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MAPPING ON ANALOGICAL REASONING TOPIC LIMIT FUNCTION: WITH AND WITHOUT INTERMEDIATE PROBLEMS

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Abstrak

Mapping merupakan salah satu tahapan penalaran analogi yang merupakan proses menemukan sebuah kesimpulan dari hubungan yang ada pada masalah sumber dan masalah target. Limit fungsi trigonometri merupakan salah satu materi matematika SMA yang sering terjadi kesalahan pada proses pengerjaannya. Penilitian ini bertujuan untuk menganalisis *mapping* siswa SMA pada penalaran analogi dengan dan tanpa masalah antara pada materi limit fungsi trigonometri. Metode penelitian yang digunakan adalah kualitatif. Subjek penelitian ini adalah 4 siswa kelas XII MIA SMA swasta di Gresik. Data dianalisis menggunakan tahapan *mapping* yaitu mengidentifikasi hubungan masalah sumber dan masalah target, mengidentifikasi suatu struktur masalah sumber yang sesuai dengan masalah target dan menggunakan masalah sumber ke masalah target. Hasil penelitian menyimpulkan bahwa siswa kesulitan pada tahapan mengidentifikasi hubungan masalah sumber dengan masalah target. Siswa yang berhasil mengidentifikasi hubungan masalah sumber dan masalah target dapat mengidentifikasi kesamaan struktur dan mengetahui cara menggunakan masalah sumber ke masalah target, sedangkan siswa yang tidak berhasil mengidentifikasi hubungan masalah sumber dan masalah target tidak dapat melanjutkan ke tahapan selanjutnya. Pemberian masalah antara dapat membantu siswa dalam mengidentifikasi hubungan masalah sumber dan masalah target, sehingga masalah antara dapat digunakan sebagai jembatan masalah sumber ke masalah target.
Kata Kunci: Limit fungsi, *Mapping*, Masalah Antara Penalaran Analogi

Abstract

Mapping is one of the stages of analogical reasoning which is the process of finding a conclusion from the relationship that exists in the source problem and the target problem. The limit of trigonometric functions is one of the high school mathematics material that often causes errors in the process. This study aims to analyze the mapping of high school students on analogical reasoning with and without intermediate problems on the limit material of trigonometric functions. The research method used is qualitative. The subjects of this study were 4 students of class XII MIA private high school in Gresik. The data were analyzed using the mapping stage, namely identifying the relationship between the source problem and the target problem, identifying a source problem structure that is in accordance with the target problem and using the source problem to the target problem. The results of the study concluded that students had difficulty at the stage of identifying the relationship between the source problem and the target problem. Students who succeed in identifying the relationship between the source problem and the target problem can identify structural similarities and know how to apply the source problem to the target problem, while students who fail to identify the relationship between the source problem and the target problem cannot proceed to the next stage. Giving intermediate problems can help students identify the relationship between the source problem and the target problem, so that the intermediate problem can be used as a bridge between the source problem and the target problem.
Keywords: *Analogical Reasoning, Fungsion Limit, Intermediate Problems, Mapping*

INTRODUCTION

The activity of doing reasoning almost always involves an analogy. Analogy is talking about two different things but they have similarities (Soekardijo, 1999). Polya (1973) stated that the essence of the analogy is similarity in characteristic. In this case, those who have in similarity will have the same characteristic in several aspects. Sastrosdudirjo (1988) states that analogy is the ability to find relationships between objects or ideas, then the relationships that have been found are used to find other objects or ideas. (Bohning & Althouse, 1997)

Novick (in English, 1999) reveals that analogical reasoning is problem solving involving source problems and target problems. The source problem is a problem that is used to help solve the target problem, while the target problem is a new problem to be solverd. Analogical reasoning occurs when someone uses the structure or idea of the source problem, to solve the target problem. Analogical reasoning is drawing conclusions from the similarity of existing data or the process used, Sumarmo (2015). Analogical reasoning is reasoning that uses analogies to form a conclusion by looking at the similarity of objects, structures, or concepts from two different elements, in analogical reasoning the two elements in question are source problems and target problems, English (1999) and English (2004). Then, Maarif (2012) states that analogical reasoning is the ability to connect two different elements based on their similarities and then draw conclusions as a basis for reasoning. So that analogical reasoning is the process of drawing conclusions from the elements in the source problem and used to solve the target problem.

Sternberg in (English, 2004) suggests that there are four stages in doing analogical reasoning, (1) encoding, identifying the similarity of the structure of the source problem and the target problem, (2) inferring, finding the relationship between the source problem and the target problem, (3) mapping, building conclusions from the similarity of the target problem and source problem, and (4) applying, applying the concepts that have been built on the mapping. Meanwhile, Rupert (2013) revealed that doing analogical reasoning has four components, that is (1) Structuring, identifying the similarity of properties and structural relationships, from each object to make a conclusion, (2) Mapping, mapping from the conclusion to the target problem, (3) applying, applying the conclusions that have been obtained to solve the target problem, (4) verifying, re-checking.

The following previous research related to analogical reasoning or its stages as done by Siswono (2016) who conducted research on analogical thinking processes to solve problems in mathematics. Then a similar research was conducted by Rahayu (2016) which discussed the ability to reasoning mathematical analogies. Furthermore, Ayu (2016), and Ayu (2019) also conducted research related to analogical reasoning to solve elementary mathematical problems, and impulsive analogical reasoning in solving geometric problems. then, Basir, et al (2018) conducted research on analogical reasoning in solving mathematical problems. Then also, I Gede (2018) examines the errors that occur when doing analogical reasoning on mathematical material.

