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Student' Argumentation Through Mathematical Literacy Problems Based on Mathematical Abilities

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Abstract: Argumentation is a mathematical skill employed in mathematical literacy to think critically and provide reasons and conclusions that solve problems. The capability of students to construct arguments and solve mathematical literacy problems is influenced by their mathematical skills. A qualitative approach is used in this study to describe students' argumentation in solving mathematical literacy problems based on mathematics ability level. The sampling technique used in this study is purposive random sampling. The research subjects were three twelfth-grade students: students with high, moderate, and low mathematics abilities. Data are collected through mathematical literacy problem tests and interviews. The data are analyzed using McNeill and Krajcik's argumentation components: claim, evidence, reasoning, and rebuttal. The results showed that students with high mathematical abilities could perform the procedures, connect information, provide general solutions, assess the mathematical solutions, and make a correct claim. Students with moderate mathematical ability could apply mathematical concepts although still make a miscalculation, connect information, provide partially correct solutions, evaluate the sufficiency of the solutions, and make a valid claim. Meanwhile, students with low mathematical ability misapplied mathematical concepts but could connect information, provide partially correct solutions and claim even cannot evaluate the sufficiency of the solutions. Based on the research result above, students with high mathematical abilities can provide excellent arguments supported by in-depth mathematical concepts in the problem-solving process. Students with moderate mathematical abilities can give good arguments but still make miscalculations in problem-solving. Meanwhile, students with low mathematical abilities can provide reasonably good arguments but misapply mathematical concepts in problem-solving.

INTRODUCTION

Argumentation is creating claims and providing evidence to support those claims (Bathgate, 2015; Toulmin, 2003). Argumentation becomes the focal point of cognitive abilities concerned with solving problems, making judgments and decisions, and formulating ideas and beliefs (Kuhn & Udell, 2003). Argumentation enables people to discover various alternative perspectives, create and select preferred and logical solutions, and back up their decisions with evidence and facts during problem-solving (Cho & Jonassen, 2002). Argumentation encouraged students to think critically and logically to construct claims by finding alternate solutions and evidence for an issue by employing relevant knowledge (Bathgate, 2015; Soekisno, 2015). Students who can provide argumentation can also leave their hesitancy in solving problems. Furthermore, they have flexibility in selecting alternative ideas or approaches and can even provide a rational solution to settling problems.

Numerous argumentation models are used to examine problem-solving processes, for example, Toulmin and McNeill. Toulmin's model (2003) describes the underlying structure of argumentation in-depth and includes a wide range of informal arguments. Meanwhile, McNeill and Krajcik's argumentation model has been adapted to students' abilities, making examining students' arguments easier and more appropriate (McNeill, 2006; Sutini, 2020). McNeill and Krajcik's argumentation model consists of a claim, evidence, reasoning, and rebuttal. A claim is a statement or hypothesis that answers a problem or explains a phenomenon. Data or information that supports a claim is referred to as evidence. The reasoning explains how the evidence presented can be used to support a claim. The rebuttal is the alternative claim that gives counter-proof and explains why the existing claim cannot be supported.

Despite the fact that argument capacity is required to solve a problem, many high school students fail to construct an argument. Even though the problems assessed have been modified to the Indonesian culture, high school students cannot provide explanations or arguments for the math problems being evaluated (Mahdiansyah & Rahmawati, 2014). In answering issues, most high school students scientific argumentation is relatively low (Noviyanti, 2019). The same idea was given by Nursyahidah's (2016) research that shows students' ability to identify mathematical conjectures, assess mathematical arguments, and create mathematical proofs in geometry learning was still relatively low. Therefore, research related to mathematical argumentation is essential to investigate students' thinking processes further to enable emerging learning innovations to improve students' argumentation. One method for improving students' argumentation is to use mathematical literacy problem-solving in the classroom (Hermawan, 2019).

The PISA framework 2018 defines mathematical literacy as an individual's capacity to formulate, use, and interpret mathematics in a variety of contexts. It includes mathematical concepts, procedures, facts, and tools to describe, explain, and predict a phenomenon (OECD, 2019). A mathematical literacy problem is a contextual problem designed to measure students' argumentation competence. The process of solving mathematical literacy problems (OECD, 2019) is classified into (1) formulating and modelling problem situations in mathematical form; (2) employing strategies, concepts, procedures, facts, and mathematical reasoning to obtain mathematical solutions; and (3) interpreting, applying and evaluating mathematical results.

Mathematical literacy problems which conducted by PISA are arranged based on content and mathematical context aspects. Context refers to a particular aspect of a person's environment in which the problems are put forward, and the mathematical content is a mathematical topic used to solve the problems (OECD, 2019). The four content categories utilized in PISA are quantity; uncertainty and data; shape and space; change and relationships.

