MATHE*dunesa*
Jurnal Ilmiah Pendidikan Matematika *Volume 8 No. 1 Tahun 2019*
ISSN :2301-9085

ALGEBRAIC THINKING OF JUNIOR HIGH SCHOOL STUDENTS IN SOLVING NUMBER PATTERN PROBLEMS BASED ON SYSTEMATIC AND INTUITIVE COGNITIVE STYLE

Firda Fiqih Arina
Mathematics Education, Faculty of Mathematics and Science, State University of Surabaya
e-mail: firdaarina@mhs.unesa.ac.id

Dr. Agung Lukito, M.S.
Mathematics Education, Faculty of Mathematics and Science, State University of Surabaya
e-mail: agunglukito@unesa.ac.id

Abstract

This research aims to describe the algebraic thinking of junior high school students, with systematic and intuitive cognitive style, in solving number pattern problem. There are four components of algebraic thinking, that are generalization from arithmetic, meaningful use of symbols, identify and extend a pattern, and mathematical modeling. The type of research is descriptive-qualitative research. Two eighth graders became the subject of this research who are determined based on systematic and intuitive cognitive styles. The data collection methode are assignments and interviews. The results show that both students understand the letter symbols as the subtitute of any number of objects in the problem when understanding the problem. When devising the plan, both students identifies the pattern by determining the difference between pattern and finding the number relationships, besides, student with systematic style look for the relationship between the term and the order by decontructing the known terms in the pattern. In carrying out the plan, student with systematic cognitive-style use n^{th} term of arithmetic sequence equation to determine the term of a number pattern and use general form that had been found to determine the order of the term, but student with intuitive cognitive styles only use n^{th} term of arithmetic sequence equation to determine the term and the order of the term of a number pattern. Both students don't do algebraic thinking in looking back.

Keywords: algebraic thinking, problem solving, number pattern, systematic cogntive style, intuitive cognitive style.

INTRODUCTION

Algebra is a branch of mathematics that studies relationships, quantities, and structures. Algebra can be used to model a real problem in abstract, with symbols and relationships between symbols, to facilitate in solving everyday problems or mathematical problems (Reys et al, 2012). The letter symbols (commonly referred to as variables) in algebra makes algebra difficult to be understood by the students. Students have difficulty to discriminate the different ways to use letters (Kieran in Manly and Ginsburg, 2010). In order to understand algebra, students need to achieve fluency using mathematical thinking tools which are one component of algebraic thinking (Kriegler, 2008). Thus, one way to understand algebra is by developing algebraic thinking.

Algebraic thinking is an appropriate way of thinking about mathematical content (Alghtani and Abdulhamied, 2010: 257). It is composed of different forms of thought and understanding of symbols (Walle et al, 2013: 259). Algebraic thinking has several components which include: 1) generalization of arithmetic, 2) use of symbols that are meaningful, 3) making explicit structures in number systems, 4) study of patterns and functions, and 5) mathematical modeling (Walle et al, 2013).

Algebraic thinking will be better develope through problem solving approaches (Barton and Katz in Booker, 2009: 11). Problem solving is a process in finding solutions to given problems by involving various knowledge that has been owned (Krulik and Rudnick, 1989: 5; Santrock, 2014: 316; Solso, 1995: 440). There are four steps in solving the problem, (1) understanding the problem, (2) devising the plan, (3) carrying out the plan, and (4) looking back (Polya, 2004). Through this approach, students can determine the steps or ways that can be used to solve problems, so that students' algebraic thinking skills can develop.

At the junior high school, problem that can be given to develop students' algebraic thinking is an algebraic problem that requires student generalization skills, that is number patterns (Windsor, 2010: 227; Drew, 1990: 2). Number patterns is study about patterns from a number sequence and uses these patterns to solve problems. Finding and understanding the patterns is very necessary to solve the problem of number patterns. The process of finding and understanding a pattern can improve students' generalization abilities. Thus, the problem of number patterns is a question related to the material of the number pattern whose solution requires to find the general form of a pattern and use it to solve the problem.

In receiving and processing information to solve problems, each people has a different way. The way people receives, processes, and responds to information is called cognitive style. Cognitive style is a habit or characteristic of a person in receiving, processing, and responding to information (Bashir et al., 2013: 93; Messick, 1969: 14; Jena, 2014: 71). Therefore, different cognitive styles will show different problem solving. Hence, it will bring differences in the one's algebraic thinking.