Holyoak & Thagard (1989) stated that the essence of doing analogical reasoning is mapping, which is looking for correspondence between the elements of the target problem to the source problem. This makes mapping very important in analogical reasoning. Mapping is the process of finding the relationship that exists in the source problem and the target problem and then conclusions can be drawn from that relationship, Sternberg (2008). Furthermore, Rupert (2013) states that mapping is a process of finding the same relationship from the characteristics that exist in the source problem and the target problem, then conclusions can be made that can be used to solve the target problem.

Siswono (2016), in previous research found the conclusion that only someone who has a high cognitive level can pass the mapping stage without any problems. This shows that students often make mistakes at the mapping stage, to reduce the possibility of errors in mapping, a bridge is needed that can connect the source problem to the target problem. Intermediate problems are problems that are given when students have worked on the source problem, and before working on the target problem. Research conducted by Manuaba (2017) revealed that students' failure to make analogies was caused by students did not know that there was a similar relationship between concepts and structures in the source problem to the target problem. Efforts that can be made to reduce these problems are to provide assistance so that the relationship between concepts and structures in the source problem becomes closer to the target problem so that students can see the relationship between the source problem and the target problem. Similar things were concluded in the research conducted by Kurniasih (2012), Scaffolding was given to provide understanding assistance to students. In this case, intermediate problems are used as scaffolding so that they can assist students in finding the relationship between the source problem and the target problem.

Previous studies related to analogical reasoning have a focus on looking at students' analogical reasoning abilities in solving problems, students' mistakes in doing analogical reasoning, analogy thinking processes, and seeing students' analogical reasoning from various points of view. So that not many studies have been conducted

focusing on one of the stages of analogical reasoning. So in this study, the research will focus on the mapping stage of analogical reasoning.

The limit of trigonometric functions is one of the high school mathematics material that often causes errors in the process. In previous research conducted (Suryana, et al, 2019) found that many students' mistake were experienced in working on trigonometric limit problems. The mistake in question are errors in understanding the problem, errors in carrying out transformations, errors in carrying out process skills and errors in writing solutions, these errors include errors in using theorems or formulas and performing arithmetic operations. A similar research conducted by Rumasoreng & Sugiman (2014) found that many students had difficulty solving trigonometric limit problems. The difficulty is the difficulty in understanding the concept and how to use the techniques to finding solutions. Therefore, the topic taken in this research is the limit of trigonometry.

The purpose of this study was to see the differences in students' abilities in mapping analogous reasoning to solve problems without intermediate problems and problems with intermediate problems on the topic of trigonometric limits.

METHOD

The method used for this research is a qualitative method. The purpose of this study was to analyze the mapping of high school students on analogical reasoning on the limit of algebraic functions as the source problem, and the limit of trigonometric functions as the target problem.

The candidate research subjects are all class XII MIA 1 and XII MIA 3 private high school students in Gresik who have studied the material on limiting algebraic functions and trigonometric concepts but have not studied the limits of trigonometric functions. The both classes were given the same treatment, that is before taking the data, the researcher reminded again things related to the limits of algebraic and trigonometric functions, this was material about the concept of limit functions and concepts from trigonometry that they had learned in class XI. Class XII MIA 1 students solve the problem of mapping analogical reasoning using the intermediate problem, and class XII MIA 3 students solve the problem of mapping analogical reasoning without using the intermediate problem. The subjects of this research amounted to 4 students who were selected from the subject who worked on the source problem correctly.

Mapping on analogical reasoning is analyzed based on the mapping stages stated by Novick (in English, 2004). With the following indicators:

Table 1 Mapping indicators on analogical reasoning

1	Students can identify the relationship between the target problem and the source problem.
2	Students can identify the structure of the source problem in accordance with the target problem.
3	Students can use the source problem structure to solve the target problem.

The first indicator states that students can find out the similarity of the relationship between the target problem and the source problem, the similarity of the relationship in question is the similarity of concepts, similarity of formulas, similarity of material, etc. The second indicator states that students can find out that there is a similarity in the structure of the source problem in accordance with the target problem, as an explanation of the meaning of the similarity in the structure is the similarity of work procedures or steps between the source problem and the target problem. The third indicator states that students can use the structure of the source problem to solve the target problem, in this third indicator students are required to know how to use the source problem to the target problem. can use the methods that exist in the source problem directly to work on the target problem, because the problems that exist in the source problem and the target problem are actually the same but only in different contexts.

The research instruments are assignments and interviews. The assignments in question is an analogical reasoning mapping test consisting of source problem, target problem, and intermediate problem for the given class. The source problem is related to finding the limit of an algebraic function, the target problem is related to finding the limit of a trigonometric function, and the intermediate problem is related to finding the limit of a trigonometric function with the same problem structure as the source problem.

