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MISCONCEPTIONS OF SENIOR HIGH SCHOOL STUDENTS' IN RADICALS

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Abstract

Although the experts already agreed on the concepts of radicals, not all of students have the same understanding and accordance with them. According to the result of pre-research, some junior and senior high school students were found to have the different understandings about the concepts of radicals and its procedures, and they were not accordance with the agreed concepts, what so called misconception. This research is a qualitative-descriptive research, which aims to describe the misconceptions of senior high school students in radicals along with the possible causes. A written test with CRI instrument and interview were used to gather the data of students' misconceptions and the causes. The research data collection was conducted in SMAN 9 Surabaya with 9 research subjects. According to the result of data analysis, they could not differentiate radicals with radicals of even and odd degree. They added or subtracted two radicals of the different root in the one same root and ignored the radicand when multiplied two multiples of radicals with the same radicands. The occurrence of those misconceptions tended to be caused by the incomplete or wrong conceptions, wrong opinion, inexact or wrong reasoning and incomplete understanding of the prerequisite topics that related to the concepts where the misconceptions occurred.
Keyword: misconception, misconception causes, radicals.

INTRODUCTION

Mathematicians create mathematical procedures based on mathematical concepts (Ben-Hur, 2006). A concept is an abstract idea used to classify objects (Soedjadi, 2000). Understanding of a concept is very important. This can be seen in the inclusion of conceptual understanding on the Indonesian curriculum of each level (Ibrahim, 2012). Moreover, understanding a concept greatly affects the student achievement (Aygor and Ozdag, 2012). However, as it is known that the object of mathematical study which includes the concept, is abstract (Soedjadi, 2000). Hence, Skemp (1971) states that mathematics cannot be studied directly from the environment, but only indirectly from other mathematicians. When studying a concept, students can have a corresponding understanding with the experts and may not. If the understanding of a concept possessed by a student is not accordance with the expert, then it is called misconception (Sheu *et al*., 2013 and Ibrahim, 2012). A single misconception can lead to a failure in progress and achievement, so the more misconceptions a student had, the greater the likelihood of student failing the test (Moore, 2006).

Radicals is studied in Grade 9, including the principal root of radicals and its operations (Kemdikbud, 2016). Some students considered that radicals is a hard topic to understand, because its operation was very complex and not common in everyday life (Ozkan & Ozkan, 2012b). According to the pre-research results, some misconceptions in radicals were found. It was found that students who had misconceptions in radicals were not only junior high school students, but also high school students, even 12th graders who would take national exams in which there would be some questions about radicals. In addition, that students had a high confidence with their answer. Therefore, it is necessary to know other misconceptions that may arise along with the factors that cause the student to have a wrong understanding of the concept, so it is known what action can be taken to correct it and reduce it. The causal factors in this study were limited to those sourced from students only. Thus, the aims of this research is to describe the misconceptions of senior high school students in radicals together with its causes.

METHOD

In accordance with the purpose of this research, the type of this research is a qualitative-descriptive research. Because of the research type, the researcher is the main instrument of this research, supported by the following instruments.

1. A radicals written test sheet with CRI (Certainty of Response Index) rubric, consist of 11 questions, was used to find the students' misconceptions.

2. Interview guidelines was used to confirm and get an explanation of the students' radicals test answers and to find the causes.

CRI is a simple and easy method for detecting misconceptions. After students were given a written test, then they were asked to determine their certainty degree in answering the test question (Hasan *et al*., 1999). Table 1 was used to know the students' certainty degree in answering the test question. Table 2 was used to classify students who lack of knowledge (LK), had correct knowledge of concept (CK) and misconception (M).

Tabel 1. CRI Criteria

Certainty Degree	Criteria
0	Tottaly guessed answer
1	Almost guess
2	Not sure
3	Sure
4	Almost certain
5	Certain

Source: Hasan *et al*. (1999)

Tabel 2. CRI Condition

	Low CRI (<2.5)	High CRI (>2.5)
Correct answer	Correct answer and low CRI (CL) **Lack of knowledge (lucky guess)**	Correct answer and high CRI (CH) **Knowledge of correct concepts**
Wrong answer	Wrong answer and low CRI (WL) **Lack of knowledge**	Wrong answer and high CRI (WH) **Misconceptions**

Source: Hasan *et al*. (1999:4)

A radicals written test with CRI rubric was given to 33 students of X MIPA 3 in SMAN 9 Surabaya. 9 of them was selected as research subjects, later would be interviewed, with following criterias.

