

STUDENTS' MATHEMATICAL REASONING IN POSING MATHEMATICAL PROBLEM IN SEMI-STRUCTURED SITUATION**Ardian Dwi Kusuma Mukti**

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e-mail : tatagsiswono@unesa.ac.id**Abstract**

Mathematical reasoning is the process of thinking to obtain new conclusions by the facts that have been owned logically about and with mathematical objects which includes activities showing understanding, making and testing the conjecture, drawing logical conclusions, and providing evidence of the truth. Problem posing can use to identify students' mathematical reasoning. Mathematical problem posing is task that asks students' to make mathematical problem from given situation. The situation that given on problem posing in this research is semi-structured situation. The purposes of the research are describing students' mathematical reasoning in problem posing in semi-structured situation

The method of the research are qualitative descriptive. Subjects of the research is 2 students of class VIII-E of SMP Negeri 22 Surabaya. Data collected by using mathematical problem posing test and interview. Mathematical problem posing test is using to know students' mathematical reasoning in problem posing and to choose subjects of research. Interview is using to show more about students' reasoning in mathematical problem posing.

The result of the research showed that in semi-structured situation students showed activities that may appear appropriate reasoning indicators for every step of posing a problem. Student in semi-structured problem posing, choose clear informations then modify the informations to pose problem.

Keywords: mathematical Reasoning, Problem Posing, Mathematics

BACKGROUND

Humans are given reason to think. Siswono (2008) states that thinking is a mental activity experienced by a person if they face a problem or situation to be solved. Keraf (1982: 5) describes reasoning as the process of thinking that seeks to relate known facts or evidences to a conclusion.

Through reasoning, new insights or meanings arise from previously possessed knowledge (Fatima, 2008). Mathematical reasoning is one of the five standard capabilities students must have in mathematics learning set out in NCTM (2000), ie problem solving skills, reasoning ability, communication skills, ability to make connections, and representational capabilities. Thus, it can be said that reasoning becomes something important in Indonesian education.

Problem solving plays a role in the development of students' reasoning, because through the filing of problems students can develop reasoning. This is consistent with the statement of Mestre (in Christou, et. Al., 2005) states that using problem posing as a tool for studying cognitive processes, asserted that problem posing can be used to

Investigate the transfer of concepts across contexts, and to identify students' knowledge, reasoning, and conceptual development'. It is also reinforced by Cunningham (in Lavy & Shriki, 2007) which states that providing students with the opportunity to pose problems enhanced students' reasoning and reflection.

There are several definitions of reasoning according to experts such as Hardjosatoto and Asdi (Rokhman, 2016) provides a definition of reasoning is the process of the human mind that seeks to arrive at a new light of something or some other information that is already known and a description of the new must necessarily constitute a sequence continuation of Something or some of the original information. While Copi (in Shadiq, 2007) states that reasoning is a special kind of thinking in which the inference takes place, in which conclusions are drawn from premise.

Based on the description of some expert opinions can be concluded that reasoning is the process of thinking to derive new conclusion through facts that have been owned logically.

The statement of mathematical reasoning is mentioned Sternberg (In English, 2004: 13), The traditional view of

mathematical reasoning as superior computational and analytical skill has been revised to accommodate processes that are important in today's era. These include gathering evidence, analyzing data, making conjectures, constructing arguments, drawing and validating logical conclusion. Brodie (2010) states that mathematical reasoning is reasoning about and with the object of mathematics.

From the above description, it can be concluded that mathematical reasoning is the process of thinking to derive new conclusions through facts that have been owned logically about and with mathematical objects.

