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ANALYSIS OF LEARNER'S CONJECTURE ABILITY IN SOLVING OPEN-ENDED PROBLEMS

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Abstract

Conjecture will always be used by learners in problem solving, because the conjecture itself is tied to activities such as logical reasoning, translating problems, analyzing and evaluating an information to obtain valid decisions related to problem solving, where the conjecture is also able to develop the learning process of the learner in making a statement, especially with the help of open problems in its application, Which can make learners morecreative. This research aims to illustrate the conjecture ability of learners in *open-ended* problems with descriptive types of research and qualitative approaches to number pattern material, especially generalizing patterns. The subjects of the study are four learners who have a high and moderate level of mathematical ability and are willing to take part in interviews. The results showed that all subjects have not been able to perform every stage on constructing the conjecture, especially in the stage of arguing the conjecture and there is one subject who does not do the stage of proof of the conjecture because it is confident in the formula that has been given by the teacher. So that learning activities are needed in which there is problem solving that collects the ability of learners' contours, open-ended problems can also be one of the problem choices that can help students build their thought processes independently, and not bound by the formula of teachers or books.
Keywords: Konjektur, Open-Ended, Generalize Patterns,NumberPatterns

Abstrak

Konjektur akan selalu digunakan oleh peserta didik dalam pemecahan masalah, karena konjektur sendiri terikat dengan kegiatan seperti penalaran yang logis, menerjemahkan masalah, menganalisis serta mengevaluasi suatu informasi untuk memperoleh keputusan yang valid terkait dengan penyelesaian masalah, di mana konjektur juga mampu mengembangkan proses berfikir peserta didik dalam membuat suatu pernyataan, terlebih dengan bantuan soal terbuka dalam penerapannya, yang mampu membuat peserta didik lebih kreatif. Penelitian ini bertujuan untuk menggambarkan kemampuan konjektur peserta didik dalam soal *open-ended* dengan jenis penelitian deskriptif dan pendekatan kualitatif pada materi pola bilangan khususnya menggeneralisasi pola. Subjek penelitian adalah empat peserta didik yang memiliki tingkat kemampuan matematika tinggi dan sedang serta bersedia dalam mengikuti wawancara. Hasil penelitian menunjukkan bahwa semua subjek belum mampu melakukan setiap tahap pada mengontruksi konjektur, terutama pada tahap mengargumentasikan konjektur dan terdapat satu subjek yang tidak melakukan tahap pembuktian konjektur karena yakin dengan rumus yang telah diberikan oleh guru. Sehingga diperlukan kegiatan pembelajaran yang didalamnya terdapat pemecahan masalah yang menagih kemampuan konjketur peserta didik, masalah *open-ended* juga dapat menjadi salah satu pilihan masalah yang dapat membantu siswa membangun proses berfikirnya secara mandiri, dan tidak terikat dengan rumus dari guru maupun buku.
Kata kunci: Konjektur, *Open-Ended,* Menggeneralisasi Pola, Pola Bilangan

INTRODUCTION

Permendikbud Number 58 and 59 of 2014 about curriculum 2013, states that the purpose of learning mathematics is to be able to communicate ideas, reasoning, and compile mathematical evidence using complete sentences, symbols, tables, diagrams or other media to clarify problems. And according to

the *National Council of Teacher of Mathematics* (NCTM) about the standard purpose of the school's mathematical reasoning process and proof is to create and investigate mathematical conjectures. Based on these two things, conjecture is needed to achieve the goal of learning mathematics. In line with statements by Garcia and Benitez (2011), the ability of the conjecture has an important role in the evidentiary process. By having conjecturing ability, learners are able to predict a pattern, change a problem to a mathematical model and predict the results obtained from mathematical activities. Similarly in problem solving, because of the ability of conjecture is related to logical reasoning, translating problems, analyzing and evaluating information to obtain valid decisions, the conjecture plays an important role in problem solving. And the conjecture is also able to develop the thought process of learners in making a statement, especially with the help of open problems in their application, which is able to make learners more creative.

The conjecture was first known as a proof, and since 2010 has changed as a conjecture ability in which there is proof as a special form (Yani & Hadi, 2020). According to Pedemonte (2001), konjektur is a statement that zis most likely considered true because there areseveral concepts that build an argument that justifies it. In his book, Daniel Shanks (2001) (in Byers, 2007) states, "conjecture is a *propotition that has not been proven, but is favored by some serious evidence"*, meaning that conjecture is an argument that can be used in solving a mathematical problem but not formally proven. Whereas according to Norton (2000), conjecture is an idea formed by learners based on his experience that meets the nature: conscious, uncertain and validation is doubtful. A conscious idea means a true, real or original idea that is logical, while uncertain is to have a foundation that does not conclude or is notconstituency or notlutive, while validated is a true mathematical statement based on observation, investigation, or exploration. Based on the above definition, a conjecture is something that even without formal proof, is still considered true, real, or original. Thus, the ability of conjecture is the ability to make a statement that can be considered true without formal proof. It is that evidence that will build a person's belief that the statements made are true.

According to Astawa et al (2018), there are 5 stages in building conjecture, one of which is generalizing and finding patterns in problems, because generalizing patterns is an important aspect in teaching and learning activities (Sutarto et al, 2016; Dindyal, 2007; Vogel, 2003; Zazkis and Liljedahl, 2002). So, researchers took issues related to generalizing patterns to reveal the ability of learners in building conjecture. To explore the idea of learners in building conjecture and generalizing, namely by inviting them to do learning with pattern images. Learners will get freedom in using complete sentences, symbols or using a convenient way for them to solve the given problems and the ability of learners in describing the generalization of problems is one of the indicators that will be seen in this study, in accordance with the stage of conjecture development and indicators in table 2.

Shiriki (2013) states that when learners have the freedom to apply ideas and find diverse strategies, they can explain their thinking in finding varied answers, learners will be able to develop their thinking and reasoning skills. , where the ability to benalar affects the process of building conjecture. Freedom to implement ideas and use diverse strategies can be implemented with an *Open-Ended*approach. *Open-ended* according to Agustianingsih and Mahmudi (2019) is a question that has more than one correct answer and strategy in solving it. In open-ended questions, learners are not limited to exploring each other's thinking skills to answer questions. According to Shimada and Brecker (1997) The open ended approach itself is considered able to providestudents withmore knowledge, experience in finding, recognizing and solving problems, because the problem has more than one method and solution. So that students can be more active and creative in finding solutions to problems.