Cognitive style which is classified based on how to evaluate and choose strategies in solving problems is a systematic and intuitive cognitive style (Keen, 1974; Martin 1998: 3). Systematic cognitive style is a characteristic of a person in evaluating information and choosing strategies to solve problems using well-formulated rules (Jena, 2014; Martin, 1998; Sagiv et al, 2013). Besides, the cognitive intuitive style of a person characteristics in evaluating information and choosing strategies to solve problems by using analytical steps and relying on experience (Jena, 2014; Martin, 1998; Sagiv et al, 2013). The basis of the classification of cognitive styles shows that systematic cognitive style and intuitive cognitive style influence the habits of thinking and the way to arrange steps in making a decision to solve a problem. Thus, the aim of this study is to describe junior high school students' thinking in a systematic cognitive style in solving the problem of number patterns.

METHOD

Based on the aim of the research, the type of research is descriptive-qualitative research. Thus, the main instrument of this research is the researcher, and supported by other instruments. They are:

1. Cognitive-Style Inventory (CSI), used to group students based on systematic and intuitive cognitive styles.
2. Mathematical ability test (TKM) is used to determine the mathematical abilities of prospective subjects.
3. Assignments adapt examples of problems to explore algebraic thinking according to Kriegler (2008). The hope, TPM is able to show the of students algebra thinking in solving mathematical problems in terms of systematic and intuitive cognitive style. TPM consists of one question with four questions.
4. Interview guidelines are used to multiply information that is not visible on the TPM sheet that students do about the algebraic thinking of the research subject.

The study was conducted in the even semester of the 2017/2018. And the data are collected from April 10 untill May 8. A total of 30 students were given a Cognitive-Style Inventory compiled by Martin (1998). In the Cognitive-Style Inventory (CSI), students were asked to determine the level of agreement for 40 statements. 20 statements related to systematic cognitive style and 20 statements related to intuitive cognitive style. Table 1. shows the level of student agreement. Furthermore, students are grouped into systematic and intuitive cognitive styles. To group students into systematic and intuitive cognitive styles, criteria are given as shown in Table 2.

Table 1. Level Agreement in CSI

Number	Level Agreement
1	Strongly Disagree
2	Disagree
3	Undecided
4	Agree
5	Strongly Agree

Tabel 2. Classification of Systematic and Intuitive Cognitive Styles

	Systematic Score	Intuitive Score
Systematic Style	81 – 100	20 – 60
	81 – 100	61 – 70
	71 – 80	20 – 60
Intuitive Style	20 – 60	81 – 100
	61 – 70	81 – 100
	20 – 60	71 – 80

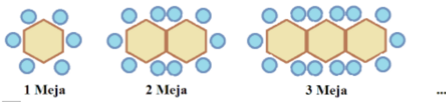
Based on the analysis of CSI, it shows that 5 students have cognitive intuitive style and 1 student has systematic cognitive style. Then, they were given a Mathematics Ability Test. This test contains five problems that is adapted from junior high school national exam. The Mathematics Ability Test results show that there are 2 students in cognitive intuitive style who have an equivalent ability with the systematic cognitive style student. Because there is intuitive cognitive style student who have different gender with systematic cognitive style student, we choose one intuitive cognitive style student with the same gender as students in a systematic cognitive style. Thus, researchers determined 2 students as the subject of the study. 1 student as subject of systematic cognitive style and 1 student as subject of cognitive intuitive style. Two students, have been selected, are given 2 assignments to find out the students' algebra thinking. Both assignments have an equivalent level. The first problem solving assignment is shown in Figure 1, and the second problem solving assignment is shown in Figure 2. Both assignments are given for triangulation purposes. Triangulation of data is needed to obtain a credible data. The type of triangulation in this study is within-method triangulation. Within-method triangulation is the use of several techniques in a method for collecting and interpreting data (Jick: 1979). This triangulation aims to test the consistency and reliability of data (Jick: 1979).

A gardener designs a garden with a pattern. If the park is 1 meter long, the garden will be surrounded by 8 ceramics. If the park is 2 meters long, then the park is surrounded by 10 ceramics, and so on. If it is drawn, it will look like the image below.

a. In general, can you find the number of ceramics for a garden of a certain length?
b. How much ceramics is needed if the length of the garden is p meters?
c. How much ceramics is needed if the garden length is 120 meters?
d. What is the maximum garden length that can be made if there are only 200 ceramics?