Permendikbud no. 58 th. 2014 (Kemendikbud, 2014:352) mentions the student indicator has met the aspect reasoning assessment and the evidence is if the student is able to do:

1. Making a conjecture
2. Drawing conclusions from a statement
3. Showing understanding
4. Providing an alternative for an argument
5. Determining patterns on a mathematical phenomenon
6. Doing math manipulation
7. Giving reason or evidence to the truth
8. Giving the validity or truth of an argument with reasoning induction
9. Proving a formula with reasoning

Sumarmo (2010) provides an indicator of reasoning in mathematics as follows:

1. Drawing a logical conclusion
2. Using the explanation by using models, facts, properties, and relationships.
3. Estimating answers and process solutions
4. Using patterns and relationships to analyze mathematical situations
5. Interesting analogy and generalization
6. Making and testing a conjecture
7. Formulating the counter example
8. Following inverse rules
9. Checking the validity of the argument
10. Composing valid arguments
11. Developing direct proof, indirect proof and using mathematical induction

Based on the description that has been described, it can be concluded that mathematical reasoning is the process of thinking to derive new conclusions through facts that have been owned logically about and with mathematical objects that include activities showing understanding, making and testing the conjecture, drawing logical conclusions, and providing evidence of the truth.

Problem posing is one that is considered important in the learning of mathematics. This argument is reinforced

by the opinion of Polya (1957) which states that the mathematical experience of the student is incomplete if he has had the opportunity to solve a problem by himself. There are several definitions of problem posing. According to Stoyanova and Ellerton (1996) problem posing is a process that is the basis of mathematical experience, students construct their own interpretations of concrete situations and formulate them into meaningful math problems. According to Silver (1994), problem posing refers to both the generation of new problems and the re-formulation of given problem.

To propose a mathematical problem, Siswono (2016) outlines steps of posing problem :

1. Understanding the information provided
2. Preparing a problem posing plan
3. Posing problem
4. Checking questions, solutions strategies, and answers

Silver (1994) provides problem posing was applied to three different forms of mathematical cognitive activity:

1. presolution posing is posing may occur before the problem is resolved. Problems can be generated from information provided in the form of stories, drawings, diagrams, and more.
2. within-solution posing is Filing occurs when students solve complex problems. In order to solve complex problems, students pose simpler new problems that can lead to solving the complex problem.
3. post-solution posing is posing occurs when the student has solved the problem. Students pose a new problem similar to the problem that has been resolved.

This research uses presolution posing. The type of problem posing before the solution requires the student to make the problem according to the situation given by the teacher and solve it himself, so that the students will be more active in the learning.

Stoyanova and Ellerton (1996) define problem posing as a process based on mathematical experience, that is, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems. There are several situations in the problem posing according to Stoyanova and Ellerton (1996), including free, semi-structured, structured.

semi-structured problem posing situation, is tasks that require students to math problems from open situations and be invited to explore structures in the situation and equip them with the knowledge, skills, concepts, and relationships of the mathematical experience they have. Semi-structured situations can also give students a unfinished problems structure and ask them to describe what problems are made of the information. a unfinished

problem structure can be images, equations, calculations, or inequalities. Examples of semi structured problem posing situations, students are posing a problem from a mango and apple picture that is given

Example of problem that can be posed:

Rohim buys 2 kg of mangoes and 1 kg of apples at Rp15.000,00. While nizar buys 1 kg of mangoes and 2 kg of apples with at Rp18.000,00. What is the price of 1 kg of mangoes?

To know the reasoning of the students needed some reasoning indicator. The reasoning indicator used in this research is to show understanding, to propose and to test allegations, to manipulate mathematics, to draw logical conclusions, and to provide evidence of truth.

Hernadi (2008) states that all information collected is built a coherence to then formulate a conjecture. Conjecture is a true mathematical statement produced based on observations or explorations, experiments, but has not been formally validated (Rosnawati, 2009). The logical reason in this study means the basic evidence (information) used to reinforce a reasonable opinion (KBBI). While Keraf (2007) states that through argument someone tries to link the information in such a way that is able to show an opinion or something that is true or not.

