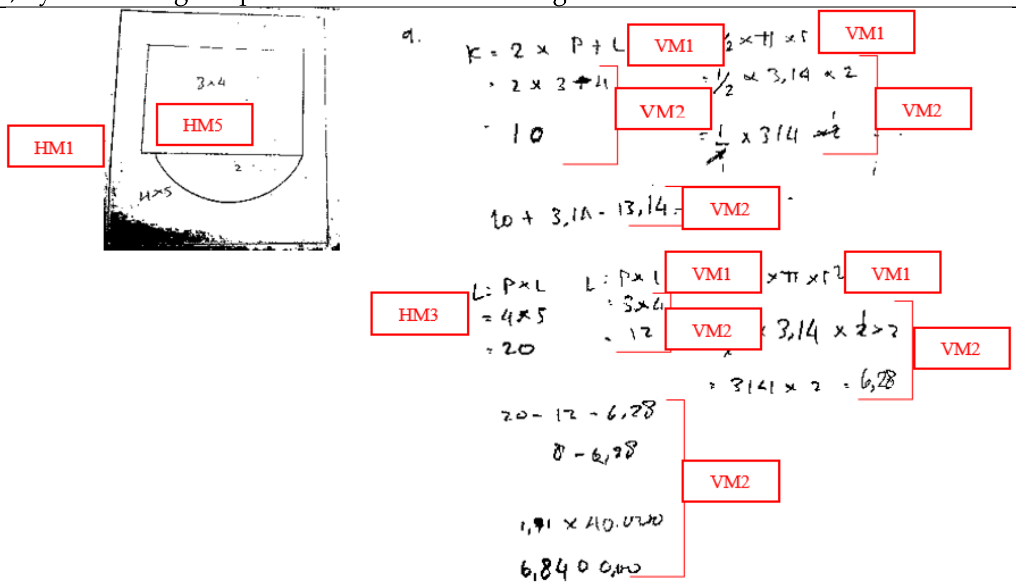

Figure 3. S3's Answer to Open-Ended Problem Test

From the interview transcript, S3 identified mathematics concepts relevant to the problem by identifying the information and the question asked in the problem about designing Mia's fish pond and flower garden on her land with only 15 m corals for the edge. S3 can also define the mathematical concept related to the problem: the perimeter and area of rectangles and circles. In the next step, the problem was represented by visualizing the problem, but S3 could not express the plan for solving the problem. Then, S3 found the relation of problem language to mathematics symbols and language by symbolizing the area of Mia's land. S3 knew the concept related to the problem but could not find the regularities; even though she did estimate the possible sizes, S3 also hesitated to say that there were some possible solutions for the design. S3 only wrote the model for the design with rectangles and semicircular shapes but did not write the left coral and the grass area. The sketch of the design already matched the information from the problem. In the answer, S3 used pre-formal representations of the situation through figures and symbols, such as K (*Keliling*/perimeter), L (*Luas*/area), r (radius), a semicircle shape for the flower garden, and two rectangle shapes for the land and the fish pond. S3 also used formal representation through the perimeter, area, and grass costs formula, but S3 wrote the wrong formula for the perimeter. The subject did some calculations incorrectly, so the solving algorithm was incomplete. Then, S3 did not customize and combine the design made in the horizontal process with the chosen size and the found perimeter and did not revise the design. S3 could not provide a logical argument to support the solution and give opinions for other possible answers. The last step, S3, could solve a similar problem using ideas from the problem given before to make general statements.

Discussion

According to data analysis from the open-ended problem test and interview result, the subject's horizontal and vertical mathematization process in solving open-ended problems can be shown as follows.

a. Horizontal Mathematization Process

All subjects identified information from the problem and expressed it in their language. They also identified questions asked in the situation and defined some mathematics concepts related to the problem. In line with research by Hayat and Yusuf (2011), students must be able to associate their mathematical knowledge with situations or problems encountered in everyday life. This follows de Lange's opinion (1987); in the horizontal mathematization process, students will express the information about mathematical concepts relevant to the given situation according to their knowledge.