Table 2. Instruments assignments analogy reasoning

Source Problem	$\lim_{x\to 2} \frac{x-2}{x^2-4}$
Intermediate Problem	$\lim_{x\to \frac{\pi}{4}} \frac{\sin x - \cos x}{sin^2 x - cos^2 x}$
Target Problem	$\lim_{x\to \frac{\pi}{4}} \frac{sin^2 x - cos^2 x}{\sin x \cos x - cos^2 x}$

Interviews were conducted after the subject worked on the assignment guided by the results of the subject's answers. Interviews were conducted with the aim of confirming the subject's answers as well as seeking information that could be obtained from the subject in the mapping of the analogical reasoning that had been done.

The main questions asked during the interview are as follows:

Table 3. Interview Guidelines

First Indicator	1. Is there a problem that you have done before that can help you in working on the target problem? 2. What knowledge of the source problem can you use on the target problem? 3. Are there any similarities between the source problem and the target problem?
Second Indicator	1. Is there a similarity in the procedure for working on the source problem to the target problem? 2. Whether the source problem work procedure can be used directly to solve the target problem? 3. If yes, please explain 4. If not, how do you modify it?
Third Indicator	1. What is the most effective way that can be used? 2. Is there another method that can be used?

At the time of the interview the researcher did not directly ask the questions in (Table 3) the interview guide, but the researcher asked the questions in the interview indirectly depending on the student's response, this was done so that the researcher could follow the students' thoughts when working on the research instrument with good.

RESULT AND DISCUSSION

Result

The results of the analogy reasoning mapping test are presented in Table 4 below.

Table 4. Results of Data Collection

Problem		XII MIA 1 Class	XII MIA 3 Class
Source Problem	Correct	4	10
	Incorrect	18	8
Intermediate Problem	Correct	2	-
	Incorrect	20	-
Target Problem	Correct	2	2
	Incorrect	20	16

Subjects selected to see their mapping ability are subjects who correctly work on the source problem. Therefore, the chosen subject is considered to have mastered the prerequisite material for working on the target problem, so that students who can become subjects in this study can be seen in (Table 4), that is 4 students from class XII MIA 1 and 10 students from class XII MIA 3.

4 subjects selected from XII MIA 1 and XII MIA 3, will continue the research by interview. The subjects were (1) SM subjects, who succeeded in working on the analogy reasoning problem without intermediate problems, (2) GM subjects, who failed to work on analogous reasoning problems without intermediate problems, (3) AZ subjects, who succeeded in working on analogous reasoning problems. with the intermediate problem, (4) MK subjects, who failed to work on the analogy reasoning problem with the intermediate problem.

Subjects with correct answers without intermediate problems (SM)

1. Students can identify the relationship between the target problem and the source problem.

R : Were you helped by the number 1 question?
SM01 : It helped
R : What kind of assistance?
SM02 : Both are $\frac{0}{0}$ so they are factored
R : $\frac{0}{0}$ how do you know?
SM03 : with the substition
R : So that's all that helps you?
SM04 : Yes sir
R : Have you ever worked on other questions similar to this one?
SM05 : I think no sir
R :Are there any other questions that you have worked on that could help you to do this?
SM06 : Yes, about the factored limit

SM subjects were helped by the source problem seen in (SM01 and SM02). SM subjects can relate the concept of limit of algebraic functions on the source problem to the concept of limit of trigonometric functions on the target problem, so that SM subjects can identify the relationship between the target problem and the source problem.

2. Students can identify the structure of the source problem in accordance with the target problem.

The following is an SM subject's work that shows how the subject identifies structure.

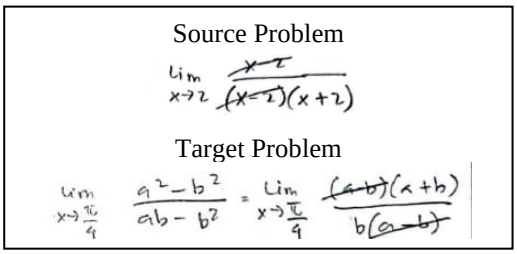


Figure 1. SM Subjects Answers

R : Where did you get the idea from for assumtion to be a and b?

SM07 : At first I had a hard time seeing question number 2, so I assumed that sin x became a, and cos x became b, it turns out that after I said that $a^2 - b^2$ is the same as the formula $x^2 - y^2 = (x + y)(x - y)$, yes, I copied it, sir, and the $ab - b^2$ because they both have b, then the b can be collected in the future so b (a – b), because the numerator and denominator are the same has (a – b) so that it can be crossed out

R : What kind of difficulties do you feel?

SM08 : Yes, I don't know what to do sir

R : Then from that difficulty, suddenly an idea emerged from where, for assumtion, to be a and b?

SM09 : Yes, at first, I tried it sir, after trying it, I found an equation similar to number 1, so finally I was able to cross it out

Seen in the interview, the SM subject can explain how he can identify the structure of the source problem (SM07 and figure 1) shows how the SM subject identifies a source problem structure that corresponds to the target problem, by doing assumtion so that it can help the SM subject to see the similarities between the source problem, with the target problem, the structural similarity that can be seen by the SM subject is when doing factorisation. Such an assumtion can be done because the SM subject finds it difficult to see the similarity of the structure of the source problem to the target problem.

3. Students can use the source problem structure to solve the target problem.

The following is an SM subject's work that shows how the subject uses the source problem structure.