1. Students who had various misconceptions in radicals.
2. Students who the answers given on the radicals written test sheet were the result of honest and earnest work.
3. Students who had a good explanation on the radicals written test sheet.

The result of radicals written test and interview was analyzed using analysis technique by Miles *et al*. (2014), that contains data condensation, data display and drawing and verifying conclusions.

RESULT AND DISCUSSION

The data were collected from February 28 to March 26, 2018. The written test was conducted on March 12, 2018, while the interview was conducted on March 13 and March 20-26, 2018. Based on the radicals written test, 9 subjects were chosen according to the criteria above, as can be seen in the following table.

Table 3. Result of Radicals Written Test

Research Subject	Name code	Number of LK	Number of CK	Number of M
RS 1	ABEM	0	7	4
RS 2	AN	0	5	4
RS 3	DPF	2	4	5
RS 4	FKS	3	4	4
RS 5	GPDY	0	4	6
RS 6	HS	1	6	4
RS 7	OJP	1	5	4
RS 8	RKW	0	5	4
RS 9	SA	1	5	4

Findings

Question 1a and 1b: Find the value of $\sqrt{49}$ and $\sqrt{-81}$.
This question is asked to know the students' understanding of the concept of principal root of even degree radicals.

Based on the results of the radicals written test and interview, it can be concluded that some research subjects had a lack of understanding of the concept of principal root of even degree radicals, because all of them provided the correct answer with a high degree of certainty in question 1a, but there were 2 research subjects, RS3 and RS7, which stated that the square root of a negative radicand has real value, i.e. negative number, as shown in Figures 1 and 2. The square root of a negative number is imaginary or not real number. They had that misconception tended to be caused by an incomplete or wrong conception about even degree radicals. This misconception was also found by Ozkan (2011).

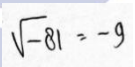

Figure 1. RS3's radicals written test result of 1b

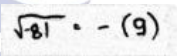

Figure 2. RS7's radicals written test result of 1b

Question 1c: Find the value of $\sqrt{(-6)^2} + \sqrt{6^2}$.
This question is asked to know the students' understanding of the concept radicals involving the degree of radicals and the power of radicand were same.

Based on the results of the radicals written test and interview, 3 of 9 research subjects, RS2, RS6 and RS8, had misconception about the concept of radicals addition. They added or subtracted two radicals of the different root in one same root. Two different radicals cannot be operated in one same root. This misconception was also found by Ozkan & Ozkan (2012a), Ozkan (2011) dan Moore (2006). Ozkan & Ozkan (2012a) stated that this misconception was the most common misconception. Then, 1 of 9 research subjects, RS3, had misconception about the concept of principal root of even degree radicals. RS3 stated that the square root of a negative radicand has real value, i.e. negative number. Again, the square root of a negative number is not real number or imaginary number. Besides, RS4 stated that the

radicals of radicand with same (even) radicals degree and radicand power was radicand. Radicals of radicand with same (even) radicals degree and radicand power is the absolute value of radicand. All of the misconceptions above tended to be caused by the incomplete or wrong conception about the concept of radicals addition operation, even degree radicals and radicals with same radicals degree and radicand power. RS3 and RS4's misconception was also found in Ozkan (2011) research. The misconceptions above is shown in the following figures.

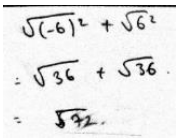

Figure 3. RS2's radicals written test result of 1c

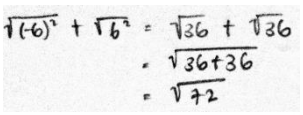

Figure 4. RS6's radicals written test result of 1c

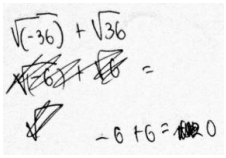

Figure 5. RS3's radicals written test result of 1c

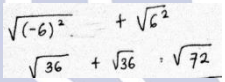

Figure 6. RS8's radicals written test result of 1c

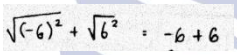

Figure 7. RS4's radicals written test result of 1c

Question 1d: Find the value of $\sqrt[3]{-8}$.