Open-Ended questions have three principles of openness: open processes,opensolutions and ways to develop open (can be developed into new problems by changing the initial conditions). The existence of these principles makes open questions provide an opportunity for learners to explore them both in terms of answers, resolution strategies and furtherdevelopment. Shimada (1997) also classifies the type of open question, namely the type of finding a relationship or mathematical formula, the type of classification that is classifying kaakteristic to find concepts and types of measuring and calculating.

Because in this study researchers want to analyze the ability of learners in building conjecture on the problem of number patterns, then the type of open question is the type of finding relationships. However, introducing open problems is not easy, especially in *textbooks* prioritize routine questions

(closed questions). This is what makes teachers and learners accustomed to problem solving without the need for high-level thinking skills. So based on Yani Supriani (2017), learners only believe in the answers or process of work provided by teachers without first looking for other strategies on other learning resources, so that they do not build their own thinking process and learning is only teacher-centered. So that creative learning is needed and requires the skills of learners in providing answers, challenging and giving learners space to solve problems according to their talent interests, physical and psychological development.

In previous research by Astawa; et al (2016; 2020) and Yuniati; et al (2018), use this type of problem routinely or closed, and are limited to only 2 research subjects so it is advisable to use other problems and expand the research subject. Therefore, the author wants to analyze how the process of building conjecture in learners in solving open-ended problems in number patterns.

METHOD

This research uses a type of descriptive research with a qualitative approach because in analyzing the process of conjecture formation in learners in completing open-ended problems required data in the form of decryption. Research that has the goal to analyze the process of learners' ability to develop their conjecture isjalan with the aim of descriptiveresearch presented by Siswono (2019), which describes a condition in a situation.

There are four stages to this study. First, the preparation stage, researchers observe and ask permission from the school, and prepare instruments that are open problems and able to charge the conjecture ability of learners. The subjects of this study were four 8B grade learners at SMP Negeri 3 Tuban, the research subjects were selected by purposive sampling method with characteristics of having high-level and moderate mathematical abilities that have been adapted to the school's KKM as attached to table 3 as well as willing in taking part in interviews. The category of mathematical ability is based on research from Nurutami; et al (2018) as stated in the table below:

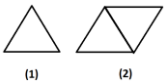
Table1. Category of Mathematical Ability

Score	Category
Score ≥ 85	High
70 ≤ score < 85	Moderate

The selected subject will perform a conjecture ability test related to number patterns, precisely determining the nth term of an object configuration. Instruments for this research there is a test of conjecture ability in open-ended problems with indicators are those at the stage of building a conjecture according to Astawa (2018), namely understanding problems, exploring problems, formulating conjectures, arguing conjecture, and proving conjecture as in table 2,and there are also interview guidelines. Here is an instrument test of conjecture ability on open-ended questions*:*

Conjecture ability test on open-ended questions

1. Take a look at the following image:

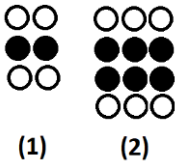


   a. Continue the figure above and determine the number pattern!
   b. Find some ways (it could be a formula) to find the nth image and explain it!

2. Take a look at the following image:

   a. If the figure above is the first figure, determine the next image and explain the pattern of numbers formed!
   b. Find a way (it could be a formula) to find the *n*th image and explain your answer!
   c. If the figure above is the second image, specify the first image and the next image and explain the pattern of numbers formed!
   d. Find a way (it could be a formula) to find the nth image of the diath of the diath number pattern and explain your answer!

3. Take a look at the following image:



   a. Take a figure and describe the fourth figure!
   b. Describe the pattern of numbers formed!
   c. Find a way (can use the formula) for the sum of all dark circles in the nth image and jelaskan your answer
   d. How many dark circles are there in stage 11?

Figure 1 Conjecture ability instruments on open-ended issues

Second, the implementation, at this stage researchers conduct data retrieval in the form of tests and interviews. The test is carried out in the mathematics laboratory room, while the interview is conducted online through *whatsapp* based on the results of tests that have been conducted by the

subject, the interview lasts a maximum of 1 hour of lessons in accordance with the fulfillment of the information needed related to the construction of conjecture.

Furthermore, in the third stage is data analysis. At this stage, the authors analyze the test results of the learners and the results of the interview. The analysis of the results of the learners' tests will be based on indicators of constructing conjecture based on Astawaet al. (2018),namely as follows:

Table 2. Indicators of building conjecture

Constructing a conjecture	**Indicators**	**Code**
Understand the problem	a. Learners read the problems given b. Learners determine whatis known and asked in the problem.	T1
Exploiting problems	a. Learners turn problems into the language of mathematics b. Learners manipulate problems c. Learners find pola in problems d. Learners determine the essentials found and connect with relevant mathematical knowledge in identifying the patterns being observed.	T2
Formulate a conjecture	a. Learners write conjecture with reference to the results of exploration problem	T3
Giving an Argument	a. Learners explain arguments for conjectures that have been formulated b. Learners generalize conjecture c. Learners recognize deficiencies and errors in formulated conjecture or in underlying arguments.	T4
Proving the conjecture	a. Establishing conjecture evidence	T5

As for the interview, the analysis is conducted with an interactive model, the analysis stage according to Siswono (2019) begins by reducing the data, which is reducing data that is not relevant to the development of student conjecture, or unnecessary data to clarify the figure of the data obtained. Furthermore presenting the data, namely interviews that have been conducted presented in the form of transcripts, then the last is to draw conclusions,the withdrawal of theseconclusions is not only based on the results of interviews but also the results of analysis of learners' answers. This stage can be done repeatedly to produce precise and accurate conclusions in accordance with the research.

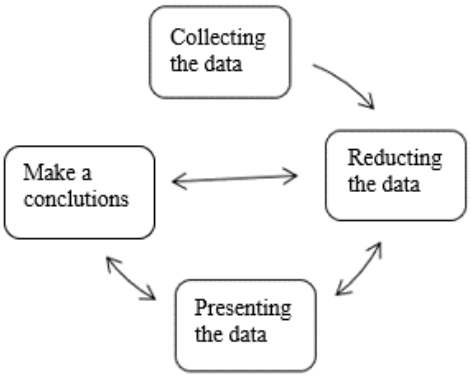


Figure2. Interactive analysis model

The last stage in this research is to compile a research report related to the ability of learners in building conjecture on open-ended problems.