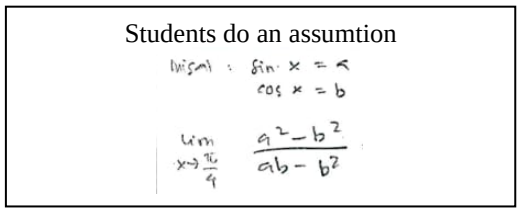


Figure 2. SM Subjects Answers

R : Do you think the method you are using is correct?

SM10 : As far as I know, if the substitution is equal to $\frac{0}{0}$, then using the factoring method

R : Isn't there another way?

SM11 : I was suspicious about identity, but if I use an identity, it becomes more complicated, sir

SM subjects can use the appropriate method from the source problem to the target problem, in (SM10, SM11, and figure 2) it is seen that the SM subjects use a method that is almost the same as the source problem to work on the target problem. The SM subject adapted by doing an assumtion so that the assumtion shows the similarity of structure with the source problem, after seeing the similarity of structure. The SM subject believed that the answer was correct based on knowledge of the $\frac{0}{0}$ form, but because the SM subject had not studied the limits of trigonometric functions, the SM subject felt that there were other ways to work on the target problem.

Subject with incorrect answer without intermediate problem (GM)

1. Students can identify the relationship between the target problem and the source problem.

R : Was you helped by question number 1?

GM01 : I don't know sir

R : Where is the lack of knowledge?

GM02 : Yes, because it's a different question, sir

R : Okay, the problem is different, now if I ask where is the difference?

GM03 : The difference between limit of algebra function and limit of trigonometric function

R : Then what are the similarities between the first question and the second question?

GM04 : Yes, it's the same with the limit, sir

R : That's all? Are you sure?

GM05 : Yes, I'm sure

R : Have you ever worked on other questions similar to this question (second question)?

GM06 : No sir

R : Then is there a question that can help you to do this question (second question)?

GM07 : Yes, about trigonometry, sir, sin $\frac{\pi}{4}$ means sin 45

The interview showed that the GM subject could not identify the relationship between the source problem and the target problem shown in (GM01 and GM02), this could happen because the GM subject did not feel helped by the source problem. Thus, the GM subject did not find any conceptual similarities between the limit of the algebraic function and the limit of the trigonometric function, so that to work on the target problem, the GM subject looked for a different way from the source problem. (GM04) also shows that the similarities that can be seen by the GM subjects are only the similarities in terms of limits.

So, the subject of GM has not been able to identify the relationship between the source problem and the target problem.

2. Students can identify the structure of the source problem in accordance with the target problem.

The following is the work of the GM subject which shows the subject did not find the similarity of the source problem structure to the target problem.

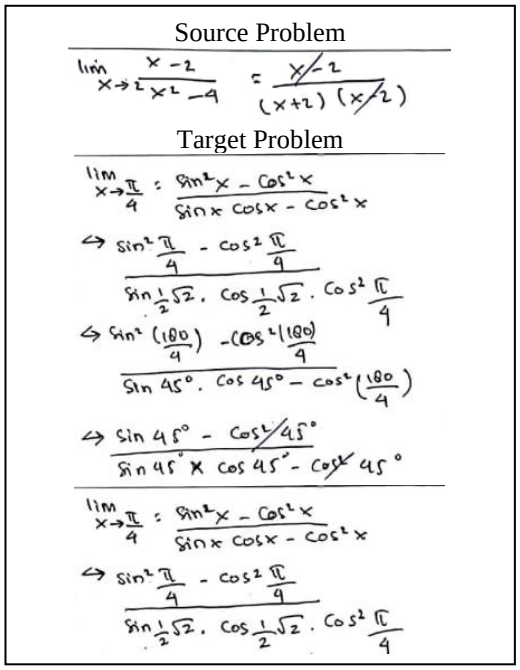


Figure 3. GM Subject Answers

R : Now try to explain where did you get $\frac{1}{\frac{1}{2}\sqrt{2}}$?
GM08 : Yes, with substitution sir
R : So the way to do the second question you did was just substitution?
GM09 : Yes, sir
R : Sure?
GM10 : I don't know sir
R : Why don't you know?
GM11 : Yes, because earlier, when you reviewed the material, you said as long as the result is not $\frac{0}{0}$, it means that it is the final result.
R : Does that mean you are sure of your answer?
GM12 : I don't know mas
R : Why don't you know?
GM13 : Yes, because I have never worked on trigonometric limits
R : So basically in your work, the first and second questions use different methods, the first problem uses the factoring method and the second method uses the substitution method.
GM14 : Yes, sir
R : Then where did you get the idea from, how come you can cross out $-\cos^2 45$?
GM15 : Yes, because it's the same sir
R : Same? same as what do you mean?
GM16 : Same as the first question, bro, you can cross out $x-2$, so $-\cos^2 45$ can also be crossed out

The GM subject has not been able to identify a source problem structure that is in accordance with the target problem which can be seen in (GM08 and Figure 3), this can be seen in GM's work, because the GM subject made an error in simplification, then in working on the source problem and the target problem the GM subject do it in a different way, that is by factorisation for the source problem and substitution for the target problem, so that when the GM subject makes substitutions on the target problem, the GM subject does not find $\frac{0}{0}$ but finds other results, from (GM11) the GM subject already believes in the answer because the results the end is not $\frac{0}{0}$, so the GM subject doesn't think about factoring or doing the same thing with the source problem. So the GM subject did not find any structural similarity between the source problem and the target problem because the structure of the answers to the two problems handled by the GM subject was different.