Figure 1. First Assignment of Algebraic Thinking Problem

In the school canteen, 6 people can sit in a table. If 2 tables are combined, 10 people can sit together. If 3 tables are combined, 14 people can sit together, and so on. If it is drawn, it will look like the image below.



a. In general, can you determine the amount of people who can sit together on certain amount of tables?
b. How many people can sit together if there are n tables?
c. How many people can sit together if there are 100 tables?
d. How many tables should be combined so that 90 people can sit together?

Figure 2 Second Assignment of Algebraic Thinking Problem

Algebraic thinking on a number pattern is made up of several components, namely 1) generalization of arithmetic, 2) meaningful use of symbols, 3) description of patterns, and 4) mathematical modeling. Based on these components, indicators were made to analyze students' algebraic thinking in the problem solving process. The analysis technique of Miles et al (2014) was used to analyze the results of assignments and interviews. The technique includes condensation of data, data display and drawing and verifying conclusions.

RESULT AND DISCUSSION

Result

The following are the results of the two subjects in solving the problem of number patterns. Figure 3 shows the results of systematic subject in solving the number pattern problem in first assignment , and Figure 4 shows the results of the solving problem number pattern in the second assignment.

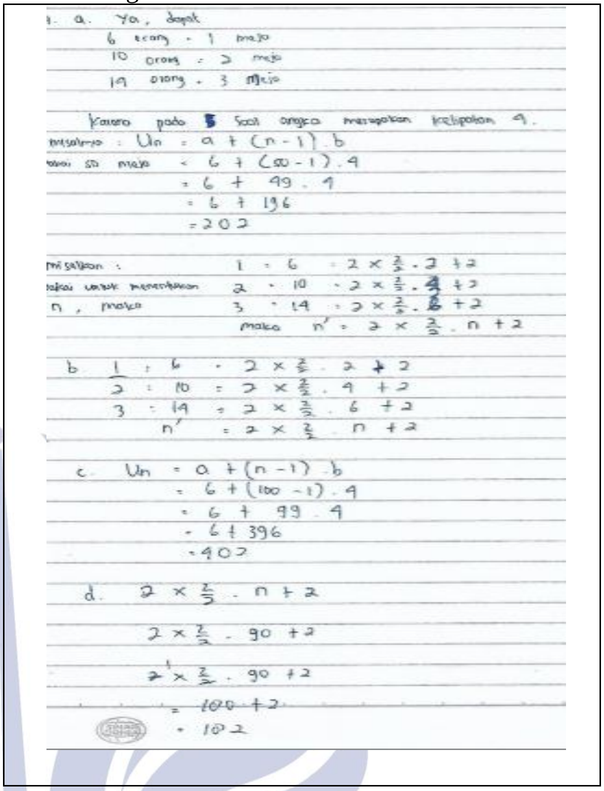


Figure 3. Result of Systematic Cognitive-Style Subject in Solving First Assignment of Algebraic Thinking Problem

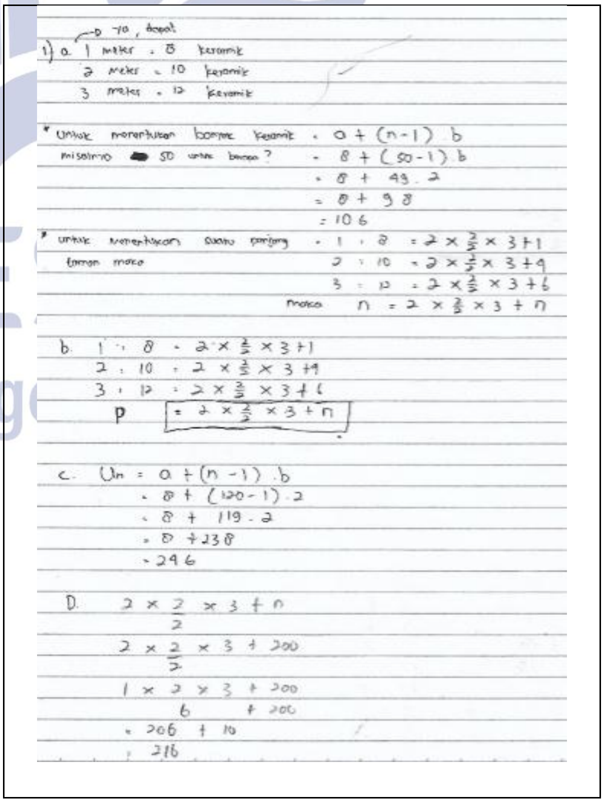


Figure 4 Result of Systematic Cognitive-Style Subject in Solving Second Assignment of Algebraic Thinking Problem

Based on the results and the interview by the systematic cognitive-style subject, the algebraic thinking of systematic cognitive-style subject is shown on the diagram in Figure 5.