This question is asked to know the students' understanding of the concept of principal root of odd degree radicals.

Based on the results of the radicals written test and interview, RS6 and RS8 stated that the cube root of a negative radicand was imaginary or not real number. The cube root of a negative radicand is a negative number. Their misconception tended to be caused by the incomplete or wrong conception about odd degree radicals. RS4 stated that the numerator of the exponents was always 2, when RS4 was asked to write radicals into exponents. When the radicals is given, the exponents form of it is radicand to the power of radicand power over the radicals degree. So, the numerator of the exponents is the radicand power. This misconception tended to be caused by the incomplete or wrong conception and inexact or wrong reasoning about the equality of radicals and exponents. The result of their radicals written test of this question is given below.

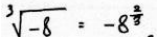

Figure 8. RS4's radicals written test result of 1d

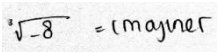

Figure 9. RS6's radicals written test result of 1d

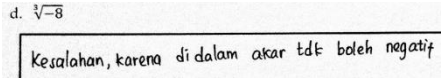

Figure 10. RS8's radicals written test result of 1d

Question 2a and 2b: Determine whether the following statements are true or false. Give a reason.

$\sqrt{75} - \sqrt{50} = \sqrt{25}$ and $\sqrt{x^2 + y^2} = x + y$, with $x, y \in \mathbb{R}$

This question is asked to know the students' understanding of the concept of radicals addition and subtraction.

Based on the results of the radicals written test and interview, 3 of 9 research subjects had misconception about the concept of radicals subtraction. RS3, RS5 and RS6 stated that different radicals with same radicals degree could be subtracted in one same root. The different radicals with same radicals degree cannot be operated in one same root. They had that misconception tended to be caused by wrong opinion, incomplete or wrong conception and inexact or wrong reasoning about the concept of radicals subtraction. Then, 5 of 9 research subjects, RS1, RS2, RS4, RS7 and RS8, had misconception about 2nd degree radicals involving the addition of 2 squared numbers. They stated that the value of that radicals was equal to the addition of the numbers. That statement was wrong, because the square of two numbers added is not equal to the addition of 2 squared numbers. That misconception was mostly tended to be caused by the incomplete or wrong conception and the inexact or wrong reasoning about radicals with same radicals degree and radicand power. This misconception was also found by Ozkan (2011). All of the subjects above stated that the statements were true. These misconceptions were the most common misconceptions (Ozkan & Ozkan, 2012a). The misconception that related with question 2a and 2b is shown below.

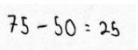

Figure 11. RS3's radicals written test result of 2a

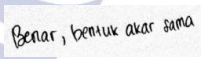

Figure 12. RS5's radicals written test result of 2a

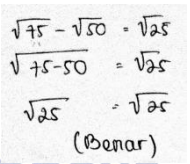

Figure 13. RS6's radicals written test result of 2a

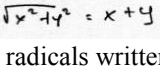

Figure 14. RS1's radicals written test result of 2b

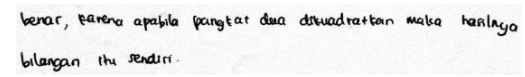

Figure 15. RS2's radicals written test result of 2b

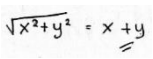

Figure 16. RS4's radicals written test result of 2b

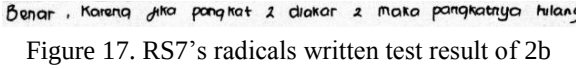

Figure 17. RS7's radicals written test result of 2b

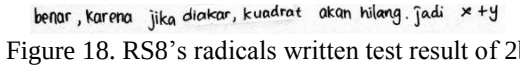

Figure 18. RS8's radicals written test result of 2b

Question 3a: Simplify $3\sqrt{24} - \sqrt{54} + \sqrt{6}$.

This question is asked to know the students' understanding of the concept of radicals addition, subtraction and multiplication.

Based on the results of the radicals written test and interview, there was 1 research subject, RS5, that had misconception. RS5 stated that different radicals with same radicals degree could be subtracted in one same root. The different radicals with same radicals degree cannot be operated in one same root. They had that misconception tended to be caused by wrong opinion about the concept of radicals addition and subtraction. The following figure is the radicals written test result of RS5.

