

## ANALYSIS OF STUDENTS' ERROR IN SOLVING HIGHER-ORDER THINKING PROBLEMS ON PROPORTION

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### Abstract

Higher-order thinking is approached as the “top end” of Bloom’s (or any other) taxonomy (Bookhart, 2010). In learning process, higher-order thinking problem is rarely used. Based on Anderson’s Taxonomy (Revised Bloom’s Taxonomy), higher-order thinking skills is in analyzing, evaluating and creating thinking process, such that not all students are able to solve higher-order thinking problems as they are having difficulties in the specific phases. Proportion problem is difficult for students, it can make students encounter errors. This research is aimed to describe students’ errors and the underline reasons that caused students’ error in solving higher-order thinking problems on proportion. The approach used in this research is a qualitative approach. Researcher uses qualitative data to describe analysis about errors committed by students of grade VIII Junior High School in solving higher-order thinking problem on proportion. This research was conducted in class VIII-D and VIII-L of SMP Negeri 2 Taman on second semester of academic year 2016/2017. Class VIII-D consists of 36 students and class VIII-L consists of 35 students. Researcher chose four students to be the research’s subjects. Four subjects consist of two students of VIII-D and two students of VIII-L. These four subjects were chosen to be interviewed according to three requirements which are students who commit many types of error in solving the problem of tests given, the variations section, type, and the underline reason of error, and openness and fluency in communicating. According to data analysis, there were five students’ errors in solving higher-order thinking problems on proportion which are, (1) reading error, (2) comprehension error, (3) transformation error, (4) process skill error, and (5) encoding error. Type of error students mostly commit is process skill error, which they made error in processing mathematical concept and arithmetical error. The underline reasons of this error are student uses correct concept but incorrect in processing it, uses wrong mathematical operation and not aware of making mistake in proportional concept.

**Keywords:** analysis of error, higher-order thinking problems, proportion.

### INTRODUCTION

Mathematical learning in junior high school encourage students to explore their knowledge from varies sources. In addition, mathematics learning process guides students to think logically and creatively, and it emphasizes on problem solving process. Problem solving is an activity that involves various actions in the mind of thought including accessing and using knowledge and experience (Lester & Kehle, 2003). Problem solving is used to solve higher-order thinking problems. A problem is a goal that cannot be met with a memorized solution. The broad definition of problem solving as the nonautomatic strategizing required for reaching a goal according to Nitko & Brookhart (2007) in Bookhart (2010). Mathematics problems in higher-order thinking as well one of them is reserved for non-routine problem (a matter which is not known the solution directly. Higher-order thinking is approached as the “top end” of Bloom’s (or any other) taxonomy: Analyze, Evaluate, and Create, or, in the older language, Analysis, Synthesis, and Evaluation by Anderson & Krathwohl (2001) in Bookhart (2010). In learning process, higher-order thinking problem is rarely

used. Teacher usually gives routine problems, which do not require student to analyze, evaluate, and create. Based on Anderson’s Taxonomy (Revised Bloom’s Taxonomy), higher-order thinking skills is in analyzing, evaluating and creating thinking process, such that not all students are able to solve higher-order thinking problems as they are having difficulties in the specific phases. One of underline reasons of students’ errors is understanding mathematics topic.

Junior high school students should master in essential subject in mathematics, if they want to solve higher-order thinking problem and avoid errors while solving it. One of essential subject in mathematics is proportion. Proportion is connected to other subject in mathematics. Such as: scale, similarity and congruence etc. “Extended research from as early as 1966 until now (Lunzer & Pumfrey 1966, Hart 1981, Hart 1984, Tourniaire & Pulos 1985, Singh 1998) in the field of proportional reasoning reveals that solving ratio and proportion problems is a very difficult task for most pupils in the middle school years throughout the world” (Misailidou, 2002). Proportion problem is difficult for students, it can make students encounter errors.

Analyzing students' error can be used a framework. This framework can be used for analyzing students' error in solving higher-order thinking problem.

