MATHE*dunesa*
Jurnal Ilmiah Pendidikan Matematika *Volume 8 No. 2 Tahun 2019*
ISSN :2301-9085

THE PROFILE OF STUDENTS' THINKING IN SOLVING THE TASKS OF MATHEMATICAL INDUCTION SEEN FROM MATHEMATICAL ABILITY

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Abstract

Every individual thinks in his life, especially when get a difficulties. Mathematics has an abstract mathematical object. This makes students have difficulty in solving math tasks. So, students will think in solving math tasks. One of the mathematical material that requires students to think is mathematical induction. Students are required to prove the truth of mathematical statements related to natural numbers using the principle of mathematical induction. Students' mathematical abilities have an influence when students solve math induction tasks. Thus, differences in mathematical abilities in solving tasks will lead to differences in students' thinking. This study aims to describe the profile of students thinking in solving the tasks of mathematical induction seen from mathemtical abilities. Thinking in this study refers to information processing model of Slavin, which is receiving information, processing the information, storing information, and recalling iformation. This study is a qualitative descriptive study. The subjects of this study is three senior high school students in 11st grade, among them one student with high mathematical ability, one student with average mathematical ability, and one student with low mathematical ability. The technique of data collection was done by giving tests related to the mathematical ability test, task solving test of mathematical induction and interviews.The results confirm that the differences in mathematical abilities have affect when the students thinking to solving mathematical induction tasks.
Keyword: thinking, mathematical induction, and mathematical ability

INTRODUCTION

Education is one measure of the state to know the quality of citizens life. Therefore, innovation in education is carried out by the government by implementing a new curriculum, namely the 2013 curriculum. The education experiences changes in competence without exception of mathematics. One of the importance of mathematics for citizens is contained in the lampiran Peraturan Menteri Pendidikan dan Kebudayaan republik Indonesia Number 21 of 2016 concerning content standards, students are able to explain patterns and use them to predict trends or examine validity of arguments.

According to Susanah (2014: 1.10) mathematics has an abstract object of study. This causes students have difficulties experience in learning mathematics (Isro'il, 2017). In solving math problems, students will think of getting answers or solutions to the problem. Therefore, how students think about getting a solution is important to know. Thinking is a cognitive process that involves some knowledge manipulation and produces behaviors that solve problems or go directly to solutions (Lailiyah et al., 2015). Thinking in cognitive psychology is using of information processing theory (Karim, 2017). In the information processing approach, thinking is a stage in processing information, controlling it, and formulating strategies that are in accordance with the information obtained (Santrock, 2010). This indicates that the information processing theory can be used to find out how students think in solving mathematic task.

One important component of mathematics that requires students to think is the process of proofing. According to Hemmi (2009: 206) "... many mathematicians mentioned that proofs helped students to learn mathematical and logical reasoning valuable in problem solving. In addition, Köğce et al. mentioned that (Köğce, Aydin, & Yildiz, 2010) proof is fundamental to doing and knowing mathematics. It is an essential part of improving thinking. The statement shows that someone will think to construct evidence. This is contained in the lampiran Peraturan Menteri Pendidikan dan Kebudayaan Republik Indonesia Number 24 of 2016 basic competencies number 3.1 and 4.1 for class XI. This competency requires students to construct evidence from mathematical statements given by using mathematical induction verification methods. In this study, the mathematical statement used is a mathematical

statement in the form of a sequence, because the form of mathematical statements has similarities to the sequence and series problems which are the prerequisite material for mathematical induction material. This is related with Baiduri (2017: 104) who gives advice, to teach mathematics induction material after series material. By giving students the question of mathematical induction, the description of students' thinking in constructing evidence can be seen when students succeed in knowing the truth value of the mathematical statements given.

In solving mathematic task, each student has a variety of ways. The questions given to students, to answer questions that have not been answered by involving existing knowledge and skills. Diana (2011: 23) states that mathematical abilities are knowledge of basic skills needed to manipulate mathematics including understanding concepts and procedural knowledge. This shows that differences in experience and knowledge of mathematics possessed by students influence the way students solve the questions given. Based on the description above, this study aims to describe the profile of students' thinking in solving the task of mathematical induction seen from mathematical abilitiy.

METHOD

This research is a descriptive study with a qualitative approach. this research was conducted in XI MIA 5 of SMAN 21 SURABAYA. Data were collcted from 33 students, 3 subjects were selected consisting of 1 student with high mathematical abilities, 1 student with average mathematical abilities, and 1 student with low mathematical abilities with the same gender, good communication skills, students' willingness and input from teacher suggestion. After the research subject was obtained, the research subject was given a test on mathematical induction and the researcher conducted an interview with the interview guidelines that the researcher had made. The test instruments of the task of mathematical induction are presented in Table 1.