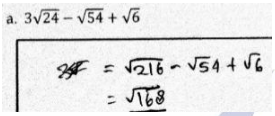

Figure 29. RS5's radicals written test result of 3a

Question 3b: Simplify $\frac{3\sqrt{6}}{\sqrt{2}} \times 4\sqrt{3}$.

This question is asked to know the students' understanding of the concept of radicals multiplication and division.

Based on the results of the radicals written test and interview, it was found that 4 of 9 research subjects, RS1, RS3, RS5 and RS9, had misconception in this question. RS1 stated that a multiplication was distributed on a multiplication. Multiplication was not distributed on a multiplication, but the multiplication distributive property was true for addition and subtraction. This misconception tended to be caused by the inexact or wrong reasoning and incomplete understanding of the prerequisite topics. RS3 and RS9 ignored the radicals when multiplied 2 radicals multiples involving same radicands. They had that misconception tended to be caused by the incomplete or wrong conception about the concept of radicals multiplication. This misconception was also found in Ozkan & Ozkan (2012a) research. Besides, equal sign (=) was used by RS9 as a multiplication operation sign when RS9 rationalized radicals denominator. The details of the misconceptions are shown on the figure below.

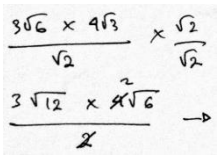

Figure 20. RS1's radicals written test result of 3b

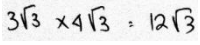

Figure 21. RS3's radicals written test result of 3b

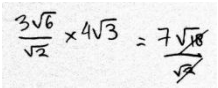

Figure 22. RS5's radicals written test result of 3b

Question 3c: Simplify $\frac{12a\sqrt{6}}{3a-\sqrt{3}}$.

This question is asked to know the students' understanding of the concept of radicals pair.

Based on the results of the radicals written test and interview, 2 of 9 research subjects had misconception. When multiplied radicals multiples with addition operation involving radicals, RS8 multiplied the multiples with the first term of the addition operation, and multiplied the radicals with the second term, by ignored the addition sign in the final result. RS9 almost did the same way, but the difference is RS9 not ignored the addition sign. Those misconception tended to be caused by the incomplete or wrong conception about the concept of radicals multiplication. Again, RS9 still used the equal sign (=) when multiplied fraction with radicals denominator with pair fraction, beside that, RS9 also stated that the pair of radicals denominator was the same form as the radicals denominator. The radicals pairs are not always same with the radicals. If the denominator is involving radicals only, then the radicals pair is the same radicals, but if the denominator is involving radicals operation, then the radicals pair is the conjugate of the radicals. In this case, RS9's misconception tended to be caused by the incomplete or wrong conception about the concept of radicals pair and operation sign. The following figures are their radicals written test results.

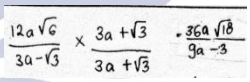

Figure 24. RS8's radicals written test result of 3c

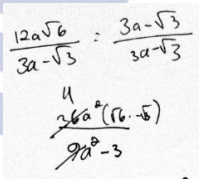

Figure 25. RS9's radicals written test result of 3c

Question 4a: Simplify the following radicals and then change its simplified form into powers.

$\sqrt[4]{64}$

This question is asked to know the students' understanding of the concept of principal root of even degree radicals and the equality of radicals and exponents.

Based on the results of the radicals written test and interview, out of 9 research subjects, RS4 was the only subject who had misconception. Same as question 1d, RS4 stated that the numerator of the power was always 2, result of the radicals symbol, when written the radicals into exponents. When a radicals is given and asked to write it into exponents, then the exponents form is radicand to the power of radicand power over radicals degree. This misconception tended to be caused by the inexact or wrong reason and the incomplete or wrong conception about the

equality of radicals and exponents. RS4's radicals written result can be seen in the following figure.

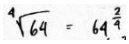

Figure 26. RS4's radicals written test result of 4a

Question 4b: Simplify the following radicals and then change its simplified form into powers.

$$\sqrt{2x^{-3}\sqrt[3]{-8x^2\sqrt[4]{x^4}}}, \text{ with } x < 0$$

This question is asked to know the students' understanding of the concept of principal root of even and odd degree radicals and the equality of radicals and exponents.

