

STUDENTS' CREATIVITY WITH INFORMAL DEDUCTION OF VAN HIELE'S GEOMETRIC THINKING LEVEL

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Abstract

This paper aims to describe students' creativity and geometric thinking informal deduction level in solving geometric problem which involves two students at informal deduction level in 10 grade. This can be reference to the other researchers or educators in developing students' creativity in learning geometry. According to the research result, the two students at informal deduction level show different result. RS1 just can show fluency as creativity's component in solving geometric problem. Besides, RS2 can show the all of creativity's components in solving geometric problem. This shows us that even they are in a same level of geometric thinking, they also can be give different skill in solving geometric problems.

Keywords: Students' Creativity, Geometric Problem, Geometric Thinking Level

INTRODUCTION

Creativity is assumed as genius people owned. This is caused creativity occupies the highest level of thinking skill, then we call it Higher Order Thinking Skill (HOTS). Haris (in Khabibah, 2006) said Creativity is defined as an ability, ability to imagine, create a new thing, ability to construct new ideas that combines, change, re-explain the idea that has been exist. Discussing about ability, we have known already that ability can be changed or improved. This is clear for us that creativity cannot only be assumed as genius people owned. Creativity can be owned by everyone. In line with this, Suherman (2003) stated that 2 important things of mathematics learning goals are critical thinking and creative thinking development. Mathematics is taught to develop critical and creative thinking skills. This supports that creativity can be owned by everyone in the world.

There are three important components in investigating creativity itself, they are flexibility, fluency, and novelty (Silver, 1997). Fluency in problem solving refers to the number of ideas generated in response to a problem given by students, and Flexibility in problem solving to apparent shifts in approaches taken when generating responses to a problem. Whereas, Novelty in problem solving refers to the originality of the ideas generated in response to a problem.

In line with this fact of mathematics and creativity, we found the other facts about mathematics in education world. Based on PISA result (2016), geometry is one of difficult materials to solve by students who are in the average 15 years old. Moreover, in TIMSS study (2011), there is the fact about Indonesian students who are

in the lowest rank in some abilities (1) to understand complex information, (2) theory, analysis and problem solving, (3) tools using, procedure and problem solving, (4) do the investigation (Kemendikbud, 2012 : 9). Then, Wardhani and Rumiati (2011) analyze too about the result of TIMSS study and explain that 20% of Indonesian students can not answer correctly the one of problem solving question about concept of quadrilateral's perimeter. These two students assessment programs (PISA and TIMSS) have shown the result in understanding mathematics especially geometry is low. Besides, Clement and Sarama (in Siew, 2013) said that students have a problem in identification quadrilateral, followed by triangle then circle. Without any exception, space as the object of geometry can be the one of difficult materials for the students. The researcher has an experience in teaching geometry. The researcher's student cannot recognize what kind of space that student actually faced. It is when the student found a problem that asked student to find the volume of a bullet. The all of facts of students' problem in learning mathematics, especially geometry, show us that students' thinking skill, can be included creativity is low. This makes researcher is interested to explore students' creativity in solving geometry problems.

There are many ways to develop students' creativity, one of them is by learning geometry. However, within teaching and learning geometry, mathematics teacher should know and recognize the importance of what is known about the ways students learn mathematics (NCTM, 2007). As an additional, the teacher should create a learning environment that provides a context that is good for development of mathematical skill.

With regard to the learning of geometry, Van Hiele (in Walle, 2001) argued that students will pass through five geometric thinking level that are hierarchical. The five level here are visualization, analysis, informal deduction, deduction, and rigor. As he had stated that this level is based on the students' experience. Students at visualization level can use properties of shape as necessary conditions to determine a shape, for example guessing the shape in the mystery shape task after far too few clues, then it will be continued by the students at higher level like analysis level. Students at analysis level can describe the types of shape by explicit use of their properties, then students at informal deduction level are able to modify definitions and immediately accept and use definitions of the new concepts. This clearly say that the more students have experiences, the more students can be in higher level of geometric thinking level. Respectively, the more of higher students' geometric thinking level, the more creativity also that they can have. Based on the above explanation, the researcher want to explore the creativity of students at the highest level of geometric thinking in 10th grade by checking and identifying flexibility, fluency, and originality of high school students at visualization level, analysis level, and informal deduction level when they solve geometric problems.