Table 1. The Task Of Mathematical Induction

Task Number 1	Use mathematical induction to prove that $2 + 5 + 8 + \ldots + (3n \quad 1) = \frac{n}{2}(3n + 1)$ for all positive integers n !
Task Number 2	Use mathematical induction to prove that $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for all positive integers n!
Task Number 3	Use mathematical induction to prove that $1 + \frac{1}{2} + \frac{1}{2^2} + \ldots + \frac{1}{2^{n-1}} = \frac{2^n \quad 1}{2^{n-1}}$ for all positive integers n!

Data analysis in this study starts from data reduction, data exposure, and conclusion. The profile of students' thinking in solving mathematical induction questions in this study was analyzed using thinking indicators that adapted from Isroil, et al (2017) is presented in Table 2.

Table 2. Student Thinking Profile Indicators in Solving Mathematical Induction Problems

Thinking Activities	Indicator
Receiving information	Access information from the task of mathematical induction is given by reading the task given.
Processing information	Linking new information to the knowledge it has by mentioning what is known and what will be proven from the task of mathematical induction given.
	Linking prior knowledge with the information provided to determine the solution.
Storing information	Repeating information by mentioning information related to mathematical induction problems faced to solve the task of mathematical induction.
	Repeating the steps of solution that are used by giving arguments when finding the relationship between mathematical induction task with the knowledge that they have before.
Recalling information	Associating between what is known with what will be proven based on the knowledge that has been possessed by mentioning the knowledge possessed related to the task of mathematical induction.
	Using prior knowledge about concepts, operations, or formulas that correspond to the information received to solve the task of mathematical induction.

RESULT AND DISCUSSIONS

1. **The Profile of Students Thinking With High Mathematics Ability in Solving The Task of Mathematical Induction**

Subjects with high mathematical abilities (T) in solving the task of mathematical induction, when receiving information, T access information from the questions given by reading questions without making a sound. T focus on the information provided without the help of other things. Slavin (2009) explains that attention is active focus on certain stimuli to the exclusion of others. T processes information by linking its prior knowledge of the known understanding that contains the formula, and the understanding proven in the form of a command word "prove" with the question given. T interprets information from the question by linking information from the problem to her prior knowledge. Thus, T also chooses to use the principle of mathematical induction as a completion step because it is listed in the question. This is related with Slavin (2009), the perception of stimuli, is influenced by our mental state, past experience, knowledge, and others.

T storing information by rehearsing information that is known and proven, then rehearsing the solution steps used and adapting it to the principle of mathematical induction. However, subject T uses the principle of mathematical induction without knowing the concept, subject T only uses the principle of mathematical induction in procedural with the principle step of mathematical induction without knowing the meaning and relationship of each step of the principle. Slavin (2009) said that rehearsal is a mental repetition of information, which can improve its retention. Subject T recalling information using information that is known and who is asked from the question and uses previous subject knowledge to correctly calculate operations of numbers, fractions, exponents to solve the problem. But subject T uses the concept of the principle of mathematical induction is not right because, she gets a same meaning between Un and Sn. This is related with Slavin (2009), that semantic memory is a part of long term memory that stores facts and general knowledge.

This is in accordance with the research conducted (Isroil et al., 2017) that students with high mathematical abilities have good and correct abilities in solving problems

2. **The Profile of Students Thinking With Average Mathematics Ability in Solving The Task of Mathematical Induction**

Subjects with average mathematical abilities (S) in solving the task of mathematical induction, when receiving information, S accesses information from a given question by reading questions without making a sound while pointing to the question. S focuses on the information provided with the help of the sense of touch. Slavin (2009) explains that attention is active focus on certain stimuli to the exclusion of others. One of study strategies help students learn is underlining, it helps students to know the most important information from the task they have. S processing information by finding what is known and what is proven without relating it to the known and proven understanding. S interprets the information from the question by linking information from the problem to her prior knowledge and it influenced S to using the principle of mathematical induction as the solution step because it is listed on the question. This is related with Slavin (2009), the perception of stimuli, is influenced by our mental state, past experience, knowledge, and others.