All subjects sketched a figure with some shapes to visualize the problem (visualizing). Subjects S1 and S2 also expressed the plan for solving the problem in their own words (formulating). In solving problems, subjects S1 and S2 could represent the problems into figures and symbols and restate the problem through the information provided and questions asked to solve the problems. However, subject S3 could not express the plan for solving the problems in her own words (formulating). Therefore subject S3 did not finish this step. Also, research by

Purnamasari (2015) showed that in solving an open-ended question, students could only reach the stage of understanding the problem, but students leave the following steps. According to Farahhadi and Wardono (2019), solving problems in designing mathematical models requires a mathematical representation to communicate certain ideas in the problem so that the right solution is obtained from the situation. This follows de Lange’s opinion (1987); in the horizontal mathematization process, students need to visualize the problem into figures (visualizing) and then express the situation in their own words in solving the plan (formulating).

All subjects found the relation of problem language to mathematics symbols and language by sketching the right shape for the design and symbolizing the sentence in the problem into a mathematical sentence. According to Laily (2014), students develop the ability to communicate using numbers and symbols and reasoning to help solve everyday life problems. This follows de Lange’s opinion (1987); in the horizontal mathematization process, students need to identify the sentences and the related mathematics symbols or words to make the problem easier to understand mathematically.

Subjects S1 and S2 found the regularity of the relation in the problem to find the solution by expressing the strategies based on the figure; they also explained the possible design according to the information from the situation. However, subject S3 could not find the regularity of the relation in the problem to find the solution. S3 could not explain the possible design according to the information from the situation. In line with Gunawan and Putra (2019), students learn to find how to solve the problem to get the solution, look for relationships, analyze patterns, and find out which methods are appropriate and which are not appropriate. This follows de Lange’s opinion (1987); in the horizontal mathematization process, students show how to find the solution using figures or models as visualization and explain the relations between the models and the information from the problem.

Subject S1 wrote some mathematical models based on the relations before. S1 could use related concepts, figures, and symbols to model the problem. While subjects S2 and S3 wrote only one mathematical model based on the relations before, subjects S2 and S3 only made the design but did not write other models for other relations. Mathematical models are created to solve real-life problems using mathematics (Pitriani, 2016). This follows de Lange’s opinion (1987); in the horizontal mathematization process, students write the mathematical models from the problem based on those relations after identifying the regularities and relations.

Based on the discussion of the horizontal process carried out by the subjects with good, medium, and poor mathematical problem-solving ability, subject S1 with good ability did all horizontal activities; S1 identified mathematics concept in a general context, formulated and visualized a problem in several ways, discovered relations and regularities, recognized isomorphic aspects in some problems, then converted a real-world problem to a mathematical problem and a known mathematical model (de Lange, 1987). Subject S2, with medium ability, had done some horizontal activities; S2 identified mathematics concepts in a general context, formulated and visualized a problem in several ways, discovered relations and regularities, then recognized isomorphic aspects in some problems, but did not finish the last step in transferring a real-world problem to a mathematical problem and a known mathematical model. Subject S3, with poor ability, had done only some stages of the horizontal activities, identifying the specific mathematics in a general context, visualizing a problem, and discovering relations, but did not finish other steps in formulating a problem in different ways, discovering regularities, recognizing isomorphic aspects in further problems, and transferring a real-world problem to a mathematical problem and a known mathematical model.

b. Vertical Mathematization Process

All subjects used the pre-formal representations of the problem through figures and symbols. Subject S1 used the formal expression through the formula of perimeter and area of circles and rectangles. Subjects S2 and S3 also used the formal representation through the formula of area and perimeter, but they used the wrong formula for the perimeter. These representations can assist students in communicating thoughts and making mathematical ideas more concrete, so if the representation follows the problem given, a complicated problem will become simpler and vice versa (Miladiah, 2020). This follows de Lange's opinion (1987) in vertical mathematization; students represent the problem with simple symbols and figures in pre-formal representation. Then, describe the situation formally, for example, with rectangles and circles' area and perimeter.

In solving problems, subject S1 calculated the chosen design and sizes and used algorithms based on the formula of perimeter and area of circles and rectangles. However, subjects S2 and S3 incorrectly calculated and used incomplete algorithms to solve the problems. An algorithm as a form of application of concepts studied and solving problems emphasizes limited procedures and algorithms (Mulyati, 2017). Subject S1 only used some symbols in the calculations, some calculations did not even use symbols or formulas, but subject S1 could still calculate the desired area and perimeter exactly. This is in line with Lestari's statement (2015) that students can remember and apply symbols or formulas in simple calculations and understand concepts separately so that they can do calculations. According to de Lange's opinion (1987), in vertical mathematization, students solve the problem using mathematics symbols and words and solve the algorithm.