**Table 1. Indicator of Students' Error Analysis on Proportion**

Error type	Sub-type
Reading	Misinterpreting key words in problem
Comprehension	Misunderstanding the instruction
	Error in selecting information
Transformation	Wrong mathematical concept
Process skill	Wrong in processing mathematical concept
	Arithmetical error
	Unfinished answer
Encoding	Error in encoding final answer
	Careless in writing the final answer

There are many factors that affect the process and results of the study which also affects the form of errors committed students in solving mathematics problems. Researcher uses opinion of Rokhimah et al (2015) and Newman (1983) as framework to analyze the underline reason of students' error in solving higher-order thinking problems on proportion. Framework of the underline reason in this research as follows.

**Table 2. The Underline Reason of Students' Error**

Types of Error	The Underline Reason
Reading	Subjects are not able to interpret the key words in question
	Subjects cannot understand vocabularies/key terms on proportion
Comprehension	Subjects cannot figure out the known thing in detail
	Subjects not identify what is known precisely, that is causing misinterpretation
	Subjects not read the questions carefully so that there is missing information
	Subjects are less of various exercises so that cannot directly be solving a different problem
	Subjects not understand the whole problems properly so that inconsistent in identifying what is known
	Subjects unable to explain the information in problems properly
Transformation	Subjects cannot plan the solution to solve the problem
	Subjects cannot determine the formula
	Subjects have not more exercises

Types of Error	The Underline Reason
	Subjects cannot determine the mathematical operations used
Process skills	Subjects use correct concept but incorrect in processing it
	Subjects not aware of making mistake in proportional concept
	Subjects cannot convert a unit into another unit
	Subjects use a correct procedure, but they did not finish it
	Subjects use wrong mathematical operation
Encoding	Subjects cannot give proper conclusion because there is an error in calculation result

The research objectives are (1) to describe students' errors in solving higher-order thinking problems on proportion, (2) to describe the underline reasons that caused students' error in solving higher-order thinking problems on proportion.

## METHOD

The approach used is a qualitative approach. Researcher uses qualitative data to describe analysis about errors committed by students of grade VIII Junior High School in solving higher-order thinking problem on proportion. The design of this research is complete it can be seen in Chart 1 below.

This research was undertaken in grade VIII-D and VIII-L SMP Negeri 2 Taman on second semester of academic year 2016/2017.

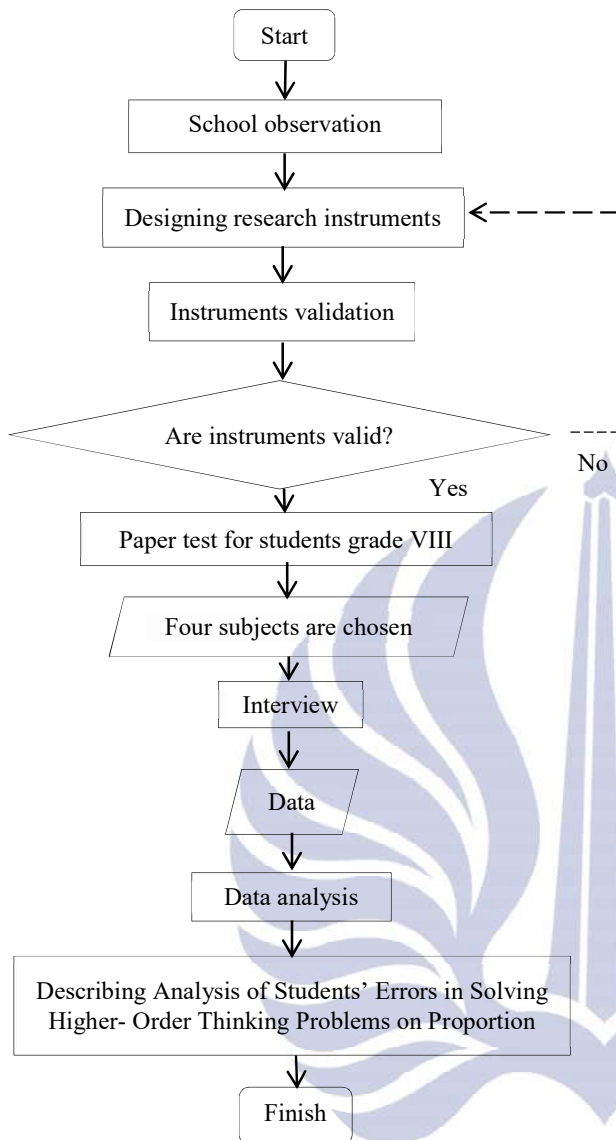
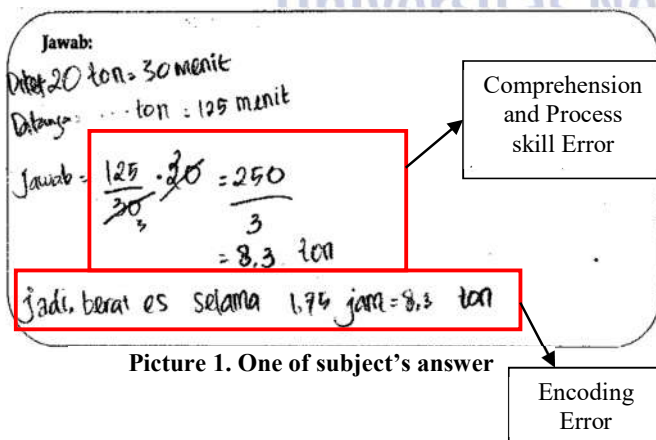