RESULTS AND DISCUSSIONS

The four subjects selected based on average assignment scores, daily quiz, and math midterm attached to the table below are learners with high- and moderate levels of math skills:

Subject	Code	Average value	Category
Subject 1	S1	89,1	High
Subject 2	S2	87,4	High
Subject 3	S3	82,5	Moderate
Subject 4	S4	78,4	Moderate

Based on the results of conjecture ability tests on open-ended questions and interviews that have been conducted by the subject, the results of the analysis are obtained as follows:

SUBJECT 1

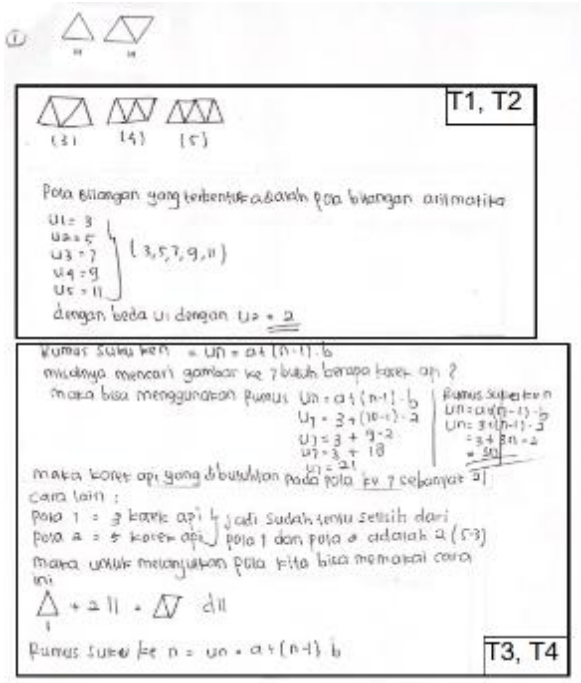


Figure 3. Answer of S1(question 1)

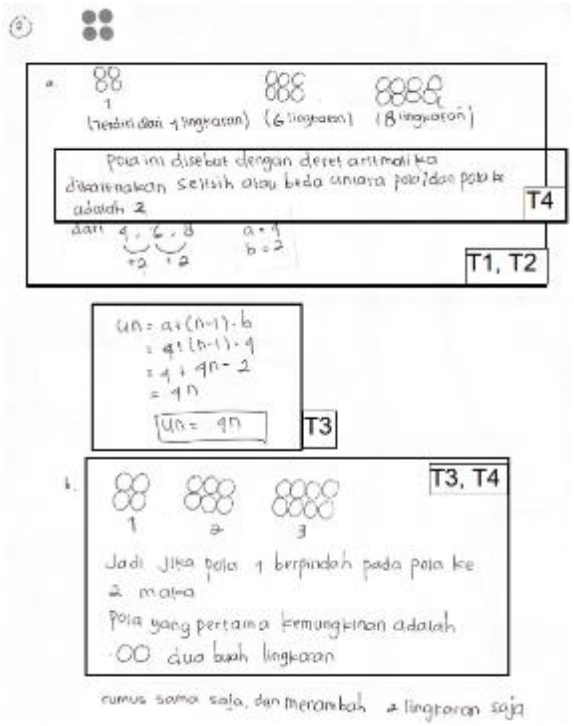


Figure 4. Answer of S1 (question 2)

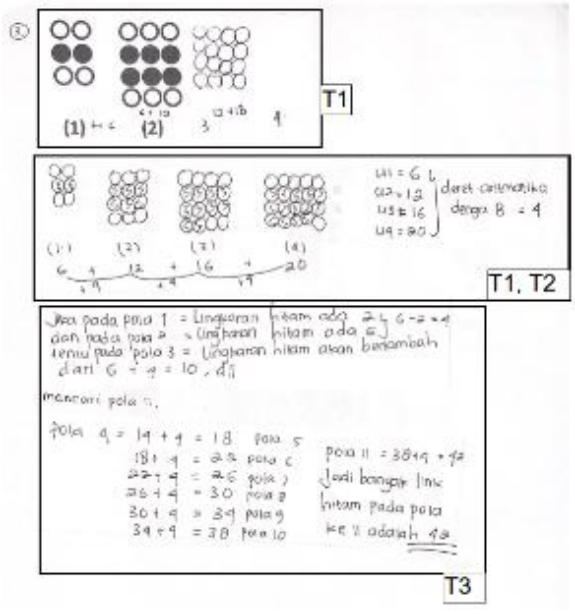


Figure5. Answer of S1 (question 3)

At the level of understanding, S1 is able to understand what is asked in the given question, in some problems she is able to give what is asked and write down the information he gets on the problem, such as in figure 2 and figure 3.

Reseacher	:What can you understand about a given problem?
S1	: the question to find the number pattern of a given image and determine the formula looking for the *n*th term.

Reseacher	: How do you understand it?
S1	: Considering what the teacher has learned and given before, it makes some of the problems that I knew before I worked like continuing a pattern,which I've seen in books or the internet before.
Reseacher	: Approximately, how many times on average to read the question to understand what it means?
S1	: to understand an easy problem at least I read the problem 3 times, but to understand difficult problems I usually read the problem more than 3 times to understand
Reseacher	: Which question takes a long time to understand? Why?
S1	: About determining the *n*th term, because in that question I also need to think how to do it, because I often only use the general formula to find the *n*th term

Figure 6. Transcript for S1 interview (T1)

In the interview, S1 explained that it is very easy to understand the problem given because he often saw it on the internet or books that he uses in learning, The excitement in reading also helps her in thinking about how to solve the problem.

Reseacher	: What information do you think is important to help you understand and answer questions?
S1	: I think it is to know the patterns formed in the image and the formula to find the *n*th term
Reseacher	: Can you estimate how to solve the problem when you read it?
S1	: There are some questions that I can estimate the answer, because it is almost the same in the book
Reseacher	: Can this information help determine the number pattern?
S1	: Yes, at the same time help to know the formulas, types, and patterns that are formed in the next terms. The exmple is on problem 1, we are asked to draw patterns to 3, 4, and 5, we have been told that the triangle pattern formed on the pattern of 3 matchsticks, while in the second pattern consists of 5 matchsticks, so the difference of each pattern is 2, then the 3rd pattern we add 2 in the 2nd term so its 7, then the 4th term we add 2 so its 9, and so does The 5th pattern = 11.