The reasoning of mathematics in posing problems here is one's process when posing mathematical problems to derive new conclusions through logically-owned facts about and with mathematical objects. Activities that may arise in the rainfall activity of the corresponding problem indicators can be seen in every step

At step understanding information provided the activities that may appear is mentioning the information that has been provided, making and testing the conjecture that the information that is given can be used to create a problem, mentioning or writing the information that used to pose a problem, giving logical reasons for writing or mentioning the information used to raise a problem, using the information on the problem posed to solve the problem, making and testing the conjecture. The information on the problem is always enough to solve the problem.

At step preparing a problem posing plan the activities that may appear is demonstrating a problem posing plans and writing the selected information to pose the problem.

At step posing a problem proposing conjecture that there are other problem that can be posed from the given situation, testing conjecture that there are other problem that can be posed from the given situation, demonstrating a problem posing plan, writing the selected information to pose the problem.

At step checking questions, solution strategies, and answer the activities that may appear is checking or providing proof of the matter that is made in accordance with the request of the task along with logical reasons,

examining or providing evidence of the solution of the problem made with logical reasons, checking or membering evidence of steps in solving the problem is appropriate with logical reasons, drawing logical conclusions from the solution problem that is posed, giving the argument that the conclusion is taken is true

METHOD

The method of the research are qualitative descriptive. Subjects of the research is 6 students of class VIII-E of SMP Negeri 22 Surabaya. Data collected by using mathematical problem posing test and interview. Mathematical problem posing test is using to know students' mathematical reasoning in problem posing and to choose subjects of research. Interview is using to show more about students' reasoning in mathematical problem posing.

At instrument problem posing test student given semi structured situation. In semi structured situations students are asked to make a question of a given brochure picture. Then the question in test is

Create a problem with the solution based on the following brochure picture



RESULT AND DISCUSSION

Grade VIII-E students are given a test for posing problem. The results of the problem posing test were analyzed to categorize the results. The results of the problem posing test are categorized into students who posing the problem correctly and solving the problem correctly (BB), students who posing the problem correctly but solving incorrectly or students who posing the problem incorrectly but solve the problem correctly (BS / SB), and students who posing the problem incorrectly and solving the problem incorrectly (SS).

Based on the results of the problem posing test was given 26 (89.6%) of 29 students categorized BB and 3 (10.4%) of 29 students categorized BS / SB and 0 (0%) of

29 students categorized SS. Based on the results of the category selected 2 research subjects that appear is 1 subject category BB and 1 subject category BS / SB.

Research subjects given code ST1 for students who are categorized BB and ST2 for students categorized BS / SB. Here the results of problem posing analysis and interview analysis for subject ST1 and ST2

Students' Mathematical Reasoning In Posing Mathematical Problem In Semi-Structured Situation for category BB

1. Understanding the information provided

2) Ani ingin membeli 18 botol air mineral aqua 600 ml. Jika di dalam supermarket tersebut terdapat "brosur promo" yaitu "Beli 5 + Gratis 1" dengan harga per botol Rp. 2100. Berapa banyak uang yang harus dibayarkan Ani?

Jawab:

- * Beli 5 gratis 1 (harga per botol 2100)
- * $5 \times 2100 = 10.500$ mendapat 6 botol.
- Membeli 18 botol.
- Kelipatan 5 yang mendapat 18 yaitu 15.
- * $15 \times 2100 = 31.500$ mendapat 18 botol.
- karena yang 3 botol gratis.

Figure 1. The result of problem posing test from subject ST1

The subject of ST1 showed understanding by way of writing information on the problem, mentioning the information provided, mentioning the information used to pose the problem, and using the information on the problem that is posed to solve the problem. Subjects made conjecture that any information selected is enough used to solve the problem and then testing conjecture by using the selected information to solve the problem and making conjecture that the information provided may be used to posing the problem

2. Preparing a problem posing plan

The stage of preparing problem posing plan subject ST1 showed an understanding by explaining the plan to make the problem of looking for information that is clear, then modify it to be more difficult then the idea of posing problem obtained after seeing the price and in making the problem should not be difficult. This result is in accordance with Stoyanova and Ellerton (1996) opinion that in semi structured situations students should use the knowledge, skills, concepts, and relationships of their earlier mathematical experience.