S storing information by rehearsing information that is known and proven and rehearsing the solution steps used and adjusting it to the principle of mathematical induction. However, subject S uses the principle of mathematical induction without knowing the concept, subject S only uses the principle of mathematical induction in procedural with the steps of the principle of mathematical induction without knowing the meaning and relationship of each step of the principle. Slavin (2009) said that rehearsal is a mental repetition of information, which can improve its retention. Subject S recalling information using information that is known and asked from questions and uses prior knowledge operations to calculate numbers, fractions and multiplication factors correctly. This is related with Slavin (2009), that semantic memory is a part of long term memory that stores facts and general knowledge.

This is in accordance with the research conducted (Isroil et al., 2017) that students with average mathematical abilities have good ability to solve problems

3. **The Profile of Students Thinking With Low Mathematics Ability in Solving The Task of Mathematical Induction**

Subjects with low mathematical abilities (R) in solving the task of mathematical induction, when receiving information, R accesses information from a given question by reading the questions with making a low sound while pointing to the question. R focuses on information provided with the help of the sense of touch and the senses of sound. Slavin (2009) explains that attention is active focus on certain stimuli to the exclusion of others. One of study strategies help students learn is underlining, it helps students to know the most important information from the task they have. R processing information by finding what is known and what is proven without relating it to the known and

proven understanding. R interprets the information from the question by linking information from the problem to the knowledge they had before, then she using the principle of mathematical induction as the solution step because it is listed on the question. This is related with Slavin (2009), the perception of stimuli, is influenced by our mental state, past experience, knowledge, and others.

R storing information by rehearsing information that is known and proven and rehearsing the solution steps used and adapting it to the principle of mathematical induction. But, subject R uses the principle of mathematical induction without knowing the concept, subject R only uses the principle of mathematical induction in procedural with the principle step of mathematical induction without knowing the meaning and relationship of each step of the principle. Slavin (2009) said that reherasal is a mental repetition of information, which can improve its retention. Subject R recalls information using information that is known and asked from the question and uses previous subject knowledge operations to calculate numbers, fractions and multiplication of factors with wrong results. This is related with Slavin (2009) that semantic memory is a part of long term memory that stores facts and general knowledge.

This is in accordance with the research conducted (Isroil et al., 2017) that students with low mathematical abilities have not good at solving problems

CLOSURE

Conclusion

Based on the analysis result and discussion, it could be concluded as follows.

1. The Profiles of Subject Thinking with High Mathematical Ability in Tasks of Mathematic Induction

   When receiving information, the subject accesses information by reading the question without sound. When processing information, the subject associates information that is known and proven from the problem with the understanding of mathematical statements in the form of a sequence and command words on the question. The subject decided to use the mathematical induction completion step according to the information on the task.

   When storing information, the subject repeats information by using the principle of mathematical induction to solve the task with an understanding of the principle of mathematical induction, but that is not appropriate, and using information from the previous completion step for the next step of completion. When recalling information, the subject relates what is known and proven correctly and uses the previous knowledge about the operation of calculating numbers, fractions, multiplying factors, and exponents correctly, even though the principle of mathematical induction used is incorrect
2. The Profiles of Subject Thinking with Average Mathematical Ability in Tasks of Mathematic Induction

   When receiving information, the subject accesses information by reading the question without making a sound while pointing to the question. When processing information, the subject associates known and proven information without giving the right argument. The subject decided to use the mathematical induction completion step according to the information on the question.

   When storing information, the subject repeats information by using the principle of mathematical induction to solve the task even with an understanding of the principle of mathematical induction that is not appropriate, and using information from the previous completion step for the next step of completion. When recalling information, the subject relates what is known and proven correctly and uses prior knowledge about the operation of calculating numbers, fractions and multiplication correctly even though it has not found a solution, even though the principle of mathematical induction used is incorrect.
3. The Profiles of Subject Thinking with Low Mathematical Ability in Tasks of Mathematic Induction

   When processing information, the subject associates known and proven information without giving the right argument. The subject decided to use the mathematical induction completion step according to the information on the task.

   When storing information, the subject repeats information by using the principle of mathematical induction to solve the task even with an understanding of the principle of mathematical induction that is not appropriate, and using information from the previous completion step for the next step of completion. When recalling information, the subject associates what is known and what is proven correctly and uses prior knowledge about the operation of calculating numbers and fractions is incorrect, and the results obtained are incorrect and the principle of mathematical induction used is less precise

Suggestion

As for the suggestions that the researchers submitted in this study are as follows:

1. The results of the study indicate that there are differences in thinking students who are capable of high, average, and low mathematics ability in receiving information and recalling information. So, the teacher

can pay attention to the level of students' mathematical abilities when carrying out learning activities.

2. This study examines the level of mathematical abilities with research subjects of female sex, researchers suggest that further research can be carried out with reviews from other aspects, and subjects with male gender.

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