Figure 7 Transcript for S1 Interview (T2)

At the stage of exploring the problem, S1 begins to associate the problem with the concept of number patterns, according to him information that is important in solving existing problems by recalling knowledge of number patterns including general formulas in number patterns, assisted by observing the given image and generalizing it into the first term, second term and so on in the number pattern. In completing the first problem, S1 looks with a different point of view from other subjects, she does not directly see the number of triangles but instead calculates the lines that exist and are associated with life, i.e. declare the line as a match. In exploring the problem to help her in finding number patterns, S1 also uses help such as in the figure 2 to get the difference in each term.

Reseacher	: What do you do to determine the term to n?
S1	: Read the problem, understand, determine the difference and then look for the appropriate formula by changing the general formula based on the first term and the difference.
Reseacher	: What kind of conclusions did you get?
S1	: By understanding the problem we can find out how to solve the problem. we see in the first term or*(a)*,different from the first and second*(b)*,and the formula used to solve the problem. On the problem 1, the formula from the substitution of *a* and *b* should be, $Un=a+(n-1).b = 3+(n-1).2 = 3+2n-2= 2n+1$
Reseacher	: If the first term is an important thing, why for a new pattern on the problem 2b, it has the same formula as the problem 2a?
S1	: Because it only adds the previous and the next terms.

Figure 8 Transcript for S1 interview(T3)

Furthermore, that is the stage of building a conjecture, S1 uses knowledge of the concept of number patterns such as terms, differences and general formulas to determine the nth term. So, in building her conjecture, S1 combines the information he gets such as how much the first term is, what is the difference, and substitutes it in the *n*th term general formula. It was applied to the first and second problems, it's just that there was an error in simplifying on problem 1. Whilefor the third problem, S1 describes one by one the existing patterns, which means that S1 cannot find the formula and rely on calculations one by one until the *n*th term. In the work of this third problem S1 makes

a different pattern also from other subjects, where he adds 4 dark circles to each increment, regardless of the number of white circles, so she managed to create a pattern for the whole, dark circle, but not to the white circle. In the second question, S1 makes a mistake with the formula for a new pattern that changes the first term even though the difference remains, contradicting its assertion that in making the formula it must look at the first term and the difference.

Reseacher	: Are you sure of your answer?
S1	: I'm very confident in the way I do the work, because I have learned to find the formula of the nth term and how to draw patterns, through the problems that the teacher gives. AlthoughI have never done this kind of problem, but I am very confident for some reason, what I like about this problem is that I have learned something new and I am very happy because I can find the appropriate formula for this problem.

Figure 9 Transcript for S1 interview (T4)

At In the stage of arguing the conjecture, S1 seems difficult in convincing that the answer he made is correct, the argument he gave is only limited to the statement that he has done almost the same problem, But he feels confident that what he's saying is true.

Reseacher	: How do you prove that the answer you Get is correct?
S1	: We have found the *n*th-term formula of the number pattern (3,5,7,9,11) is 2*n*+1. Based on previous information, we know that *U*1=3, *U*2=5, *U*3=7, *U*4=9, *U*5=11. So you can try for example looking for *U*3, then 2*n*+ 1 = 2(3)+1= 6 +1 =7. So, the conclusion of the formula that we have made is **TRUE**

Figure 10 Transcript for S1 interview (T5)

Although she could not argue for an answer, S1 was able to prove his formula by looking for the term he had found based on the figure when conducting the interview and admitted to miscalculating or simplification when working on the problem.

SUBJECT 2

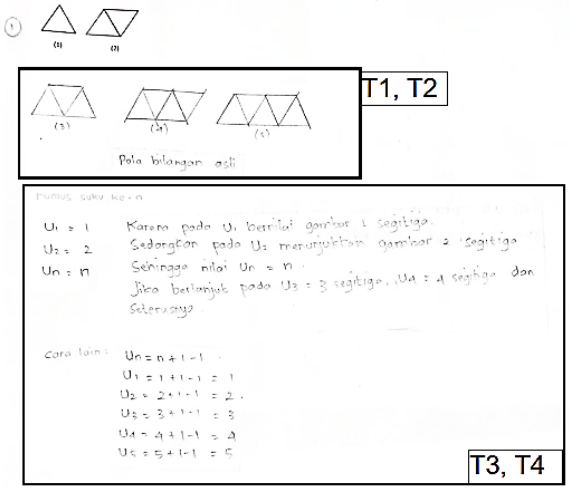


Figure 11 Answer of S2 (question 1)

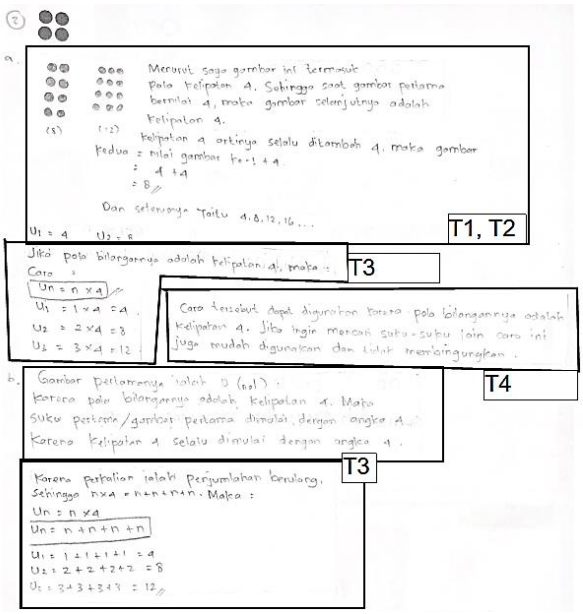


Figure12. Answer of S2 (question 2)