3. Posing problem

Budi sedang pergi ke supermarket. Di sana ia mendapat brosur promo dari supermarket tersebut. Budi ingin membeli nugget Friesia karena ada promo yaitu "Uang Pembelian ke-2 Diskon 30%". Harga per bungkus yaitu Rp. 32.000. Budi ingin membeli dua bungkus. Berapa uang yang harus dibayarkan?

Jawab:

- * Bungkus pertama : Rp. 32.000
- * Bungkus kedua : $32.000 \times \frac{70}{100}$
- $= 9.600$

Jadi, uang yang harus dibayar Budi yaitu

Rp. $\begin{array}{r} 32.000 \\ + 9.600 \\ \hline 41.600 \end{array}$

$\begin{array}{r} 320 \\ + 960 \\ \hline 1280 \end{array}$

figure 2. The result of problem posing test for subject ST1 in interview

At posing a problem, subject ST1 made and tested conjecture that there are another problem that can be given. Subject ST1 showed understanding with explaining the plan to pose the problem that is still the same as before that is estimate the answer. For ideas is modifying the product with the product.

4. Checking questions, solutions strategies, and answers

At checking question, the solution strategy and the result, ST1 provide evidence of the truth that the tasks are in accordance with the task requests, provide evidence of the correctness of the steps of the problem, and provide evidence of the results. The subject of ST1 draws the conclusion that in making the question of the type of brochure such as the image given is to seek clear information and write the problem because it should be detailed with logical reasons because if done by others, others can understand about the problem.

Students' Mathematical Reasoning In Posing Mathematical Problem In Semi-Structured Situation for category BS/SB

1. Understanding the information provided

Pisang: Rp. 1.390,00 / Aqua: Rp. 2.000,00 beli 5 gratis 1

3) Ali ingin membeli 2 buah pisang dan 10 Aqua botol. berapa uang yang harus dia dibayarkan oleh Ali agar dapat membeli semua barang tersebut?

Jawab:

- $2 \times 1.390 = 2.780$
- $10 \times 2.000 = 21.000$
- 23.780 = uang yang harus dibayarkan Ali

figure 3. the result of problem posing test from subject ST2

The step understanding the information given subject ST2 showed understanding by way of mentioning and explaining the information provided. The subject ST2 mentioned the selected information to make the problem and gives a logical reason because the selected information already has its price. Subject ST2 described the information on the problem is made used to solve the problem. Subject ST2 made conjecture that the information provided can be used to create a problem. Subject ST2 made and tested conjecture that the information on the problem is enough to solve the problem.

2. Preparing a problem posing plan

At preparing a problem posing plan, subject ST2 showed an understanding of the problem plan. The plan to pose problem is the subject of ST2 is thought out how to make the answer reasonable, then make the problem according to the selected information and explained that the problem posed must accordance with general condition. problem posing idea that is thinkable from matter ever done then subject of ST2 in making problem need not always difficult, in accordance with existing in mind. This result is in accordance with Stoyanova and Ellerton (1996) opinion that in semi structured situations students should use the knowledge, skills, concepts, and relationships of their earlier mathematical experience.

3. Posing a problem

Ala membeli 2 buah pisang dan 2 buah air mineral seharga Rp. 6.980,
 Ani membeli 1 buah pisang dan 2 buah air mineral seharga Rp. 5.980.
 Jika Toni ingin membeli 3 buah pisang dan 1 buah air mineral
 maka uang yang harus dibayarkan seharga !

Jawab : x = pisang
 y = air mineral

$$\begin{aligned} \text{Ala} &: 2x + 2y = 6.980 \\ \text{Ani} &: x + 2y = 5.980 \end{aligned}$$

$$\begin{array}{r} 2x + 2y = 6.980 \\ - (x + 2y = 5.980) \\ \hline x = 1.390 \end{array}$$

$$\begin{aligned} y &= 1.390 + 2y = 5.980 \\ 2y &= 4.590 \\ y &= 2.295 \end{aligned}$$

Ala: $2x + 2y = 6.980$
 $2(1.390) + 2(2.295) = 6.980$
 $2.780 + 4.590 = 6.980$
 $7.370 = 6.980$
 Salah!