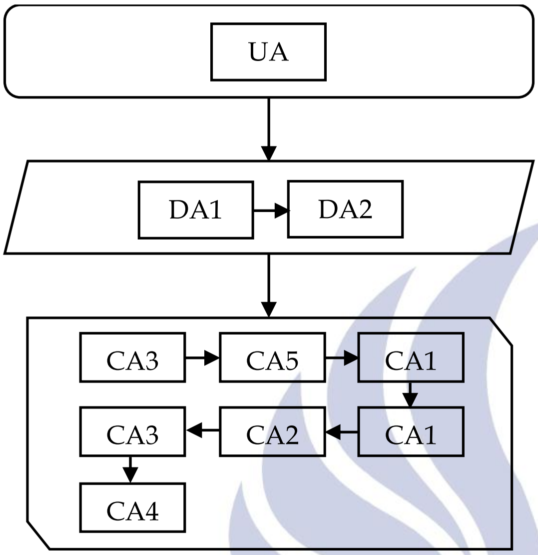
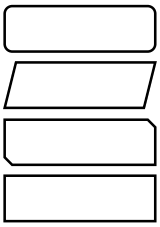


Figure 5. Algebraic Thinking of Systematic Cognitive Style Subject

Information:



: Understanding the problem

: Devising a plan

: Carrying the plan

: Indicators of algebraic thinking that are carried out

UA : Reiterate or explain questions or statements related to symbols

DA1 : Identify the pattern

DA2 : Determine relationships between numbers

CA1a : Deconstructing numbers

CA1b : Using relationships between numbers that have been found to determine the general form of a number pattern of numbers

CA2 : Presenting a general form of pattern that have been found using images, words, or algebraic forms

CA3 : Reiterate or explain statements related to symbols

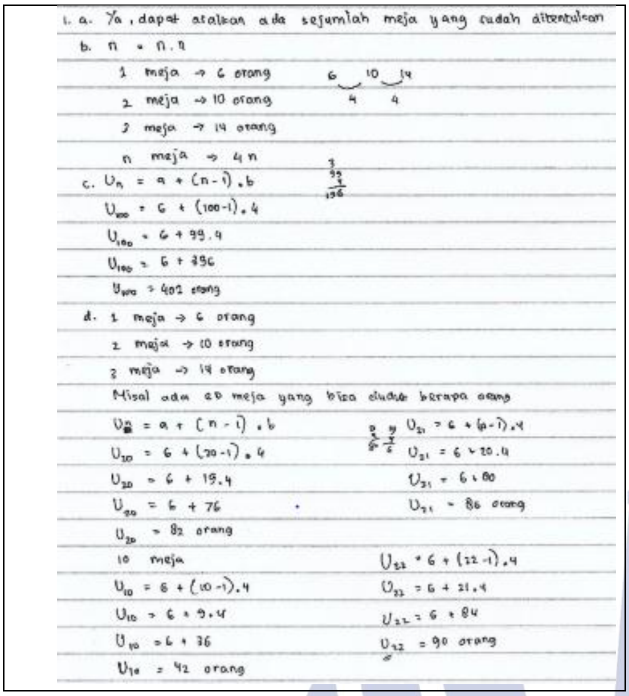
CA4 : Use pattern that have been found to determine the requested term or order.

CA5 : Determine the value of a variable as unknown thing

$\longrightarrow$: Activity's flow

The results of the cognitive intuitive-style subject in solving the problem of number patterns is shown in Figure 6 shows the results of problem solving for number patterns in first assignment. Figure 7 shows the results of problem solving for number patterns in second assignment.