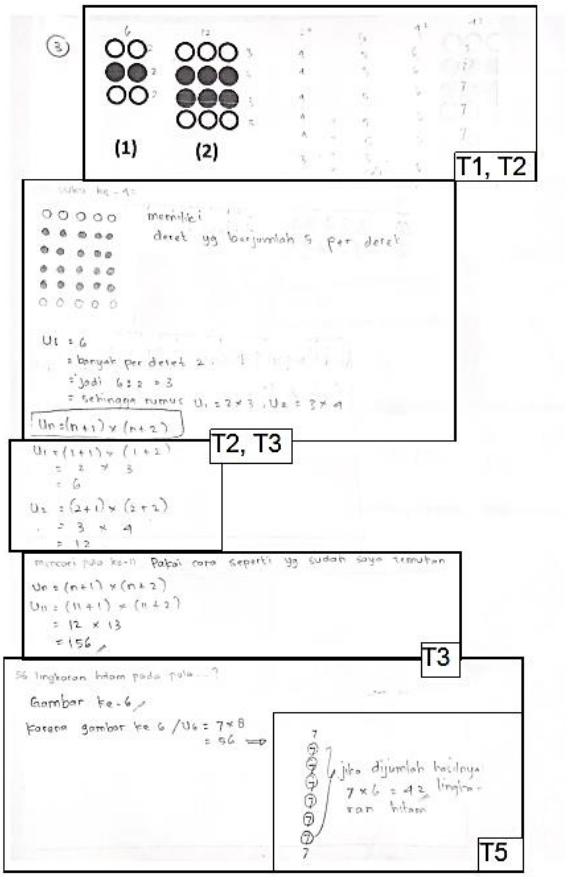


Figure 13. Answer of S2 (question 3)

As with S1, S2 is able to carry out the stage of understanding the problem, it's just that there is still an error of understanding caused by not focusing on reading the third problem, S2 without reading directly concludes the same question as the previous question, and there is a misconception related to the first term in the form of zero, it uses the second term (non-zero number) as the first term.

Reseacher	: How do you understand what you're asking?
S2	: I can understand the problem when I read it carefully, focused and repeatedly about 3-4 times.
Reseacher	: What do you understand about the problem?
S2	: We are told to continue the image until it forms a pattern and look for the *n*th formula.
Reseacher	: In question 3 asked the *n*th-term formula for dark circles only, why the formula you make for the whole?
S2	: Sorry, I'm less focused on problem 3, because I think it's hard to know the pattern of the figure, so I don't focus on 'dark circle'.
Reseacher	: What information do you think is important for understanding and solving problems?
S2	: Information about the known pattern of the image given to each problem, because with this information we can more easily know the purpose of the problem and the figure of how to solve.

Figure 14 Transcript for S2 interview (T1 and T2)

Based on the interview, S2 reads the problem carefully, focused and repeated 3-4 times to understand the problem. There was also a misconception in problem 3, she thought the problem would be the same, so she looked for formulas for the whole pattern, not just for the dark circle pattern.

At the stage of exploring the problem, S2 focuses on finding patterns from a given image and then changing them in the form of rows of numbers directly, simply put, S2 calculates the number of elements in each image. According to her, by observing the image and obtaining the pattern, it will be easier to know the purpose of the problem and an overview of how it is done. But for the third problem, S2 does not count as many whole forms as the problem of 1 and 2, but pays attention to many series and rows, so that the pattern of numbers formed is sequential multiplication (2x3), (3x4), (4x5), and so on.

Reseacher	: What do you do to determine the term to n?
S2	: Knowing the term, as in the first question, $u1= 1$, $u2= 2$, $u3= 3$, then I know that the value and term are the same, so $Un = n$. Then in the third question, because $u1 = 2x3$, and $u2$ is $n+ 1$ while $u3$ is $n+ 2$ then the corresponding formula is $n=(n+1)(n+2)$
Reseacher	: But the formula is for the overall pattern, how about dark circles only?
S2	: Can do manual calculations, because the pattern of the compound formed 2, 6, 12, 20, then it can be further added different before plus 2. Then there will be many dark circles in the 11th term is 132 and 42 dark circles are the 6th term.
Reseacher	: What kind of considerations do you do to get these formulas?
S2	: I'm just trying to find the right results and ways to do it.

Reseacher	: Are you sure of your answer?
S2	: Must be sure, because I have tried my best to test the formula I created with the information I found earlier.

Figure 15 Transcript for S2 interviews (T3 and T4)

Unlike the S1, S2 looks to do a simpler process in instructing the conjecture in finding a way to find the nth term. She simply observes the pattern of rows formed and then dabbled in the right formula to look for known terms, then changed them for the nth term. As in figures 11, 12, and 13. As well as when asked another way, S2 simply modifies by adding a number operation that has a zero result as in figures 11 and 12. In figure 11, the first formula he finds is *Un=n* and then S2 simply modifies by adding an operation saying 1-1 which results in a zero, meaning that the operation of the number does not change the initial formula he found. Similarly for figure 12, the initial formula found is *Un=n*x4,S2 only changes multiplication to repeat summation so that the new formula obtained is Un =*n*+*n*+*n*+*n*. For the error she made at number 3, S2 chose to work manually because it forgot the formula of the multilevel number pattern, following the answer S2 in the interview:

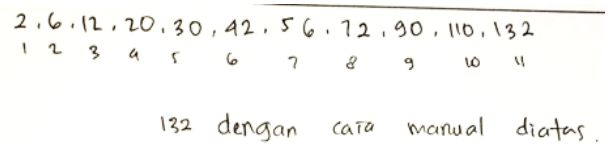


Figure16. Answer of S2 (revision of question 3)

The stage of arguing the conjecture is also still less achieved by S2, but she is better able to convince others about the answer compared to other subjects, because she writes the reason for the answer in each answer as in figure 12. In her interview, S2 said he was confident in the answer because he had tried /tested various ways to get the right formula, and tried it on other terms as well.

Reseacher	: How do you prove that the answer you wrote is correct?
S2	: The proof, in the first case, the value and number or term are the same, then if *U*1 = 1, then the value is 1, if the value is *n*,then *Un=n*. Another example of the second problem, *U*1 = 4, the value is equal to one and the number is 4, so the number is a multiple of 4 of the value, then *U*1 = 1x4 = 4, then *Un=n*x4 = 4*n*

Figure 17 Transcript for S2 interview (T5)

To prove its conjecture, S2 did the same way as S1, which is to try it on the term he found from the figure given to the problem.