Chart 1. Research Design Chart

## RESULT AND DISCUSSION

The type of error and its underline reason are discussed as follows.



Picture 1. One of subject's answer

### 1. Type of errors

#### a. Reading error

Subject reads 1,75 hours for problem number 1 as 1 hours and 75 minutes or 135 minutes, such that subject commits error while converting the unit time. It means subject commits reading error which is misinterpreting key words in problem.

#### b. Comprehension error

Subject wrote the given and what question asked in problem number 1. Subject did not write all the given, only "20 ton = 30 menit". Subject's answer is 125 minutes, it is fine that subject used unwritten information in the given which is 10 minutes for machine warming up. Machine warm up for 10 minutes then it starts the production, if 20 ton of ice cube are produced in 30 minutes since the machine turn on thus the production period is only 20 minutes. But, subject did not use it while processing to get the final answer. It can be concluded that RS1 commits comprehension error which is error in selecting information.

#### c. Transformation error

Subject does not use proportion concept and in some cases subject guessed with trial and error strategy. According to indicator of students' error analysis, subject cannot be able to identify the proportion concept and operation should be used, thus subject commits transformation error.

#### d. Process skill error

Subject commits error while converting unit of time from problem number 1. Subject converts 1,75 hours into minutes but the answer is wrong. Subject converts 1,75 hours into 135 minutes. Subject thought that 1,75 hours be 1 hours (60 minutes) and 75 minutes, such that subject adds 60 minutes and 75 minutes into 135 minutes, that is incorrect. If 135 minutes convert into hour, it will be 2 hours 15 minutes. So, converting hour into minute by multiplying 60, because 1 hours equals 60 minutes such that 1,75 hours equals 105 minutes. It can be concluded that subject commits error in processing skill which is arithmetical error.

#### e. Encoding

Subject commits error in encoding final answer.

### 2. The Underline reasons

#### a) The underline reason of reading error

Subject misinterprets key word in number 1. According to indicator of students' error analysis, subject commits reading error which is misinterpreting key word in problem. the underline reason of this error is subject not able to interpret the key words in question