Figure 6 Result of Intuitive Cognitive Style in Solving First Assignment of Algebraic Thinking Problem



1. a. Ya, dapat atalkan ada sejumlah meja yang sudah ditentukan
b. n = n . n
1 meja → 6 orang   6 10 14
2 meja → 10 orang   4 4
3 meja → 14 orang
n meja → 4n
c. $U_n = a + (n-1) . b$
$U_{100} = 6 + (100-1) . 4$
$U_{100} = 6 + 99 . 4$
$U_{100} = 6 + 396$
$U_{100} = 402$ orang
d. 1 meja → 6 orang
2 meja → 10 orang
3 meja → 14 orang
Misal ada 20 meja yang bisa duduk berapa orang
$U_n = a + (n-1) . b$ $U_{21} = 6 + (21-1) . 4$
$U_{20} = 6 + (20-1) . 4$ $U_{21} = 6 + 20 . 4$
$U_{20} = 6 + 19 . 4$ $U_{21} = 6 + 80$
$U_{20} = 6 + 76$ $U_{21} = 86$ orang
$U_{20} = 82$ orang
10 meja $U_{22} = 6 + (22-1) . 4$
$U_{10} = 6 + (10-1) . 4$ $U_{22} = 6 + 21 . 4$
$U_{10} = 6 + 9 . 4$ $U_{22} = 6 + 84$
$U_{10} = 6 + 36$ $U_{22} = 90$ orang
$U_{10} = 42$ orang

Figure 7 Result of Intuitive Cognitive Style in Solving Second Assignment of Algebraic Thinking Problem

Based on the results and the interview by the intuitive cognitive-style subject, the algebraic thinking of intuitive cognitive-style subject's is shown on the diagram in Figure 8.

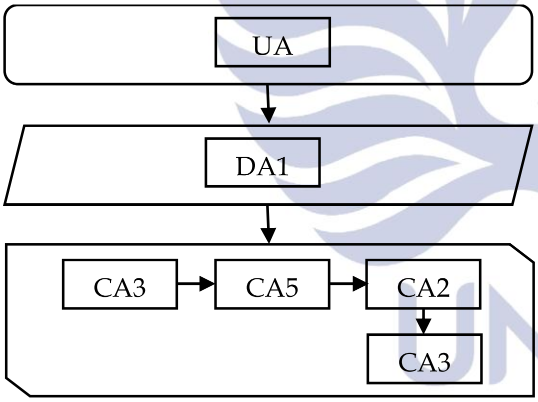
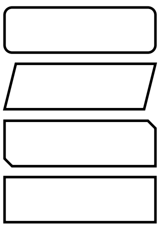


Figure 8 The Algebraic Thinking of Intuitive Cognitive Style Subject

Information



(rounded rectangle)	:	Understanding the problem
(parallelogram)	:	Devising a plan
(cut-corner rectangle)	:	Carrying the plan
(rectangle)	:	Indicators of algebraic thinking that are carried out
UA	:	Reiterate or explain questions or statements related to symbols
DA1	:	Identify the pattern
CA2	:	Presenting a general form of pattern that have been found using images, words, or algebraic forms
CA3	:	Reiterate or explain statements related to symbols
CA5	:	Determine the value of a variable as unknown thing
→	:	Activity's flow

Discussion

At the stage of understanding the problem, when there is a symbol on the problem both subjects consider the symbol as a general form of varying quantity. This finding was also found in Maulidiah (2016). This activity shows that subjects understand the meaningful use of symbols, componen of algebraic thinking.

At the stage of determining the plan, the two subjects identified the pattern by finding differences between the terms in the pattern. An important concept in working on patterns is to identify the core of the pattern (Walle et al, 2013). This activity shows that the subjects satisfy algebraic thinking component, namely identification of the pattern. Furthermore, there is algebraic thinking activity be done by the subject with systematic cognitive style but not be done by the subject of intuitive cognitive style that is looking for the relationship between the term's order and the term. This is consistent with the opinion of Sagiv, Amit, Ein-Gar & Arieli (2013: 403), which is "individuals with a systematic (rational) style tend to apply rule-based thinking. They analyze the situation and logically evaluate various approaches to discover underlying rules." The systematic cognitive-style subject looks for the relationship between the terms' order and their terms by deconstructing known terms in the pattern. This is consistent with the opinion of Walle et al (2013) "Exploring numerical situation rather than just computing, is an effective way to infuse algebraic thinking and strengthen understanding of numbers". Hence, systematic cognitive-style subject satisfy components of algebraic thinking, that is generalizations from arithmetic.

At the stage of carrying out the plan, systematic cognitive-style subject does it in order. This is different from the subject of intuitive cognitive style that works according to the level of difficulty. This is because the subject of systematic cognitive style tends to work in a structured manner while the intuitive cognitive-style subject uses unpredictable rules that rely on experience based on cues or hunches, and explores and leaves the other way quickly (Martin, 1998).