Shape and space are content that includes phenomena about the visual world that entail patterns, characteristics of objects, locations, and orientations, representations of objects, encoding visual information, navigation, and dynamic interactions connected to natural forms (OECD, 2019). The research conducted by Trapsilasiwi (2019) found that Indonesian students' ability to solve mathematical literacy problems on shape and space is still in the low to medium range. This study uses mathematical literacy problems with the shape and space content to describe students' arguments in the problem-solving process because shape and space content not only makes up one of the PISA contents but also contains elements of curriculum-based mathematics.

Mathematical literacy problems could enhance students' argumentation because the context used is related to daily problems so that students can better comprehend the problems. In addition, the context of daily problems that are close to the students can encourage the students to be able to solve any given problem (Hermawan, 2019). The capability of students to solve mathematical literacy problems is influenced by their mathematical skills (Yulia, 2021). Each individual may have a different solution, strategy, or implementation of mathematical principles while addressing a problem. Differences in problem-solving are conceivable owing to differences in each individual's mathematical skill (Isroil, 2017).

Prior researchers have used McNeill and Krajcik's argumentation McNeill (2006), Sadieda (2019), and Sutini (2020). McNeill (2006) utilized the approach as a rubric to assess the strengths and weaknesses of students' arguments about lever problems. Meanwhile, Sadieda (2019) describes students' argumentation in proving sub-group problems. Sutini (2020) identifies students' argumentation structure in covariational reasoning of function graph problems. However, the prior studies have yet to address the problem of mathematical literacy. Based on the above explanation, this research aims to describe students' argumentation through mathematical literacy problems based on mathematics abilities. It is critical to provide educators with feedback in constructing appropriate learning so that students at all levels of mathematics ability can demonstrate good learning outcomes in mathematical literacy problems by developing arguments.

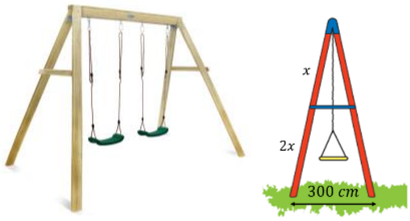
METHOD

This research is a qualitative descriptive study. The technique used in taking samples is the purposive random sampling technique. The sample of the research subjects was taken from 12th-grade students who have various mathematics abilities, gender backgrounds, and communication skills. The Mathematics Abilities Test (MAT) was used to identify three research subjects with different levels of mathematical capacity, namely high, moderate, and low.

There are three instruments used to collect data in this research: the mathematics abilities test (MAT), the mathematical literacy problems test (MLP), and interview guidelines. The mathematics abilities test (MAT) consists of five essay questions adapted from the senior high school National Examination (UN) questions. The mathematical literacy problems (MLP) test consists of two essay questions about shape and space. Those questions are originally taken from mathematics textbooks and then adapted to Hasanul Anshori (2021) and Hermawan (2019), which are presented in contextual and true-false questions. It aims to determine students' argumentation in providing and deciding the validity of the given statement in the questions. The mathematical literacy problems test can be seen in Figure 1.

1. Andrew has 700 cm of wood. He plans to make a regular hexagon raised-garden bed with a length of 50 cm on each side. Andrew planned to add a few congruent triangles to each side of the hexagon after realizing plenty of wood left. Andrew believes there will be enough wood to make various triangular models, such as equilateral and isosceles triangles. Is Andrew's statement correct? Explain yours.

2. Louis wants to build a backyard swing set for his child, so he designed a swing set considering the backyard space at his house. He designed the ends of the swing set to look like the letter A, as shown in the diagram below.



In order to maintain balance in the swing set, the legs of the frame must be the same length, and the horizontal supporting bars should be parallel to the ground. Louis states that the length of the wood for horizontal supporting bars will be affected by whether an isosceles or an equilateral triangle frame is used. Is Louis's statement correct? Explain yours

Figure 1. The Mathematical Literacy Problems

Interviews were conducted to obtain more detailed descriptions related to the process of solving students' mathematical literacy problems. Experts validated the instruments before they were tested.

In this study, the indicators were created utilizing McNeill's argumentation (2006) and argumentation indicators in the process of solving PISA mathematical literacy (OECD, 2019). The indicators were used to analyze the outcomes of the interviews and the students' mathematical literacy problem-solving examinations. The research indicators can be seen in Table 1.