METHODOLOGY

This research is qualitative research which involves two students as research subjects. They are students at informal deduction level of geometric thinking. This research goal was to describe students' creativity in solving geometric problem based on their geometric thinking level. This research aims to show the difference between creativity of two students at informal deduction level in solving geometric problem. 10th grader students in SMA Negeri 8 Surabaya are given van Hiele Geometric Thinking Level Test at first to investigating their geometric thinking level. After that students are given Geometry Problem Task to explore more their creativity in solving geometry problems. The two top score of students at informal deduction level is selected as research subject than will be interviewed to explore more about their creativity in solving geometric problem.

Geometry Problems Task

Problems used were taken from PISA items which consists of 2 numbers of problems. Those 2 problems are as follow:

1. In modern architecture, buildings often have unusual shapes. The picture below shows a computer model of a 'twisted building' and a plan of the ground floor. The compass points show the orientation of the building. The ground floor of the building contains the main entrance and has room for shops. Above the ground floor there are 20 storeys containing apartments. The plan of each storey is similar to the plan of the ground floor, but each has a slightly different orientation from the storey below. The cylinder contains the elevator shaft and a landing on each floor. (OECD, 2016) Estimate the total height

of the building, in metres. Explain how you found your answer.

2. Estimate the area of Antarctica using the map scale. Show your working out and explain how you made your estimate. (You can draw over the map if it helps you with your estimation) (OECD, 2016)

Interview

Subjects selected were interviewed to explore more about students' creativity in solving geometry problems. The questions of this interview is based on the result of students' geometric problem task. Based on interview data analysis of Miles and Huberman (1994), the result analysis was done by data reduction, data display, and drawing and verifying conclusion.

Based on Silver (1997), Fluency refers to the number of ideas generated in response to a problem. Ideas generated in response to a problem of thi case include giving many interpretations or solution methods or answers. Flexibility to apparent shifts in approaches taken when generating responses to a problem. Then, novelty refers to the originality of the ideas generated in response to a problem.

From this explanation, researcher does the research that refers to this three key components to analyze students' answers. Silver summarized the indicator of creativity in solving problem as follows :

Table 1. Indicators of Students' Creativity

Creativity	Indicators
Fluency	Students explore open – ended problems, with many interpretations, solution methods, or answers
Flexibility	Students solve (or express or justify) in one way, then in other ways. Students discuss many solution methods.
Novelty	Students examine many solution methods or answers (expressions or justifications); then generate another that is different

(Source: Silver, 1997)

The table below is the encoding of students' creativity in solving geometry problems used in this research :

Table 2. Encoding Indicators of Students' Creativity

Creativity Aspects	Indicators	Code
Fluency	Students can give more than two interpretations in exploring problem	FC.1
	Students can give more than one solution methods in exploring open – ended problems.	FC.2
	Students can give more than one answers in exploring open – ended problems.	FC.3

Flexibility	Students do shifting in approaches taken when generating responses to a problem.	FX.1
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Continue of Table 3. Encoding Indicators of Students' Creativity

Creativity Aspects	Indicators	Code
Novelty	Students examine two or more solution methods then generate another that is different.	N.1
	Students examine two or more answers then generate another that is different.	N.2

Result of Geometric Thinking Level Test

Two students with their optimal work in showing creativity are chosen for the subject. Not only their optimal work in showing creativity, but also the subjects are chosen based on their geometric thinking level which is at informal deduction level and their communication ability. From 35 students of 10th grader in SMA Negeri 8 Surabaya, there are 14 students at visualization level, 14 students at analysis level, and 4 students at informal deduction level. Whereas, the 3 remained students cannot be in this geometric thinking level at all. There is no students at level 3 (formal deduction) and level 4 (rigor).