Ani: $x + 2y = 5.980$
 $1.390 + 2(2.295) = 5.980$
 $1.390 + 4.590 = 5.980$
 $5.980 = 5.980$
 Benar!

Jika Toni membeli 3 buah pisang dan 1 buah air mineral
 maka uang yang harus dibayarkan seharga !

$$3x + y = 3(1.390) + 2.295 = 4.170 + 2.295 = 6.465$$

Figure 4. the result of problem posing test from subject ST2 in interview

At step posing a problem, subject ST2 making and testing conjecture with explained that there were another problem that can pose. Subject ST2 showed understanding with explained the plan of pose problem that us thinkable the solution, before write the problem.

4. Checking questions, solutions strategies, and answers

At Checking questions, solutions strategies, and answers, subject ST2 can't providing evidence of the truth because subject ST2 wrong in calculating when subject ST2 solve the problem it make subject ST2 also can't providing evidence of the truth the solution. But, subject ST2 can providing evidence of the truth that the problem posed is true. Subject ST2 drawing logical conclusions that pose a problem and solve the problem need to think and must check the solution is right. Then subject ST2 Give a logical reason because although making your own problems and already understand how to solve it can still miscalculate.

CONCLUSION

Based on the result of this research, we have conclusions that students' mathematical reasoning in posing mathematical problem in semi-structured situation.

The stage of understanding the information provided, the students mentioned and explained the information provided, mentioned and wrote information on the problem made, made and tested the conjecture that the information given enough to solve the problem

At Preparing a problem posing plan, student explained plan pose problem is looking for information that clear and then modify

At step posing a problem, students made and tested the conjecture that there are another problems that can be made from the information provided and do the plan that is prepared.

At step checking questions, solution strategies, and solution. Students provide evidence of the truth of the posed problem is in accordance with the request, provide evidence of the correctness of the steps in solving the problems posed, provide evidence of the truth of solution and draw the conclusion that in making the problem type brochures need to find clear information with logical reasons that solver can understand the problem that is posed

SUGGESTION

Based on the result of this research, we have suggestion as follows.

1. Based on the results of research students show reasoning in the pose problem. This suggests that the problem-posing task can be used to identify students' reasoning.
2. To other researchers who will follow up this research should minimize the weaknesses, in order to obtain better and more accurate results by providing more in-depth interview questions. So it can further explore information about students' mathematical reasoning in posing the problem