SUBJECT 3

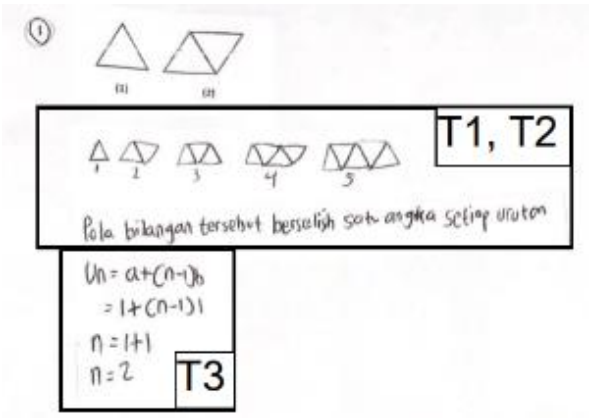


Figure 18. Answer of S3 (question 1)

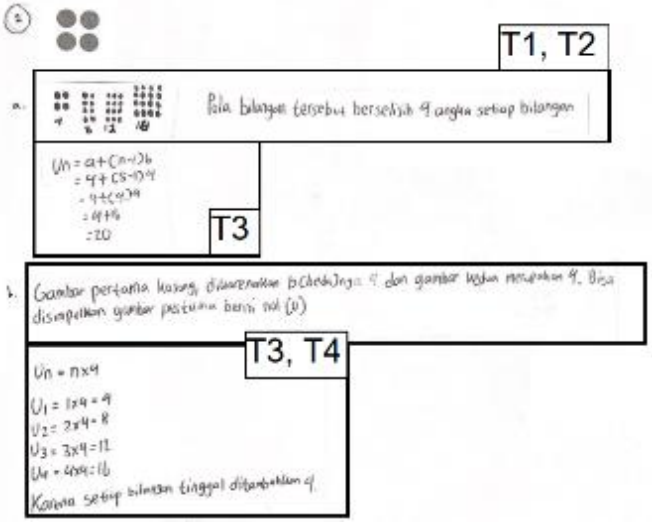


Figure 19 Answer S3 (question 2)

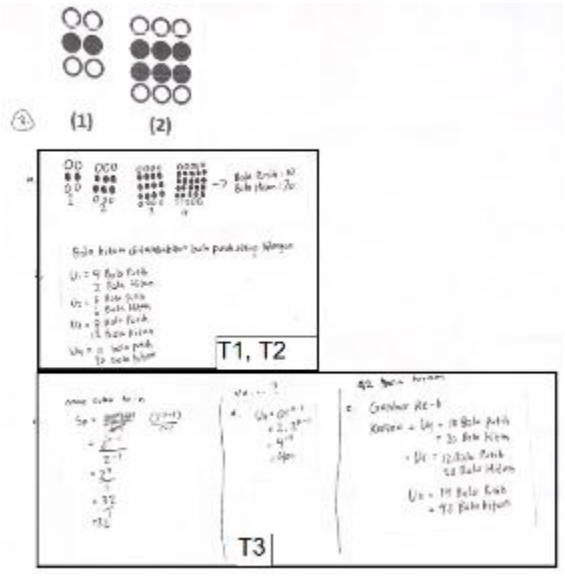


Figure 20. Answer S3 (question 3)

Reseacher	: What do you understand and how do you understand what is being asked?
S3	: Creates a number pattern and looks for a formula for that number pattern. I understand by understanding subjek, predicate and inti about what is asked.
Reseacher	: How many times have you read it until you understand the problemr?
S3	: Read it 2 times, and addition to number 3, because it requires modeling for the next pattern look like.

Figure 21 Transcript for S3 interview (T1)

At the stage of understanding the problem, S3 looks able to understand because writing the known information from the problem in the answer sheet, even during the interview S3 understands by looking for the point of what is asked about the question, because what is asked is how to find the *n*th term, it can be started by determining the pattern of numbers formed from the image. But there is still a slight misunderstanding in the third question, because in the problem there is the word 'all dark circles' makes S3 use the formula of geometric series, which is a contradiction with the number pattern he made.

Reseacher	: What information do you think is important for understanding and solving existing problems?
S3	: Problems that use easy languange for learners to understand and with a direct question for example on this problem is given a figure. So when reading, it will be directly think what the next pattern, the pattern of questions and how to find the answer.

Figure 22 Transcript for S3 interview (T2)

And in the stage of exploring the given problem, S3 also looks to have no problems, he is able to turn the image that forms a pattern into a row of numbers. Based on the results of the interview, through the images given, he immediately thought about how the next pattern and how to find the answer. The number pattern that is made is almost the same as the answer S2, it's just that the point of view is different. In the third question, S2 looks at the whole through the number of rows and columns, while for S3 it focuses more on dark circles and white circles, so there are two number patterns. This can be seen in figures 13 and 19.

Reseacher	: What do you do to determine the term to *n?*
S3	: Specifies the *n*th term based on the existing general formula, so we can subtitute *a* and *b* in the formula.
Reseacher	: What kind of considerations do you do?
S3	: No consideration, directly using the formula that has been practiced
Reseacher	: For the formula of course there are variables used, why does your answer produce a number instead of a variable?
S3	: Since I thought when *a* and *b* had subtitue by a number, then the next *un* would be equal to 0, I was confused as to how to calculate it.
Reseacher	: Why for problem 3 you use the geometric sequences formula?
S3	: Because it is a geometric sequences and the problem is told to count the number of dark circles.
Reseacher	: How can you conclude that the rows formed are geometric sequences?
S3	: Because, the *b* in arithmetic have the same sum/number.

Figure 23 Transcript for S3 interview (T3)

At the stage of constructing the conjecture, it appears that S3 has difficulty, although in the interview he said that entering or substituting information from the number pattern he got earlier in the general formula, but it appears on the answer sheet that S3 has difficulty in simplifying the formula, where it should be looking for *Un*,S3 actually looks for n. It happens repeatedly in every given problem, for the second problem, S3 enters *n*= 5, which results in Un not being a parameter but a number.

S3 was also fooled by the pattern of multilevel numbers, he thought that it was a geometric sequence, and because there was a word 'number' then he used the formula Sn geometric rows.

Reseacher	: Are you sure of your answer?
S3	: Sure, because it has tried and tested correctly the results
Reseacher	: How do you prove the truth?
S3	: Because the formula used is a general formula given during learning, so the answer is definitely correct.