Result of Geometric Problem Task

The maximal score that can be gotten by students is 50. Here is the research subjects who have been chosen to explore more about their creativity based on their each level of geometric thinking :

Table 4. Research Subjects

Name	Geometric Thinking Level	GPT Score	Subject's Initial
1. AA (Male)	Informal Deduction Level	20	RS1
2. SB (Female)	Informal Deduction Level	27.5	RS2

The discussion of the students' creativity difference based on their geometric thinking level is shown as follows :

1. RS1

RS1 had score 40 of Geometric Problem Task. This score is not too far from RS2's score. The difference is just 7.5. Based on RS1's answer sheet below, we can see that actually RS1 only wrote the answer without showing what is given and asked of the problem :

$$4 \times 21 = 84 \text{ m}$$

Figure 1. RS1's Answer for Problem 1

From RS1's answer we have already known that the estimation of building's height is correct. This actually can show us that RS1 can estimate the total height of building fluently. But this is not enough for us to state that RS1 had shown fluency component. Then, we explore more by interviewing RS1. Here is the interview transcript of RS1's answer for problem 1 :

Table 4. Interview Transcript of RS1's answer for Problem 1

Label	Conversation	Code
....		
R - 2	From problem 1, what are the informations you can get?	
RS1 - 2	Twisted building has rotated design drawn as the given picture, doesn't it? The ground floor has main entrance and stores. Each floors rotate with same rotation. There are 20 floors above the ground floor. So in total, there are 21 floors. Then, we ask to estimate the total height of the building.	FC.1
R - 3	From your explanation before, so how you can get this answer?	
RS - 3	From the asked, we should estimate, right. Then I assume the height of each floors is 4 m. directly, I multiply it by 21, since there are 20 floors above the ground floor. So, the total height of the building is 84 m.	
....		

For problem 1, RS1 can give more than 2 interpretations which are there is twisted building that have same rotation degree in each level of floors, the ground floor contains main entrance and stores, and this twisted building consists of 21 floors in total. RS1 also said that from this problem he should estimate the total height of this twisted building. This shows that RS1 have fluency component, especially FC.1.

Paying attention on RS1's solution method in answering this problem, RS1 only

used one method, which is Think it. There is no shifting on method in estimating the total height of twisted building. Since there is no more than one solution method, then RS1 cannot show the different method after examining two or more solution method. This shows that RS1 cannot fulfil flexibility and novelty component in solving this problem 1.

The consistence of RS1's solving geometry problem can be shown in his answer for problem 2. Clearly, we state that RS1 just can show fluency component. Here is RS1's answer for Problem 2 :

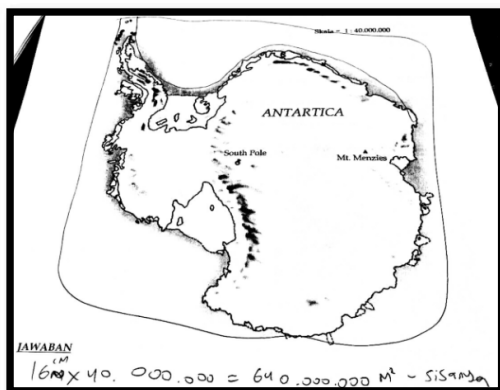


Figure 2. RS1's Answer for Problem 2

RS1 showed that the area of Antarctica in real can be found as above. RS1 made a mistake in finding the area of continent in map. RS1 stated that actually 16 cm meant to be the length of each continent's side in map. RS1 assumed that this continent in square shape, then RS1 forgot that he should find the square's area with length of 16 cm so that he can multiply this area with 40,000,000 directly based on the definition of scale. RS1 also stated that there's the one step left which is subtracting by the area in square that is. Eventhough actually RS1's explanation was close too correct, but RS1 still had a mistake in his way to find the area of Antarctica. RS1 didn't realize that actually the map scale shows the ratio of the length in map to the length in real. The creativity's component that RS1 can show in this problem is just fluency. The fullfilment of fluency component can be shown by interviewing RS1. Here is the transcript of the interview :

Table 5. Interview Transcript of RS1's answer for Problem 2

Label	Conversation	Code
....		
R - 2	What are the informations you can get?	