To determine the term order, subjects apply the formula of the n^{th} Term of arithmetic sequence. Both of them describe letter symbols in the general formula of the

n^{th} term of arithmetic sequence, symbol n as the order of terms asked, and the symbol b is different in an arithmetic sequence. Systematic cognitive-style's subjects add that a symbol symbolizes the first term in the pattern. This activity shows that students do components of algebraic thinking, namely the meaningful use of symbols. Next, the two subjects determine the value of U_n as something unknown by substituting a, n and b according to their understanding of the symbol. The process of finding unknown values requested in an equation expressions is the most important activity in algebra (Lew, 2004). This shows that the subject understand the meaningful use of symbols, the algebraic thinking component.

Subject of intuitive cognitive style present a general way of determining the terms in number patterns using algebraic form. Symbols in the form of letters in statements or equations made by the subject are represented as any number This is consistent with the opinion of Walle et al. (2013: 266). "Expression of equations with variables means expressing patterns and generalization". This activity shows that the subject understands the meaningful use of symbols, the component of algebraic thinking.

To determine the order of the terms, subjects use different ways. Intuitive cognitive-style subject do trial-and-error. She chose any numbers to represent the order of the term, then check it using the formula of the n^{th} Term of arithmetic sequence to determine whether the term is apropriate with the known or not. On the other hand, systematic cognitive-style subject deconstruct the known terms and try to relate them to the order of terms. This is accordance with the opinion of Walle et al (2013) "Young students explore families and in the learn process how to decompose and recompose numbers". When the subject concludes the results of deconstruction, there are some mistakes. In first assignment, the subject concludes that to determine the length of the park, it could be done by adding the number of ceramics with $2 \times \frac{2}{2} \times 3$ and in second assignment concludes that to find the length of the garden can be done by multiplying the number of people with $2 \times \frac{2}{2}$ then the result was added with 2. However, if we observed, the numbers that were added to produce 8, 10, 12 in first assignment and were multiplied to produce 6, 10, 14 in second assignment are 2, 4, and 6 which are the results of 2×1, 2×2, and 2×3. 1, 2, and 3 are terms' order that are known in the number pattern. Thus, the subject describes the general form of the term order in first Assignment by adding 2 times the terms' order with $2 \times \frac{2}{2} \times 3$ and describing the general form of the order of term in second assignment by multiplying 2 times the term order with $2 \times \frac{2}{2}$ then add it with 2. Regardless of the mistakes that have been made by the subject, the subject's activities when identifying the results of deconstructing the known term to find a general form is accordance with Walle et al (2013) that the right method for generalizing is finding patterns that grow and represent it. Thus, the subject satisfies the algebraic thinking component, That is generalization from arithmetic.

General forms that have been found be presented in the form of mathematical equations in which there are letter symbols on it. This letter symbols represent any number. This general form is used to determine the order of terms when the term is known. This is in accordance with Walle's opinion (2013: 280) "Mathematical models allow us to find values that cannot be observed in the real phenomenon". Thus, students perform algebraic thinking components that are mathematical modeling and meaningful use of symbols. Both subjects don't do activities that satisfied the algebraic thinking component at the looking back stage.

CLOSING

Conclusion

Based on the discussion, the two subjects use or explain the symbol as a generalization of a number in the process of understanding the problem. Thus, each subject performs the component of algebraic thinking which is the meaningful use of symbol. Both subjects identify patterns by determining the difference in patterns in the process of devising plans. Thus, both of them perform algebraic thinking components, namely identification and description of patterns. At this stage, there is algebraic thinking activities carried out by the subject with systematic cognitive-style, but not carried out by the subject with intuitive cognitive-style, that is looking for the relationship between the order of the tribe and the tribe. In the process of solving the problem, the two subjects use the general formula of arithmetic sequence to determine the term of the number patterns. Both subjects did not perform the algebraic thinking component at the look back. In addition there are also differences in subject algebraic thinking in solving the problem of number patterns. In working on problem solving, systematic cognitive-style subjects do it in order. This is different from the subject of an intuitive cognitive style that works according to the level of difficulty. To determine the sequence of terms, the subject in a systematic cognitive style tries to deconstruct numbers. Unlike the subject of a cognitive-intuitive style that does the trial and error.