REFERENCES

- Amalina, Ijtihadi Kamilia. 2016. *Investigasi Kemampuan Berpikir Kreatif Siswa dalam Pengajuan Masalah Matematika*. Skripsi tidak diterbitkan. Surabaya: Universitas Negeri Surabaya
- Barlow, Angela T., & Cates, Janie M. 2006. "The Impact of Problem Posing on Elementary Teachers' Beliefs about Mathematics and Mathematics Teaching". *School Science and Mathematics*. Vol 106 (2) : pp 64—73.
- Bonotto, Cinzia, & Sato, Lisa Dal. 2015. "On the Relationship Between Problem Posing, Problem Solving, and Creativity in the Primary School". Dalam Florence Mihaela Singer, Nerida F. Ellerton dan Jinfa Cai (Ed). 2015. *Mathematical Problem Posing : From Research to Effective Practice*. New York : Springer.
- Brodie, Karin. 2010. *Teaching Mathematical Reasoning in Secondary School Classroom*. New York: Springer.
- Christou, Constantinos, et.al. 2005. "An Empirical Taxonomy of Problem Posing". ZDM, (online), vol 37 (3), (<http://miwalab.cog.human.nagoya-u.ac.jp/database/paper/2006-12-19.pdf>, diakses pada 12 Januari 2017)
- English, Lyn D. 2004. *Mathematical and Analogical Reasoning of Young Learners*. London: Lawrence Erlbaum Associates, Publisher.
- Fatima, SK dan Rao, D. Bhaskara. 2008. *Reasoning Ability of Adolescent Students*. New Delhi: Discovery Publishing House PVT.LTD.
- Kemendikbud. 2014. *Permendikbud 58 Thn 2014 tentang Kurikulum 2013 SMP dan MTs*. Jakarta: Kementrian Pendidikan dan Kebudayaan
- Keraf, Gorys. 1982. *Eksposisi dan deskripsi*. Ende-Flores: Nusa Indah
- Keraf, Gorys. 2007. *Argumentasi dan Narasi*. Jakarta: PT Gramedia Pustaka Utama
- Lavy, I. & Shriki, A. 2007. *Problem posing as a means for developing mathematical knowledge of prospective teachers*. Makalah disajikan pada Proceedings of the 31th Conference of the International Group for the Psychology of Mathematics Education, di Oranim Academic College of Education pp 129 – 136.
- Leung, S. S., & Silver, E.A. 1997. "The Role of Task Format, Mathematics Knowledge, and Creative Thinking on Arithmetic Problem Posing of Prospective Elementary School Teachers". *Mathematics Education Research Journal*. Vol 9(1) : pp 5—24.
- National Council of Teachers of Mathematics. 2000 . *Principles and Standards for School Mathematics*. Reston, VA : NCTM.
- Polya, G., 1957. *How To Solve It*. Princeton : Princeton University Press.
- Shadiq, Fadjar. 2007. *Penalaran atau Reasoning. Perlu Dipelajari Para Siswa di Sekolah?*. Diakses tanggal 9 Januari 2017 dari: <http://prabu.telkom.us/2007/08/29/penalaran-atau-reasoning/>
- Silver, Edward A. 1994. "On Mathematical Problem Posing". *For the Learning of Mathematics*. Vol 14 (1): pp. 19-28.
- Silver, E. A. and Cai, J. 1996. "An analysis of Arithmetic Problem Posing by Middle School Students". *Journal for Research in Mathematics JSTOR*.
- Siswanti, Rina Elok. 2016. *Penalaran dalam Pemecahan Masalah Matematika ditinjau dari perbedaan Gender*. Skripsi tidak diterbitkan. Surabaya: Universitas Negeri Surabaya
- Siswono, Tatag Yuli Eko, & Budayasa, I Ketut. 2006. "Implementasi Teori tentang Tingkat Berpikir Kreatif dalam Matematika". Makalah disampaikan pada *Seminar Konferensi Nasional Matematika XIII dan Kongres Himpunan Mahasiswa Indonesia*, Semarang, 24-27 Juli 2006.
- Siswono, Tatag Yuli Eko. 2007. "Meningkatkan Kemampuan Berpikir Kreatif Siswa Melalui Pengajuan Masalah dan Pemecahan Masalah". Makalah disampaikan pada *Simposium Nasional Penelitian Pendidikan yang diselenggarakan oleh Pusat Studi Kebijakan Departemen Pendidikan Nasional*, Jakarta, 25-26 Juli 2007.
- Siswono, Tatag Yuli Eko. 2010. *Penelitian Pendidikan Matematika*. Surabaya: Unesa University Press.
- Siswono, Tatag Yuli Eko. 2016. "Diklat Guru Matematika SMP Se-Kabupaten Jember". Makalah disajikan dalam, Jember, 25 – 29 Januari 2016
- Stoyanova, Elena, & Ellerton, Nerida F., 1996. *A Framework for Research into Students' Problem Posing in School Mathematics*, (Online),
- Sumarmo, Utari. 2010. *Berfikir & Disposisi Matematika: Apa, Mengapa, dan Bagaimana Dikembangkan pada Peserta Didik*. Bandung: FPMIPA UPI.
- Suriasumantri, Jujun S. 2010. *Filsafat Ilmu (Sebuah Pengantar Populer)*. Jakarta: Pustaka Sinar Harapan