Continue of Table 5. Interview Transcript of RS1's answer for Problem 2

Label	Conversation	Code
RS1 - 2	So, this problem gives the map of Antarctica continent. This map has scale 1 to 40.000.000. Map in this problem can be used to measure the area of continent in real.	FC.1
R - 2	So, what is the asked of the problem?	
RS1 - 2	Estimation of continent's area in real.	FC.1
....		

Based on the interview and geometric problem task result, we can conclude that the component of creativity that is owned by RS1 is just Fluency component.

2. RS2

In problem 1, students should estimate the total height of the twisted building by using their experience in visiting an apartment or mall and or building else and maybe their experience can use to estimate the total height of the building follows their knowledge about trigonometry. The last subject is in informal deduction level, who can give the best and maximal work for geometric problems task. In the first problem of geometric problems task, RS2 can give the best answer which if we compare to the all students in class, there is no student can answer this problem like her. Then, here is the answer of RS2 for problem 1:

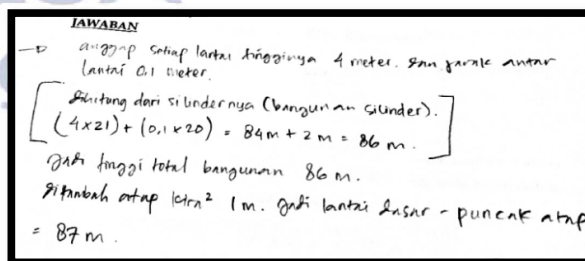


Figure 3. RS2's Answer for Problem 1

From RS2's work, we can see that RS2 made sure that her estimation is close to reality and logics. RS2 assumed the height of each floor is 4 meter and she paid attention to the distance between each floor is 0.1 meter. RS2 estimated the total height of the building from the cylinder

building. RS2 shows her understanding about the height of the building correctly. This answer is different than the other answers in this class. Then, to explore more about RS2's work, here is the interview result of RS2 for problem 1 :

Table 6. Interview Transcript of RS2's answer for Problem 1

Label	Conversation	Code
....		
R – 2	What are the informations you can get from problem 1?	
RS2 – 2	Modern architecture nowadays has unusual shape, nah here named Twisted Building. This twisted building has 21 floors contains of apartments. Each floors has same design but has a slightly different orientation in each floors. Then, we should estimate the total height of the building.	FC.1
R – 3	Then, how you solve this?	
RS2 – 3	So for the first, I assume the height of each floor is 4 m and the length of the distance in each floors is 0,1 m.	
R – 4	What does the mean of the distance in each floors?	
RS2 – 4	Ok, it's like when you see your building in the game of the sims, kak. There is a gap between the first and second floor. The distance I mean is like we see our home or this school in real, we see there's distance between the first and second floors. I think that's it.	N.1
R – 5	Is it fix 0.1 m?	
RS2 – 5	I think ya.	
R – 6	How long is 0.1 m converted in to cm?	
RS2 – 6	Ya 100 cm kak.	
R – 7	Okay, just continue your explanation!	
RS2 – 7	I calculate from the cylinder one, so that I am not confused.	
R – 8	Why ?	
RS2 – 8	Just to make it clear only, cause even this building is rotated whatever it is, the height is the perpendicular orientation of the ground floor right. Just that.	

Continue of Table 6. Interview Transcript of RS2's answer for Problem 1

Label	Conversation	Code
R – 9	Okay, next?	
RS2 – 9	Since, there are 21 floors, so the total height of the building is 84 m then is added by 0.1 is multiplied by 20, so it is 86 m. and we add it by 1 m, the height of the roof top, then the total height of the building is 87 m.	
....		