All subjects customized and combined the model made in the horizontal process based on the calculation in the previous step. Subject S1 did not revise the model because it had met the conditions required by the problems. Subjects S2 and S3 also did not change the model, but their solutions were wrong. Therefore, they did not finish this step correctly. This is de Lange's opinion (1987) about vertical mathematization; if needed, the previous model in horizontal mathematization will be revised and customized with the given problem to make other appropriate models.

Subjects S1 and S2 gave logical arguments to support the design and the solution and opinions on other possible solutions. In comparison, subject S3 could not provide logical arguments to support the design and the solution and gave views on other possible solutions, in line with Wetson (2007), who states that reasoning is the basis for preparing argumentative written discourse, which can be conveyed through arguments with examples. This is by de Lange (1987) about vertical mathematization; students need to provide logical arguments to support the statement or reasons to show that the answer to the problem is correct and give rational ideas for other possible solutions to the problem.

All subjects would be able to solve a similar problem to the problems given; they used ideas from the situation presented to make a valid opinion and general statement about how to solve other similar problems. According to Hermanto (2011), generalization is a process of concluding, starting by examining specific situations towards general conclusions. This is in de Lange's opinion (1987) about vertical mathematization; facts or ideas from the problem will be used to make valid opinions in a different situation and a general statement of the problem in the broader case.

Based on the discussion of the vertical process carried out by the subjects with good, medium, and poor mathematical problem-solving ability, subject S1 with good ability did all vertical activities, S1 represented a relation in a formula, proved regularities, refined and adjusted models, used different models, combined and integrated models, formulated a new mathematical concept, and generalized (de Lange, 1987). Subject S2, with the medium ability, only had done some steps in the vertical activities, S2 combined and integrated models, formulated a new mathematical concept, then generalized but did not finish the steps to represent a relation in a formula, prove regularities, refine and adjust models, and use different models. Subject S3, with poor ability, only did the last step in the vertical activities in generalizing. However, S3 did not finish other steps to represent a relation in a formula, prove regularities, refine and adjust models, use different models, combine and integrate models, and formulate a new mathematical concept.

CONCLUSION AND SUGGESTIONS

Conclusions

The horizontal mathematization process in solving an open-ended problem topic area and perimeter starts with identifying the information and the question asked from the problem and identifying the mathematics concepts related to the problem, which are the area and perimeter of rectangles and circles. Then, the subjects represent the problem into some rectangle and circle figures according to the problem (visualizing) and then express the problem in the subject's own words (formulating). After that, students write the mathematics symbols and language related to the information in the situation, such as the symbols of the perimeter, the area, the length and width of the rectangle, and the radius or diameter of the circle. Based on the mathematical symbols and language before, students find the regularity of the relations between the information in the problem to find the possible solutions according to the subject's creativity. Then the last step in horizontal activities is translating the situations in the problem into mathematical models; every subject may have different models according to their preference for the problem. The mathematical models in the horizontal mathematization process will be used in vertical mathematization.

The vertical mathematization process in solving an open-ended problem topics area and perimeter starts with using mathematical representations, such as; pre-formal representation using some rectangle and circle figures and symbols related to the figures, and formal representation using formal mathematical formulas for the area and perimeter of rectangle and circle. Then, students use formal mathematics symbols and algorithms according to the previous model to find the possible solution to the problem. The algorithms and the solutions of every subject may differ. Based on the earlier models in horizontal mathematization, students customize and combine some models to get the correct answers; if needed, the earlier models can be revised or developed and customized with the given problem to make other appropriate models that suit the problem. After that, students make logical arguments to support the solution and other possible solutions that fit the situation. Then the last step is generalizing the solution using the concepts of area and perimeter of rectangles and circles to solve similar problems. Every student may have different strategies and solutions when solving open-ended problems. Therefore, they may have various activities in their horizontal and vertical mathematization processes in solving open-ended problems.

Suggestions

Based on the results of this research, it is better for other reserachers to emphasize mathematics learning based on problem-solving. Moreover, teachers should use contextual problems, especially open-ended ones, to stimulate students to carry out horizontal and vertical mathematization processes in solving open-ended problems. They can also teach area and perimeter material (or other materials) by emphasizing concepts rather than just memorizing formulas so that students can work on problems even if they forget the formulas. Besides, students should be more accustomed to solving contextual questions, especially open-ended ones, so that students have more skills in processing problem-solving planning to solve non-routine questions. For future research, the researcher can choose subjects of the same gender to control the subjects in the subject selection.

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