Table 1. Indicators of Students' Argumentation Abilities

Argument in Mathematics Literacy Problem Processes	Code	Indicator
Evidence - Formulating	EF	Students can formulate mathematical forms according to real problems.
Evidence - Employing	EE	Students can use processes, concepts, facts, or mathematical reasoning to determine results or solutions in mathematical form.
Reasoning - Employing	RE	Students can relate existing information and provide reasons as justification for statements accompanied by evidence.
Rebuttal - Employing	RBE	Students can make generalizations or provide arguments for alternative solutions from mathematical forms.
Rebuttal - Interpreting	RBI	Students can reflect on the mathematical solutions obtained in the context of real problems together with explanations that support or refute the answer to the problem.
Claim - Interpreting	CI	Students can conclude the final solution for a given real problem.

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RESULT AND DISCUSSION

The research was started by conducting a mathematics ability test for class 12th grade at a high school in Sidoarjo, which comprised 36 participants to choose research subjects. After that, the outcomes of the test answers were assessed and grouped based on mathematical ability. The details of the selected research subjects are presented in Table 2 below.

Table 2. Details of the Selected Research Subjects

No	Initial Name	MAT Scores	Mathematics Abilities Categories	Code
1	AVAP	93	High	S1
2	BAF	80	Moderate	S2
3	ADS	62	Low	S3

Argumentation of student with high mathematics abilities (S1)

The answer sheets of S1 for both questions can be seen below.

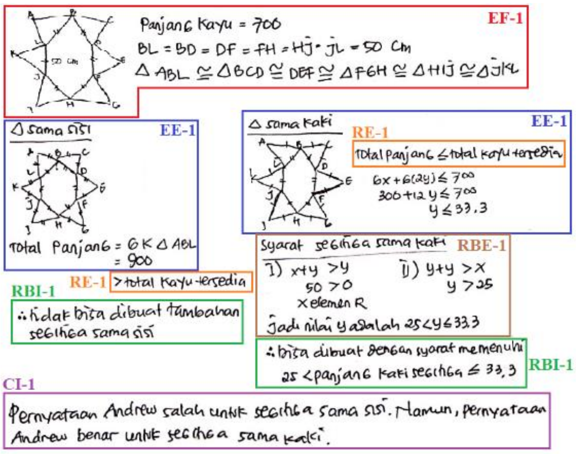

Figure 2. S1's Answer Sheets for Number 1

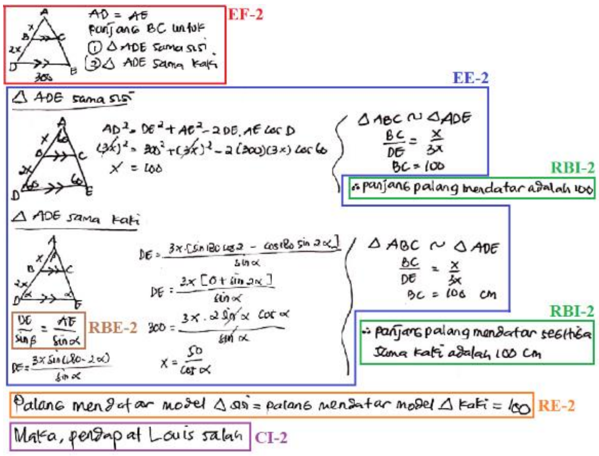

Figure 3. S1's Answer Sheets for Number 2

Based on the student's answer sheets in Figure 2 and Figure 3, it can be seen that the student's process of solving mathematical literacy problems in forming their arguments. In order to get in-depth descriptions of students' arguments, interviews were conducted as follows.

Table 3. Interviews Transcript of S1

R	:	Try to explain the steps to solve the problems you have written for question number 1
S1	:	First, I made a geometric illustration and wrote down the information known **(EF-1)**. In order to find the equilateral triangle model, I added up the perimeters of the triangles and hexagons **(EE-1)**, it turned out that the result was more than the available wood **(RE-1)**, so the equilateral triangle model could not be made since an insufficient amount of wood **(RBI-1)**. Meanwhile, for the isosceles triangle model, suppose that the length of the sides of the hexagon is x, the sides of the triangle are y, then the total wood is 6x+6(2y) **(EE-1)** because the amount of wood available is only 700 cm, so 6x+6(2y)≤700 **(RE-1)** then y ≤ 33.3cm. In order to ensure that the conditions for forming a triangle are also fulfilled, I'm looking for the value of y with triangle inequality. Since the triangle is isosceles, we obtain two inequalities, namely x + y > y and y + y > x; by solving algebraically, we obtain 25 < y ≤ 33.3 **(RBE-1)**. Thus, the isosceles triangle model can be