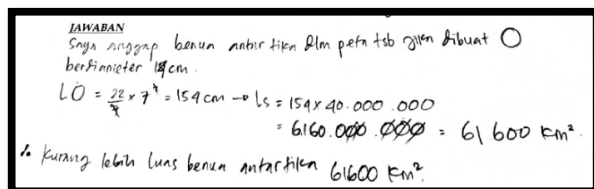
From the interview result, we finally know RS2's interpretations of this problem. RS2 has more experience that the others so that she can give an answer like that. RS2 relates one situation to another situation so that she can think to estimate it she needed to estimate the distance from one floor to another floor. This distance for RS2 is the space for the thickness of the story of the floor above the ground floor and also for the next floor. RS2 paid attention to for the height of the rooftop in this building. RS2 really understand every statements that she had written.

According to the test and interview result, RS2 gave more than two interpretations correctly and explain what is asked in problem 1 correctly (FC.1). the interpretations that has been given by RS2 are :

- The twisted building of this problem i rotated building.
 - There are 21 floors in this building.
 - We should estimate the total height of this building.
 - There are distances between each of floor that we can estimate as 0.1 m or 100 cm at least.
 - The height of each floor at least or should be 4 m.
 - There should be roof top with height of 1 m at least.
- Eventhough, RS2 didn't give more than one solution or answer for this problem, RS2 still has a fluency component.

Since, RS2 didn't shift their approaches taken when solving this problem. This shows that RS2 has no flexibility. Eventhough, RS2 didn't give more than one solution method or answer in problem 1, but RS2 has given different solution method than the other because RS2 compare her experiences when playing the sims and seeing the building in real life than she has a conclusion about how to estimate the total height of the building (N.1). This way shows that RS2 has novelty.

The last problem in Geometric problems Task, students should estimate the exact area of the continent of Antarctica. RS2 had not given correct answer for this problem, but in her way to estimate it is proper to appreciate. RS2's work below shows her different way than the other students in class :



JAWABAN
Saya anggap bentuk antar tika itu pda tsb gmn dibuat O
ber diameter 14 cm.
 $L O = 28 \times 7 = 154 \text{ cm} \rightarrow Ls = 154 \times 40.000.000 = 6160.000.000 = 61.600 \text{ km}^2$
Kurang lebih luas benua antartika 61600 km².

Figure 4. RS2's Answer for Problem 2

RS2 assumed that Antarctica has a shape of circle with diameter of 14 cm. Her way in assuming the shape of this continent is unique. This is different than the others. But this is not enough to judge the work of RS2, so that the interview below shows the reason of RS2's work in solving this problem :

Table 7. Interview Transcript of RS2's answer for Problem 2

Label	Conversation	Code
	
R - 2	From the problem, what are the informations you can get?	
RS2 - 2	The map scale is 1 : 40.000.000 and this one is the map of Antarctica continent. This map is drawn to measure. Then, from this, we should estimate the area of this continent in real.	FC.1
R - 3	So, how you solve this?	
RS2 - 3	I assume this continent in map has the shape close to circle. Then, I measure the diameter of the circle that this continent lay on is 14 cm. From this, we can find the area of this continent in map directly. The area of this continent in map is 61.600 km ² .	
R - 4	Can you explain again, how you get this idea first?	
RS2 - 4	I think just that is it.	
R - 5	Really? Can you explain it more again?	

Continue of Table 7. Interview Transcript of RS2's answer for Problem 2

Label	Conversation	Code
RS2 - 5	Ok, honestly, I did cheating at that time. I saw my friend's answer. She made a square shape in this map. Ok, then I got it. She just think that this continent's shape is close to square shape. But then, I think it again. And I feel that it is far to square shape, cause there are many gap remained. So yah, then I used the circle shape assumed that this continent lay on it. That's it.	FX.1 N.1
	

From the interview result, RS2 had cheated in solving this problem because she tried to see her friend's work. But the positive thing is she can find different way to estimate the exact area of this continent. RS2 tried to find the area of its map by using plane of circle with diameter of 14 cm. Even, the way of RS2 estimated the area of its map is correct, but RS2 didn't realize that actually her understanding in definition of map scale is wrong. RS2 cannot multiply that scale with the area of the map directly, because the scale is the ratio of distance not area. This causes RS2 given wrong answer for this problem.