Figure 24 Transcript for S3 interview (T4 and T5)

In the next stage, namely arguing and proving the conjecture, S3 feels confident in the answer because it has obtained results, but does not explain

why the results are said to be true. And he could not prove the truth of the formula he had made.

SUBJECT 4

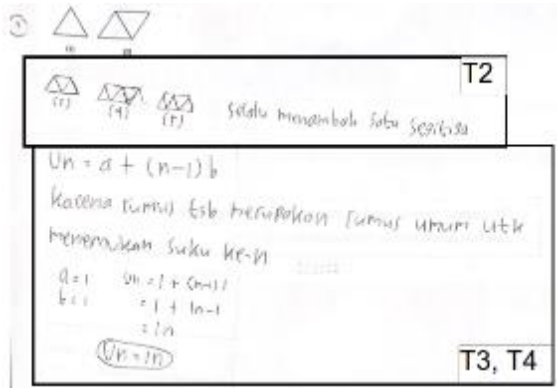


Figure25. Answer of S4 (question 1)

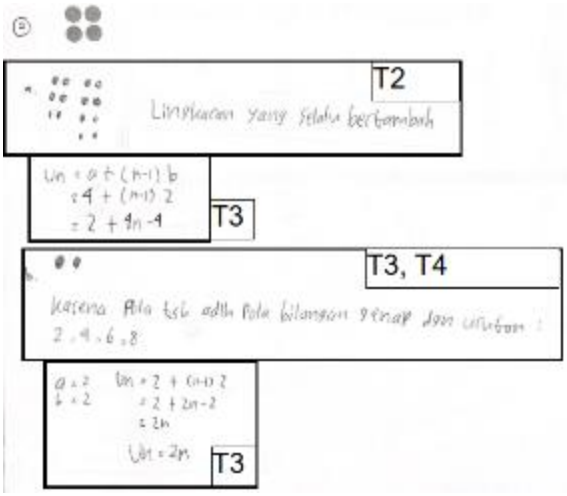


Figure 26. Answer of S4 (question 2)

Figure 27. Answer of S4 (question 3)

Reseacher	: What do you understand and how do you understand what is being asked?
S4	: With the given image, we are told to make a next term and create the formula *Un*. I read more than 3 times and observe the image given to the problem to understand that.
Reseacher	: In each question, asked the pattern of numbers that are affected, what do you think the problem means?
S4	: The pattern, or the difference from one image to the next image.
Reseacher	: Pay attention to your answer to the third question, are there any irregularities?
S4	: Yes, I shaded the entire circle, so that in the figure there is no white circle. I'm not focused because it's the last question.

Figure 28 Transcript for S4 interview (T1)

The first stage is to understand the problem, at this stage S4 is able to understand the given problem but there is also a misconscital when asked about the pattern of numbers formed, S4 tends to answer as in figures 25, 26, and 27, even though he is able to continue the given image. The answer given by S4 is less precise because it is still too common. S4 said that he thought what was asked was the difference of each image in each term.

Reseacher	: What do you think is useful information to answer questions?
S4	:*a*, *b*, as well as the formula given by the teacher in class.
Reseacher	: How did you get that information?
S4	: Pay attention to the image and remember the formula.

Figure 29 Transcript for S4 interview (T2)

In the exploring stage, S4 is able to determine important information such as the first*(a)*,and thedifference*(b)*known from the number pattern. So S4 can be said to be successful in converting a given image pattern into a mathematical sentence. It's just that because of the rush, S4 is less exploring the third problem, where there should be dark and white circles, he actually makes the next term is black because it only pays attention to the number of elements without seeing others.

Reseacher	: What do you do to determine the term to *n?*
S4	: Since we know the first term and the difference, then we can include it in the general formula that has been studied, so as to get the formula *Un.*
Reseacher	: What kind of considerations do you do until you reach that conclusion?
S4	: since I know that the general formula *Un=a+(n-1)b*,and I know *a* and *b* on each problem, then I use that method to search for *Un*
Reseacher	: When generalizing, why do a and b change places?
S4	: I'm only remembered cross-multiplication.
Reseacher	: Pay attention to the formula *Un* on the third question, you say the pattern is a 6-step number pattern, what does it mean?
S4	: A number pattern that has a difference of 6
Reseacher	: But in finding *Un* you use *a*= 6 and *b*= 12 and then in the next step *a* becomes 12 and *b*= 6.
S4	: Should *be a*=6 and *b*=6, then *Un*=6+(*n*-1)6=6+6*n*-6=6*n*
Reseacher	: How do you find the 11th term and the term that has 42 dark circles in the third question?
S4	: Because it is different 6, then 11 I multiply by 6. Divide also for the circle numbering 42 I for 6 then produce 7 (term 7). But I was wrong for forgetting the white circle.

Figure 30 Transcript for S4 interview (T3)

The construction stage of the conjecture was not well done by S4, but he became the only subject who managed to construct a new formula for the pattern that changed its first term in figure 26. The thing he did to determine the nth-term formula like S1, which uses the existing general formula accompanied by the subtitute of information about patterns such as the first term and the difference in the formula, but experienced errors in simplifying the formula. As explained in the interview, that understanding and workmanship on the third problem is in a hurry, S4 seems unfocused in working, and he is able to explain the answer he meant in the interview.

Reseacher	: Are you sure of your answer?
S4	: Sure, because I use commonly used formulas as well as the information I get on the problem.
Reseacher	: How do you prove the truth?
S4	: for example, after the first question *of U*1 = 1, with the formula Un = *n* if *n* we replace 1, then you get *U*1 = 1. Then for the second problem, *a=U*1 = 2, with the formula *Un*=2*n* then *n* we change with 1, then *U*1 = 2.1 = 2. Proven right.

Figure 31 Transcript for S4 interviews (T4 and T5)

Like other subjects, S4 is also still lacking in explaining his arguments regarding the answers given although he is confident in the answer that uses the general formula of the
term to *n*. The proof is also the same as S1 and S2, which uses the terms that have been obtained in the image given to the problem.