3. Students can use the source problem structure to solve the target problem.

The following is a GM subject's work which shows the subject cannot use the source problem structure to solve the target problem.

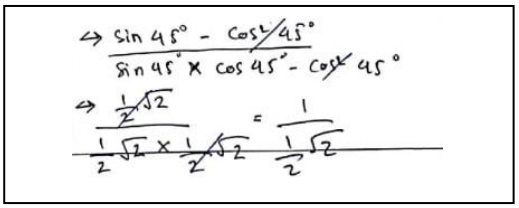


Figure 4. GM Subjects Answer

R : Do you think the method you are using is correct?
GM17 : I don't know mas
R : Why don't you know?
GM18 : Yes, because you haven't learned how to do trigonometric limits
R : So you yourself are not sure about the way you have done?

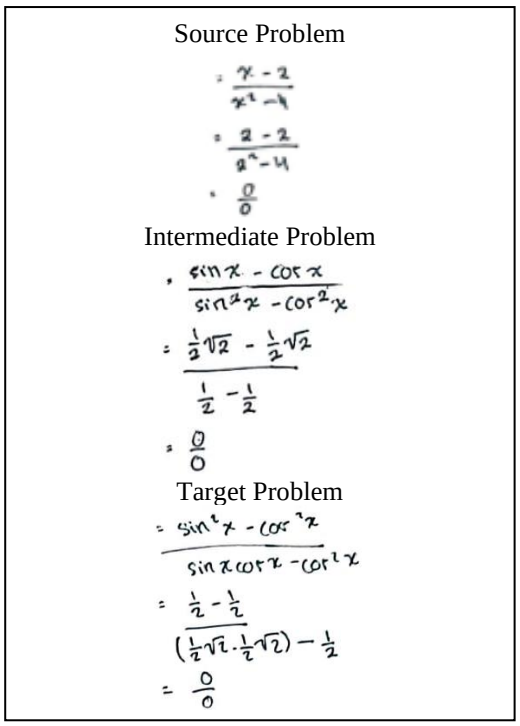
GM19 : Yes, sir
R : Then do you think there is another way to do this problem?
GM20 : Yes sir
R : What kind of way is it?
GM21 : The method that you will teach at the next meeting
R : What way will I teach?
GM22 : Yes, I don't know sir

At this stage the GM subject can be declared unable to use the source problem to the target problem, because the GM subject cannot identify the structural similarity between the source problem and the target problem so that there is no concept of a source problem that will be used to solve the target problem by the GM subject. It can be seen in (GM17 and Figure 4) that the GM subject does not believe in his own answer, meaning that the GM subject already feels that the answer is still not quite right, from (GM20) it shows that the GM subject has other ways that can show the correct results, but not the same way the GM subject did. Then from this information, it can be concluded that the GM subject does not know how to apply the source problem to the target problem.

Subjects with correct answers with intermediate problems (AZ)

1. Students can identify the relationship between the target problem and the source problem.

The following is the work of AZ subject which shows AZ subject can identify the relationship between the target problem and the source problem.



R : Was number 1 helped you to do question number 2?
AZ01 : Yes sir
R : Where did you help?
AZ02 : The first question and the second question are the same if they are substituted to produce $\frac{0}{0}$, because the first question can be solved by factoring, so I tried the same way to work on second question
R : Did questions 1 and 2 help you to do question number 3?
AZ03 : Yes sir
R : Where did you help?
AZ04 : Yes, with same way factorisation

Subject AZ felt helped by the source problem and the intermediate problem shown in (AZ01, AZ03, and Figure 5), because subject AZ found it helpful, it can be stated that subject AZ knows that there are similarities in the concepts that exist in the limit of algebraic functions with the limits of trigonometric functions, so it can be concluded that the subject of AZ can identify the relationship between the source problem, the intermediate problem and the target problem. (AZ02) shows that subject AZ can explain correctly how the relationship between source problems, intermediate problems and target problems is correct, so that AZ subjects can be stated to know the relationship between source problems, intermediate problems, and target problems.

2. Students can identify the structure of the source problem in accordance with the target problem.

The following is the work of AZ subject, which shows that AZ subject can identify a source problem structure that matches the target problem.

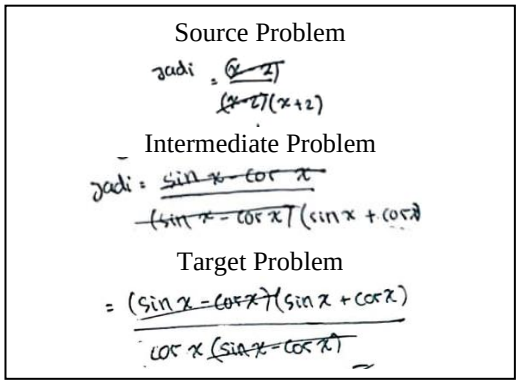


Figure 6. AZ Subject Answers

R : Where did you help?