According to the test and interview result, RS2 can give more than two interpretations of this problem correctly and understand what is the asked of this problem correctly (FC.1). First, RS2 can show that from this problem RS2 get the information of Antarctica Continent map with the map scale of 1 : 40.000.000. then, RS2 said that to find the exact area of this continent, we should find this area of this continent in this given map. RS2 also said that many ways that we can use to find the area in this map. But then, in solving this problem, RS2 didn't give many solutions or possible answer for this problem. RS2 is enough to be said as student whose fluency component. Eventhough RS2 did cheating, but RS2 has two ways in solving this problem. This can be legal cause in RS2's cheating, RS2 still think it first (first way) then guess new ways which is better and check it, guess it again then RS2 can improve (second way). Then, we said RS2 has flexibility component, because RS2 shifted their approaches. Even, in point 2) we know there is no using of same or different method to get same or different answer in this problem, then RS2 also cannot examine it then generate in different way, but from the interview result, RS2 stated that from her cheating in a way looking her friend's work, we know that she had examined

solution method then generate her own way that is different from the others. This shows that RS2 has novelty component.

Comparing RS1 and RS2, they have different behavior in solving geometric problem task. Not only, behavior in solving geometric problem task, but also they behaved in interview activity differently too. Based on the interview activity and geometric problem task, RS2 was more active, had big antusiast in answering and explaining what she had done in geometric problem task. RS2 also had good enough self confidence. This can be seen in the interview activity. RS2 was also dilligent. This can be proven in RS2's answer sheet. RS2 explained her answer step by step. This contradicts with RS1, eventhough they have same level of geometric thinking. Since, RS1 had different antusiast level and positive behavior, then RS1 cannot give better result than RS2.

CONCLUSION AND SUGGESTION

Conclusion

Based on research result that has been explained above, we know that eventhough they are in the same level of geometric thinking level, then they give different result in solving geometric problem. RS2 is more capable in giving and showing creativity's component in solving geometric problem task than RS1. Globally, we can conclude that RS2 has better creativity than RS1. RS2 can give the all of three components of creativity in solving geometric problem task. Besides, RS1 only can give and show the fluency component. In the first, has mentioned that RS1 is male and RS2 is a female student. Comparing RS1 and RS2, based on the interview activity and geometric problem task, RS2 was more active, had big antusiast in answering and explaining what she had done in geometric problem task. RS2 also had good enough self confidence. This can be seen in the interview activity. RS2 was also dilligent. This can be proven in RS2's answer sheet. RS2 explained her answer step by step. This contradicts with RS1, eventhough they have same level of geometric thinking. Since, RS1 had different antusiast level and positive behavior, then RS1 cannot give better result than RS2. From this explanation, we can conclude that the positive behavior can help them in having better creativity.

Suggestion

According to the research that has been done, researcher suggests:

1. For mathematics educators to focus to students' geometric thinking level whenever educator will teach them about geometry to give meaningful learning based on students' ability in learning geometry. This is not only educators have transfered it yet or not but this is also about how meaningful educators give them a learning. Because the more meaningful teaching and learning will foster students' creativity too. Also, the positive behavior can help them in having better creativity, then this can be parameter for educators in fostering their positive behavior more again, such as self

confidence, communication ability, antusiast in learning mathematics, etc.

2. For the other researcher, since, the weakness of this research is on the geometric problem task, then the researcher suggested the future researchers who have similar research to prepare a problem which can guide them well in showing more their creativity in solving problem so that the analysis of the result will be more optimal.

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