Based on the results of the radicals written test and interview, there was 2 research subjects who had misconception in this question. RS1 and RS6 stated that the 4th radicals degree of a negative number to the power of 4 is a negative number. In this case, both the radicals degree and the radicand power are even, then the value of that radicals is the absolute value of radicand, because it was different with odd degree, which could result a negative number. Besides, RS6 also stated that the last factor of radicand was the only one that raised to the power, when written the radicals involving multiplication operation, into exponents. It was wrong, if a radicals involving multiplication operation is given, to write it into exponents, all of radicands should be raised to the power. The details of their radicals written result is shown in the figure below.

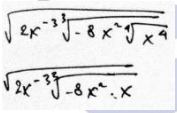

Figure 27. RS1's radicals written test result of 4b

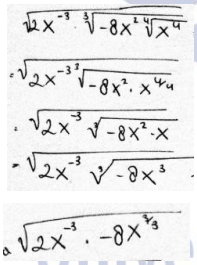

Figure 28. RS6's radicals written test result of 4b

CLOSING

Conclusion

Based on the analysis data research result and discussion, the misconceptions of senior high school students in radicals are given as follows:

1. The even degree radicals of negative radicand had real number. That real number was negative number. The value of radicals with even radicals degree of negative radicand is imaginer. This misconception tended to be caused by the incomplete or wrong conception about the even degree radicals.
2. The odd degree radicals of negative radicand had no real number or imaginer. The odd degree radicals of negative radicand have real number which is negative number. This misconception tended to be caused by the incomplete or wrong conception about the odd degree radicals.
3. The value of radicals with same and even radicals degree and radicand power was radicand. The value of radicals with same and even radicals degree and radicand power is the absolute value of radicand. This misconception tended to be caused by the incomplete or wrong conception about the radicals with same radicals degree and radicand power.
4. The addition and subtraction of different radicals with same radicand could be operated in one same root. Two or more radicals can be operated when they have same radicand and radicals degree, but the radicands are not operated in one same root. This misconception tended to be caused by the wrong opinion and incomplete or wrong conception about the radicals addition and subtraction.
5. The value of n degree radicals of the sum of 2 n-powered numbers was the sum of those 2 numbers. The value of n degree radicals of the sum of 2 n-powered numbers is not equal to the sum of those 2 numbers, because the n power of 2 numbers added is not equal to the sums of 2 n-powered numbers. This misconception tended to be caused by the inexact or wrong reasoning and incomplete or wrong conception about the radicals with same radicals degree and radicand power.
6. Multiplication operation was distributed on a multiplication. Multiplication operation is not distributed on a multiplication, but it is distributed to addition and subtraction. This misconception tended to be caused by the inexact or wrong reasoning and incomplete understanding of the prerequisite topic, which is multiplication.
7. When two radicals multiples with same radicand was multiplied, the radicands was ignored or not multiplied. When two radicals multiples with same radicand is multiplied, both the radicands and the multiples are multiplied. This misconception tended to be caused by the incomplete or wrong conception about the radicals multiplication.
8. When two radicals multiples with same radicand was multiplied, the multiples was added and the radicands was multiplied. When two radicals multiples with same radicand is multiplied, both the radicands and the multiples are multiplied. This misconception tended to be caused by the wrong opinion about the radicals multiplication.

9. When multiplied radicals multiples with addition operation involving radicals, the multiples and the first term of the addition operation was multiplied, and the radicals and the second term was multiplied. This misconception tended to be caused by the incomplete or wrong conception about the radicals multiplication.
10. The pair of radicals denominator was the same form as the radicals denominator. The radicals pair is same with the radicals, when the denominator is involving radicals only. This misconception tended to be caused by the incomplete or wrong conception about the radicals pair.
11. The equal sign (=) was used to rationalize denominator as multiplication operation sign. The multiplication operation sign in rationalizing denominator is multiply (×) or dot (·) sign. This misconception tended to be caused by the incomplete or wrong conception about the radicals pair and operation sign.
12. The numerator of the power was always 2, result of the radicals symbol, when written the radicals into exponents. When a radicals is given and asked to write it into exponents, then the exponents form is radicand to the power of radicand power over radicals degree. This misconception tended to be caused by the inexact or wrong reason and the incomplete or wrong conception about the equality of radicals and exponents.

Suggestion

According to the research result, there are some suggestion for teachers as follows:

1. This result need to be considered to determine the learning strategy used to teach radicals topic.
2. This result need to be considered to identified other misconceptions related to misconception causal factor in radicals.

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