Figure 5. AZ Subject Answers

AZ05 : The first question and the second question are the same if they are substituted to produce $\frac{0}{0}$, because the first problem can be solved by factoring, so I tried the same way to work on question number 2

AZ subjects in the previous stage were able to explain and identify the relationship between source problems, intermediate problems, and target problems appropriately, so that at the next stage, that is identifying structure, AZ subjects did not feel difficult. (AZ05 and Figure 6) show if the subject of AZ can find out the similarity of the existing structure in the source problem, the intermediate problem and the target problem by using a factorisation method, so that the AZ subject does not find it difficult at this stage.

3. Students can use the source problem structure to solve the target problem.

R : In your opinion, is the method you are doing is right?
AZ06 : I don't know if I'm right or wrong, because I haven't studied the material
R : Even you haven't studied this material, are you sure about your answer?
AZ07 : yes, sure
R : Do you think there are other ways that can be used to work on question number 3?
AZ08 : I don't know sir

AZ subjects can use the source problem to solve the intermediate problem, and use the intermediate problem to solve the target problem, it can be seen from (Figure 6) where the method used by the AZ subject to solve the source problem, the target problem, and the problem is the same, and The concepts used are also the same, except for the differences in algebra and trigonometry. Statement (AZ06) shows that subject AZ feels confident in his answer, this can happen because at the stage of identifying the relationship and identifying the structure of subject AZ there are no obstacles so subject AZ feels confident in his answer, but even though subject AZ is confident in his answer, subject AZ also still feel doubtful, because the material is new material that has never been received by subject AZ, in (AZ08) it shows that subject AZ does not know other ways to work on the target problem, this is analogical to subject AZ who has never studied the limit material of trigonometric functions before.

Subject with incorrect answer with intermediate problem (MK)

1. Students can identify the relationship between the target problem and the source problem.

R : Was question number 1 helped you to do question number 2?
MK01 : Actually, I don't know sir, I don't understand this
R : But can you do this?
MK02 : Yes sir, I was taught before but it was a long time ago, now I forget about it
R : Okay, do you think questions number 1 and 2 have any similarities or not?
MK03 : I don't know sir
R : What's the difference?
MK04 : Number 1 is about ordinary limits, number 2 is about limits of trigonometry mas
R : What do the numbers 2 and 3 have in similarities ?
MK05 : Same as trigonometric limit sir
R : What's the difference?
MK06 : I don't know sir
R : So on questions 2 and 3, did you do it with your memory in class?
MK07 : Yes, sir
R : Are you sure about your answer?
MK8 : No sir

MK subject has not been able to identify the relationship between the source problem It can be seen in (MK01), the intermediate problem, and the target problem correctly, because the MK subject does not know that there is assistance from the source problem to work on the intermediate problem and the target problem. MK subject cannot see any similar concept between the limit of

algebraic functions and the limit of trigonometric functions. However, (MK02) shows that in fact the subject had studied the trigonometric limit material in the tutoring place, but the subject had forgotten and only remembered how to solve it by "crossing out" or simplifying it, so at this stage the subject still cannot identify the relationship problem. source, intermediate, and target problems correctly.

2. Students can identify the structure of the source problem in accordance with the target problem.

The following is the work of the MK subject, which shows the subject's error in identifying the structure.

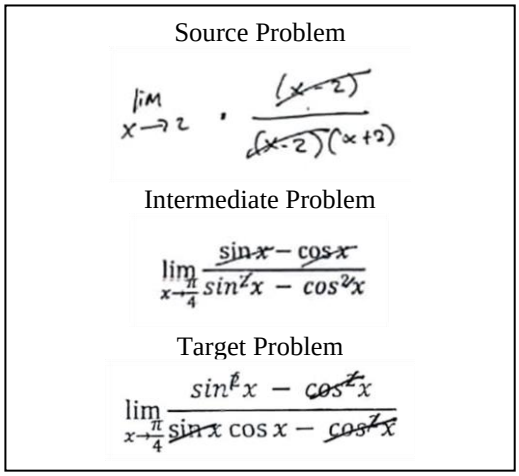


Figure 7. MK Subject Answers

R : Where did you get your inspiration can do it ?
MK09 : I just cross it out
R : Sure?
MK10 : I don't know sir, I don't understand
R : Did you see from the way that you crossed out number 1 like that?, so you also crossed out numbers 2 and 3
MK11 : It could be sir, because in the past there was also a way of doodled remembering me when I was taught in the lesson.

MK subject knows the structure of the source problem, the intermediate problem, and the target problem, (MK09 and figure 7) shows that the MK subject knows the same structure for crossing out, but this is still not correct because the appropriate structure is to simplify it by factoring. (MK11) explained that the real idea why MK subjects crossed out was because MK subjects had studied the limit of trigonometric functions, but MK subjects only remembered the crossed out parts, not accompanied by right simplifications, which resulted in MK subjects still unable to identify the structure of the source problem, intermediate problem, and target problem correctly.

3. Students can use the source problem structure to solve the target problem.

R : In your opinion, is your method correct?
MK12 : I don't know sir, I don't understand
R : What do you think?
MK13 : I don't think so, I just don't understand the problem
R : What is the correct way for numbers 2 and 3?
MK14 : I don't know sir

MK subject felt unsure of his work as shown in (MK12, MK13, and MK14), this could be seen if the MK subject still did not understand the problem. And from the previous stage, the MK subject could only write off without correct simplification, and the subject admitted that he had indeed forgotten the material that had been taught in his tutoring place, so that the subject only did it the wrong way. So it can be concluded that the MK subject does not know how to properly use the source problem to the target problem.