Based on the above analysis, the four study subjects had never known about *open-ended* before, so they did not know that there were various kinds of solutions to the problem. But the results of the work of the four subjects vary according to how they view the given image, one of the four, namely S1, has a more complicated way of looking than the other. But that doesn't make the S1 lag behind in terms of building a conjecture. Disobedience to *open-ended* problems makes learners more confident that the problem only has one pattern that can be formed and a fixed n-term formula for each number. Although in the question asked to provide other patterns and formulas.

In building the conjecture, it is found that each subject is able to understand the given problem by reading it repeatedly until it is thought of what is said and how to solve it, S1 and S2 only do the reading in focus, while S3 looks for the core of the question to understand, and S4 is more on reading and observing the given image. There is a misunderstanding in the second question, which when billed to create an nth-term formula after the pattern is changed (the image that was originally as the first term shifted to the 2nd term), some subjects assume that the previous formula was only manipulated, but actually the same. Whereas as is known, if the first term *(a)* changes then the formula will change, because the first term is part of the formula. There is also a misunderstanding due to lack of focus in reading and thinking that the question is the same on every question.

At the stage of exploring the problem also looks no problem, each subject is able to find and explain information that can be used to solve the problem, such as finding the first term, difference, pattern of numbers formed and general formula to find the formula of the*nth* term based on the pattern of numbers he made. S1 explores by converting the given image into a number, then determining the difference and remembering the general formula to find the nth term, the same is also done by S4 which determines the first, different and recalls the general formula of the nthterm. Meanwhile, S2 and S3 are more concerned with images and converting to mathematical sentences to get important information. But surely every subject is also able to manipulate or convert the information into mathematics.

Furthermore, the stage of building a conjecture, in determining the*nth,*S1, S3, and S4terms*recalls* his knowledge that uses the general formula that he gets when learning in the class, and substitutes the information he gets such as the first and different terms in the formula. However, S3 was unsuccessful in constructing its conjecture due to its inability to simplify the formula, as well as S1 and S4 which also have misconceptions in calculations and simplification of formulas,it's just that S1 is able to correct it when conducting interviews. While S2 only pays attention to the information he gets and then manipulates it or converts it into a mathematical form.

The next stage, yesItu give an argument, here every subject has difficulty in explaining his argument to make sure that the answer he finds is correct. Each subject uses a common formula that has been used and studied before as a shield to state the answer is correct. S1 gives a personal argument because he believes but does not know the reason for his beliefs other than ever working on a problem that is almost similar, S2 also gives a personal argument butmore logicalbecause he thinks it is a necessity to be sure of the answer he made, he has also tried several things to find the answer. But the data is not given,S3 and S4 are sure because it usesageneral formula that has been in thebook. This argument is still not good and there are some mistakes. This is in line with that stated by a study that learners tend to give less formal arguments (Cervantes-Barraza, et al. 2010). So it can be concluded that the ability of learners in giving arguments is still not good, this can happen because of the unfamiliarity of students exposed to problems that demand their conjecture skills, because as it has been explained that teachers often give routine problems such as problems that can directly use formulas on books / internet or can be guessed about those that have procedural or arranged / defined ways of workmanship. This certainly prevents learners from working independently to build answers according to their level of thinking and knowledge that he got before. This is in accordance with supratman, et al., (2016) that learners are not directed to find problem solving with the knowledge they have but only for them to work in the way that has been provided.

At the stage of proving the conjecture, only one learner cannot prove the formula he has made, namely S3 because he believes that by using the general formula that he got and he changed with the information on the problem, then he was sure without proof that the formula to find the nth term that he found was true. The other three subjects, managed to prove the formula he made by using the information he got or known on the problem. That is, these three learners are able to prove the conjecture they have built, in line with research conducted by Wayan, et al (2010).

It can be concluded that the four subjects have the ability to build their conjecture, it's just not perfect, because there is still a stage that they cannot do, namely arguing the conjecture. That is, they have been quite good at constructing conjecture and it would be even better if the teacher gives a question that collects the ability of the conjecture and the position. This is in line with previous research by Supriani, Yani; et al (2019), that the ability of learners in building mathematical conjecture is still not good. However, that does not mean there is no conjecture construction capability at all. As well as research by Yunianti, Suci; et al (2018) that learners are able to perform stages in the conjecture construction process in sequence only not all stages are carried out.

CONCLUSION AND SUGGESTION

Conclusion

Research subjects have different abilities in constructing conjecture on *open-ended*problems, it's just that each subject cannot perform all stages in the conjecture development process. At the stage of understanding the problem, all subjects do the same to understand the problem, i.e. by reading it repeatedly to find out and establish what is being asked and needed to find answers. Each subject is able to understand a given problem, but there are some problems that take more time to understand

because they unconsciously understand how to complete it. In the stage of exploring the problem, each subject has been able to find important information such as first and different terms and translation or manipulate the image of the problem into numbers to form a number pattern. At the stage of constructing the conjecture, two subjects are able to construct precisely, while the rest ofthenyes is not successful dueto incomprehension in simplifying a formula and miscalculation. At the stage of giving arguments, some subjects have been able to give their arguments logically but informally, and others still present arguments that are personal. And finally, at the stage of proving the conjecture, there is one subject that cannot prove, only based on the formula that he uses is a general formula so that the answer is correct, while others do the same activity, namely substitute data or information that has been known in the formula yesngthey make.

Open-ended problems can make the subject have different answers, but it also makes doubt in determining which answer is correct. These doubts are also increasing when discussing the problem after the test takes place. But subjects also feel happy because they can work on the problem according to their imagination. However, due to the unfamiliarity, the subject is still fixated that there is only one correct answer, and when billed another way of completion, they cannot find it.

Suggestion

The study subjects are still less familiar with the problem that collects the ability of the conjecture, so it makes them difficult in several stages in the process of constructing the conjecture, such as building a conjecture, arguing and proving it. Therefore, it is expected that teachers pay more attention to the thought process of learners, one of which is by providing problems that charge the ability of conjecture. The problem of *open-ended* types can also be one of the alternative problems that can improve the independent thinking process of learners and stimulate the ability to build their conjecture.

And teachers must instill confidence in learners, so that learners use their thought process more without feeling doubt between right and wrong, and able to argue or give opinions on the answer. This research is still limited to the ability of participants' conjecture in solving *open-ended* problems in number pattern material, and limited to the number of research subjects, so for researchers can further conduct research with different materials and types of problems and more include many research subjects to obtain more varied results.

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