Suggestion

Based on the analysis, discussion and conclusions of the research, the suggestion are show below.

1. For the further researcher should provide more detailed questions and respond more to the

answers given by students in order to obtain better research.

2. Teacher can use it to predict students' algebraic thinking in a systematic cognitive style and intuitive cognitive style in solving problems of number patterns

REFERENCES

Alghtani, O. A., dan Abdulhamied, N. A. 2010. “The Effectiveness of Geometric Representative Approach in Developing Algebraic Thinking of Fourth Grade Students”. Dalam International Conference on Mathematics Education Research 2010 (ICMER 2010). Procedia Social and Behavioral Science 8 pp. 256-263.

Bashir, T., Shafi, S., Ahmed, H. R., Jahangir, S., Saeed, H., & Zaigham, S. 2013. “Impact of Cognitive and Decision Making Style on Resilience: An Exploratory Study”. European Journal of Business and Management Vol. 5, No. 29, pp. 92-108.

Booker, G. 2009. “Algebraic Thinking: Generalising Number and Geometry To Express Patterns and Property Succinctly”. The Mathematical Association of Victoria, Annual Conference.

Drews, D. 1990. Patterns and Relationships in Mathematics. Some Activities for Primary Years. St Martins College.

Jena, P. C. 2014. “Cognitive Styles and Problem Solving Ability of Under Graduate Students”. International Journal of Educational Psychological Research (IJEPR), 3(2), pp. 71-76.

Jick, T.D. 1979. “Mixing Qualitative and Quantitative Methods: Triangulation in Action”. Administrative Science Quarterly, 24, pp. 602-611.

Keen, P. G. W. 1974. Cognitive Style and The Problem Solving Process: An Experiment.

Kriegler, S. 2008. “Just What is Algebraic Thinking?” Submitted for Algebraic Concept in the Middle School A Special Edition of Mathematics Teaching in The Middle School.

Krulik, S., dan Rudnick, J. A. 1989. Problem Solving: A Handbook for Senior High School Teachers. Boston, MA: Allyn and Bacon.

Lew, H. C. 2004. “Developing Algebraic Thinking in Early Grades: Case Study of Korean Elementary School Mathematics”. The Mathematics Educator. Vol. 8 No. 1, pp. 88-106.

Manly, M., dan Ginsburg, L. 2010. “Algebraic Thinking in Adult Education”. National Institute for Literacy. Washington, DC.

Martin, L. P. 1998. The Cognitive Style Inventory. The Pfeiffer Library, volume 8, 2nd Edition.

Maulidiah, N. 2016. Profil Berpikir Aljabar Siswa SMP dalam Memecahkan Masalah Matematika Ditinjau dari Kemampuan Matematika. Skripsi tidak diterbitkan. Surabaya: Universitas Negeri Surabaya.

Messick, S. 1969. The Criterion Problem In The Evaluation of Instruction: Assessing Possible, Not Just Intended Outcomes. New Jersey: Princeton University.

Miles, M. B., Hubberman, A. M., & Saldana J. 2014. Qualitative Data Analysis: A Methods Sourcebook. 3rd Edition. California: Sage.

Polya, G. 2004. How to Solve It, a New Aspect of Mathematical Method. Princeton University Press, Princeton Jersey.

Reys, R., Lindquist, M., Lambdin, D., & Smith, N. 2012. Helping Children Learn Mathematics. Brisbane: John Wiley and Sons Australia.

Sagiv, L., Amit, A., Ein-Gar, D., & Arieli, S. 2013. “Not All Great Minds Think Alike: Systematic and Intuitive Cognitive Styles”. Journal of Personality, 82(5), 402-417

Santrock, J. W. 2014. Educational Psychology. Fifth Edition. New York: McGraw-Hill

Solso, R. L. 1995. Cognitive Psychology. MA: Allyn and Bacon.

Walle, J. A. V. D., Karp, K. S., & Williams, J. M. B. 2013. Elementary and Middle School Mathematics: Teaching Developmentally. Eight Edition. New Jersey: Pearson Education, Inc.

Windsor, W. 2010. “Algebraic Thinking: A Problem Solving Approach”. Dalam Len Sparrow, Barry Kissane, dan Chris Hurst (Ed.), Shaping the Future of Mathematics Education Proceeding of the 33rd Annual Conference of the Mathematics Education Reaserch Group of Australia. John Curtin College of the Arts, Freemantle, 3-7 Juli 2010.