Discussion

The following is a flow chart of the mapping process for each subject, with descriptions :

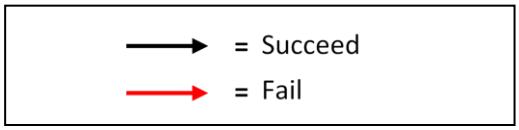


Figure 8. Description of the flow chart for each subject

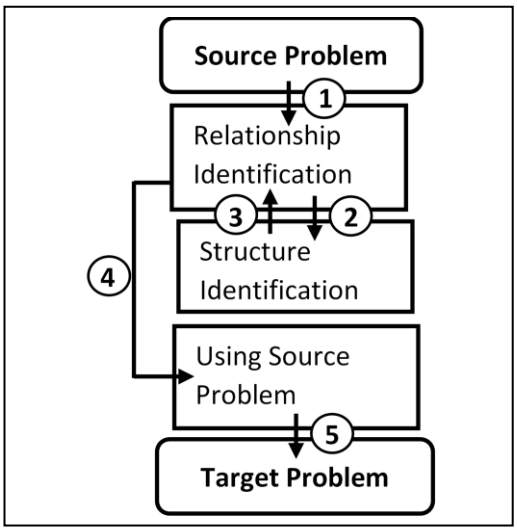


Figure 9. Mapping process flow chart (SM)

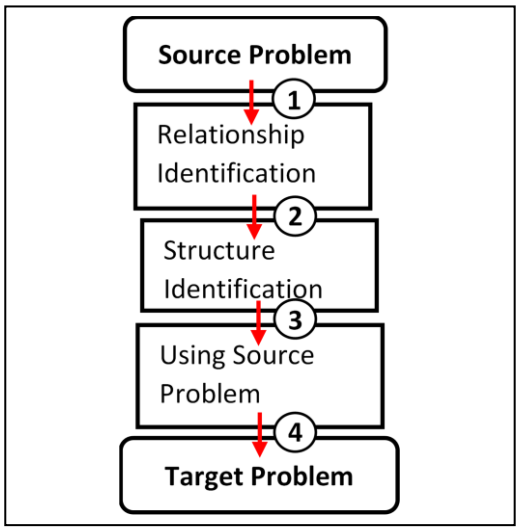


Figure 10. Mapping process flow chart (GM)

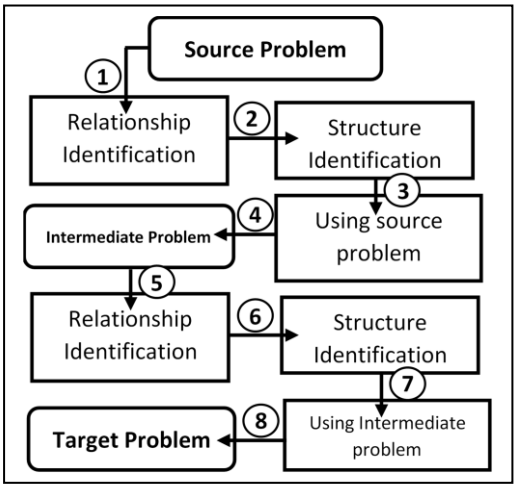


Figure 11. Mapping process flow chart (AZ)

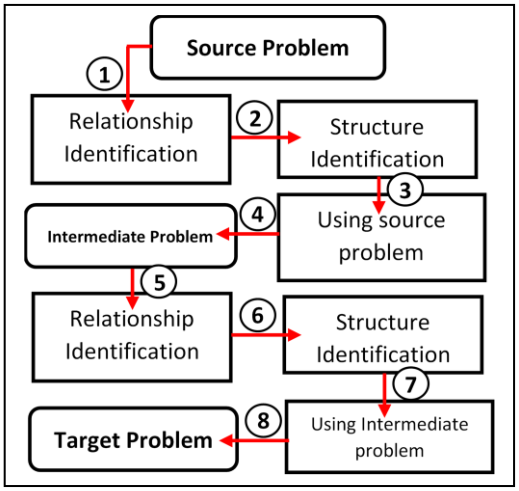


Figure 12. Mapping process flow chart (MK)

The difficulty of the subject in working on the analogy reasoning problem lies in the process of mapping the stages of identifying the relationship between the source problem and the target problem, it can be seen in the research results of the subject who has identified the relationship between the source problem and the target problem correctly, will be able to work on the intermediate problem and the target problem. on the otherwise, subjects who have not been able to identify the relationship of the source problem to the target problem correctly, then the subject will not succeed in the next stage of the mapping process, that is the stage of identifying the similarity of the structure, and using the source problem to the target problem. So that, it can be concluded that in the mapping process, the stage of identifying the relationship is a very important and crucial stage, because it is at that stage that determines whether the subject can continue the mapping process to the next stage or not. In a previous research that discussed the stages of analogical reasoning conducted by Ayu (2016) and Basir (2018), it was found that students who fail at one stage of analogical reasoning will also fail at the next stage of analogical reasoning. The results of this research are analogical to the results of this research, which found that students who fail at one stage of mapping will also fail at the next stage of mapping.