- b) The underline reason of comprehension error
  - 1) Subject is less of various exercises so that cannot directly be solving a different problem.
  - 2) Subject cannot figure out the known thing in detail.
  - 3) Subject does not read the questions carefully so that there is missing information.
  - 4) Subject does not identify what is known precisely, that is causing misinterpretation.
- c) The underline reason of transformation error
  - 1) Subject cannot plan the solution to solve the problem.
  - 2) Subject's confusion in using direct proportion or inverse proportion.
  - 3) Subject cannot determine the mathematical operations used
  - 4) Subject cannot distinguish between direct and inverse proportion.
  - 5) Subject makes mistake while understanding the problem.
- d) The underline reason of process skill error
  - 1) Subject cannot convert a unit into another unit.
  - 2) Subject is not aware of making mistake in converting unit.
  - 3) Subject uses correct concept but incorrect in processing it.
  - 4) Subject uses wrong mathematical operation.
  - 5) Subject is not aware of making mistake in proportional concept.
  - 6) Subject makes mistake while understanding the problem.
  - 7) Subject is careless in calculation.
  - 8) Subject is rushed in calculation.
- e) The underline reason of encoding error
  - 1) Subject cannot give appropriate conclusion because there is an error in calculation result.
  - 2) Subject is wrong because of the previous error.
  - 3) Error in encoding final answer.
  - 4) Subject is rushed in calculation.

### 3. Additional Result

According to explanation of term, the underline reason of students' errors in solving mathematics problems divides into two kinds, which are cognitive and non-cognitive. The underline reason in this research only involves non-cognitive. Researcher did not use cognitive underline reason to be part of research.

## CONCLUSION

According to the result of research data analysis and discussion,

1. Students' error in solving higher-order thinking problem on proportion as follows.
  - a. Encoding error is committed by most of students in solving higher-order thinking problems on proportion. Sub-type of encoding error which students committed in solving higher-order thinking problems on proportion is error in encoding final answer.
  - b. Process skill error is committed by students in solving higher-order thinking problems on proportion. Sub-types of process skill error which students committed in solving higher-order thinking problems on proportion are arithmetical error, wrong in processing mathematical concept, and wrong in processing the right answer.
  - c. Transformation error is committed by students in solving higher-order thinking problems on proportion. Sub-types of transformation error which students committed in solving higher-order thinking problems on proportion are wrong mathematical concept and wrong in planning the right solution.
  - d. Comprehension error is committed by students in solving higher-order thinking problems on proportion. Sub-types of comprehension error which students committed in solving higher-order thinking problems on proportion are error in selecting information and misunderstanding the instruction.
  - e. Reading error is committed by most of students in solving higher-order thinking problems on proportion. Sub-type of reading error which students committed in solving higher-order thinking problems on proportion is misinterpreting key words in problem.
2. The underline reason of each error which subject committed in solving higher-order thinking problem on proportion as follows.
  - a. The underline reasons of encoding error are subject cannot give appropriate conclusion because there is an error in calculation result, the answer is wrong because of the previous error, error in encoding final answer, subject is rushed in calculation.
  - b. The underline reasons of process skill error are student cannot convert a unit into another unit, not aware of making mistake in converting unit, uses correct concept but incorrect in processing it, uses wrong mathematical operation, not aware of making mistake in proportional concept, makes mistake while understanding the problem,

- careless in calculation, being rushed in calculation.
- c. The underline reasons of transformation error are student cannot plan the solution to solve the problem, student' confusion in using direct proportion or inverse proportion, cannot determine the mathematical operations used, cannot distinguish between direct and inverse proportion, makes mistake while understanding the problem.
  - d. The underline reasons of comprehension error are student less of various exercises so that cannot directly be solving a different problem, cannot figure out the known thing in detail, not read the questions carefully so that there is missing information, not identify what is known precisely such that causing misinterpretation.
  - e. The underline reason of reading error is subject not able to interpret the key words in question. Simpulan menyajikan ringkasan dari uraian mengenai hasil dan pembahasan, mengacu pada tujuan penelitian. Berdasarkan kedua hal tersebut dikembangkan pokok-pokok pikiran baru yang merupakan esensi dari temuan penelitian.

## RECOMMENDATION

According to the research committed, researcher's suggestion as follows.

### 1. For teacher

After reviewing the result of this research, researcher suggests to give students higher-order thinking problem in daily routine activity in class such that they can improve their higher-order thinking skill and decrease or minimize errors they possibly commit.

### 2. For other researchers

Researcher suggests to other researchers who are going to commit the similar research, to commit mathematical ability test before giving higher-order thinking problems, and use another indicators to complete the report

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