One of the reasons of the subject's difficulty in finding the relationship between the source problem and the target problem is the large knowledge gap between the source problem and the target problem. Efforts to overcome these difficulties is to provide assistance to the subject in the form of intermediate problems. The results of data collection indicate that giving intermediate problems can help the subject, it can be seen in (Table 4) that subjects who successfully work on intermediate problems will also succeed in correctly working on target problems, so the results of this research show the effect of giving intermediate problems in working on analogical reasoning problems is very positive, because it can help shorten the distance of knowledge that exists in the source problem to the target problem. Previous research related to the provision of question assistance conducted by Kurniasih (2012) concluded that the provision of scaffolding can help students' understanding in finding the relationship between the

SM subjects can find the relationship between the source problem and the target problem, but it's still difficult to find structural similarities between the source problem and the target problem ican be seen in the flow chart (SM) where SM subjects work on instrument questions without intermediate problems, so in the process of working on it the target problem, the SM subject did an assumtion that was felt to make it easier for the subject to see the relationship between the source problem and the target problem. While it can be seen in the flow chart (AZ) where subject AZ works on instrument questions with intermediate problems, subject AZ does not find it difficult to find similarities in the structure of the source problem and target problem, so subject AZ does not need to make an assumtion. It can be concluded from these observations that the difference seen in the subjects who work on the instrument questions with intermediate problems and the subjects who work on the instrument questions without intermediate problems is the distance of knowledge from the source problem to the target problem..

Another difference that occurs to subjects who work on instrument questions with intermediate problems and subjects who work on instrument questions without intermediate problems is behavior during the data collection process, where classes who work on instrument questions without intermediate problems find it difficult, many of them are hard to find the relationship between the source problem and the target problem, even many of them did not find a relationship between the source problem and the target problem, they also complained about the instrument questions which they thought the instrument questions could not be done, on the reasons that they had never been taught, even though they were working on the target problem, the subject can look for the relationship between the source and target problems, therefore finding a similiar structure or concept that can be used to solve the target problem. Whereas in the class that was given an instrument with intermediate problems, not many complained about the instrument, and when the researcher asked something related to the instrument, their minds were more organized than the subjects who were given the instrument without intermediate problems, many of them also said directly to the researcher, the intermediate problem was helping them in working on research instruments, it shows that the intermediate problem can be a bridge for students to find the relationship that exists in the source problem to the target problem. Similar things have been found in research conducted by (Rahayuningsih, 2014; Fatahillah, 2017; and Lutfia, 2019) which states that giving additional problems to help students solve the main problem is a form of scaffolding, and the provision of appropriate scaffolding can reduce mistakes that are usually made by students, so the results of this research are in line with previous research. Intermediate problems or scaffolding are additional problems that can shorten the knowledge gap between the source problem and the target problem, so that students can more easily work on the target problem. These additional problems can be in the form of a source problem that has been modified in such a way that it is closer to the target problem, for example, a modified source problem so that its structure is almost the same as the target problem, a modified source problem so that the

context of the problem looks the same as the target problem, and a source problem that is similar to the target problem modified by analogy.

The similarity seen in the subjects who worked on the instrument questions with intermediate problems and the subjects who worked on the instrument questions without intermediate problems was the difficulty experienced by the subjects in finding the relationship between the source problem and the target problem, this can be seen in (Table 4) where there are many subjects failed to work on the target problem, so this research shows that there are still many students who have difficulty in doing mapping, this invention is similar to research conducted by Ayu (2019) and Purwanti (2016) where the results of both research show the same thing, that is the number of students who are still having difficulties and there are even some students who fail to do the mapping. Siswono (2016) also found that only students with high cognitive levels could pass mapping without feeling difficult, while students with moderate cognitive levels were having difficulty on the mapping, and for students with low cognitive levels failed to do the mapping.

CONCLUSION

Mapping is a process of looking for similarities in the relationship between the source problem and the target problem. The difficulty of students in mapping in analogical reasoning is identifying the relationship between the source problem and the target problem, this can be seen in the results of research which show that student failure in mapping occurs at the stage of identifying the relationship between the source problem and the target problem. Students who can identify the relationship of the source problem with the target problem, can identify the similarity of the structure of the source problem with the target problem, and know how to apply the source problem to the target problem. On the otherwise, students who cannot identify the relationship between the source problem and the target problem cannot proceed to the next stage of the mapping process. At the next mapping stage, that is identifying the structure of the source problem in accordance with the target problem, students who work on the instrument with intermediate problems have no difficulty at this stage while students who work on the instrument without problems find it difficult to find a similar structure to the source problem that is in accordance with the target problem. so as to make it easier to see the similarity of the structure of the students doing an assumtion. At the last mapping stage, that is knowing how to use the source problem to the target problem, students who can go through the previous stage, that is identifying the relationship between the source problem and the target problem and identifying the similarity of the source problem structure in accordance with the target problem, can find out how to use the source problem to the target problem. , while students who have not been able to go through the previous stages do not know how to use the source problem to the target problem. The results showed that giving intermediate problems can help students identify the relationship between the source problem and the target problem, so that the intermediate problem can be used as a bridge between the source problem and the target problem.

SUGGESTION

Suggestions for further research on mapping, (1) can be done using quantitative methods with a larger number of samples to ensure the effectiveness of using intermediate problems in problem solving analogy reasoning,

(2) Further research can also be carried out to find the characteristics of the appropriate intermediate problems to assist mapping on analogical reasoning in problem solving.

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