		made if that the length of the wood chosen for the sides of the triangle is $25 < y \leq 33.3$ **(RBI-1)**. So, Andrew's statement is wrong for equilateral triangles and true for isosceles triangles **(CI-1)**.
R	:	Try to explain the steps to solve the problems you have written for question number 2
S1	:	First, I made a geometric illustration and wrote down the information known **(EF-2)**. In order to find a model of the equilateral triangle, I use the cosine rule with an angle of 60 degrees. With the similarity of two triangles **(EE-2)**, I can obtain the length of BC equal to 100. So, the length of the supporting bars of the equilateral triangle model is 100 cm **(RBI-2)**. Meanwhile, for the isosceles triangle model, I use the sine rule. Since the model is an isosceles triangle, then there are two different angles, namely alpha and beta **(RBE-2)**. After obtaining the value of x **(EE-2)**, I'm looking for the length of BC with the similarity of two triangles which is equal to 100. Thus, the length of the supporting bars in the isosceles triangle model is 100 cm **(RBI-2)**. Thus, whatever the model that will be used, the length of the horizontal bar is always equal to 100 cm **(RE-2)**. Therefore, Louis' opinion is wrong **(CI-2)**.
R	:	Do you think the answer you found is correct and sufficient to answer the problem?
S1	:	Yes. I think my answer is already correct and sufficient to answer the problems. Furthermore, the final result is an integer, so it will make the manufacturing process easier. In addition, earlier, I also double-checked the answer by counting every length found in the question.

Based on S1's answers and interviews, the components of the mathematical argument can be seen as follows. In boxes EF-1 and EF-2, S1 can identify and write mathematical models based on contextual information for each problem. S1 can express information using relevant mathematical symbols to solve the first problem, such as naming a triangle and declaring congruent triangles, as seen in the EF-1 indicator above. Besides, S1 can write parallel symbols based on the information provided in writing the second mathematical model, as seen in the EF-2 indicator above. As for the two mathematical models formulated by S1, those expressed in geometric representations and relevant mathematical symbols.

In boxes EE-1 and EE-2, it can be seen that S1 can apply relevant mathematical concepts and perform procedures well. S1 can use the perimeter of a polygon and the triangle inequality theorem in the first problem, as seen in the EE-1 box. While in problem number 2, S1 can employ numerous trigonometric ideas, such as sine rules, cosine rules, and the similarity of triangles, as seen in the EE-2 box.

As seen in boxes RE-1 and RE-2, S1 can connect existing information to obtain a mathematical solution. In the first problem, S1 connects the length of the lumber to create an equilateral triangle model with the length of available lumber. So, it can be observed that the length of the lumber to form an equilateral triangle model is greater than the length of available lumber. Meanwhile, to determine the length for the isosceles triangular models that fulfill the problem, S1 connects the length information for additional isosceles triangular lumber with the length of lumber left by determining the relationship of both is less than. While in the second question, S1 was able to connect the information that the two horizontal supporting bars of the two models were the same size, which was 100 cm.

In solving the mathematical model problem in the first problem, S1 can provide alternative solutions, as shown in the RBE-1 in Figure 2. In the first problem, S1 provides several alternative solutions that fulfil the triangle inequality theorem requirements. In addition, S1 can clearly explain the underlying reason for applying the triangle inequality theorem, which aims to ensure that the lengths of the sides can be formed into triangles. Furthermore, in solving mathematical model problems in the second problem, S1 can think critically and abstractly by allowing various possible values, which can be seen in the RBE-2 box in Figure 3 that the S1 uses alpha and beta angles instead of particular angles.

S1 can reflect the mathematical answer to the real-world situation after acquiring the mathematical solution, which can be seen on the RBI-1 and RBI-2 boxes. In the first question, S1 was able to represent the mathematical results that lumber was not enough to model an isosceles triangle but was sufficient to model an isosceles triangle with a note using a length that met the requirements, as seen in the RBI-1 box. Whereas in the second question, S1 was able to write down the results of the representation that the length of the two models is always 100 cm.

Furthermore, based on the interview results, S1 can also explain and provide an argument that supports or rejects the mathematical solution that has been given. The the interview script in Table 3 indicates that S1 can state whether the solution interpretation found fits the given problem or not. S1 determine that the solution interpretation found fits the problem by double-checking the answer. S1 substituted the length of the sides obtained in each problem to recheck her answer. Therefore, the solution of S1 is completely correct.

S1 can make valid claims utilizing the data calculations for both problems, which can be seen in the CI-1 and CI-2 boxes. In the first problem, S1 states that Andrew's viewpoint is wrong for an equilateral triangle but correct for an isosceles triangle. Meanwhile, S1 states that Louis's opinion is wrong in the second problem.

Based on student answer sheets and interviews, an S1 argumentation diagram can be made as follows.

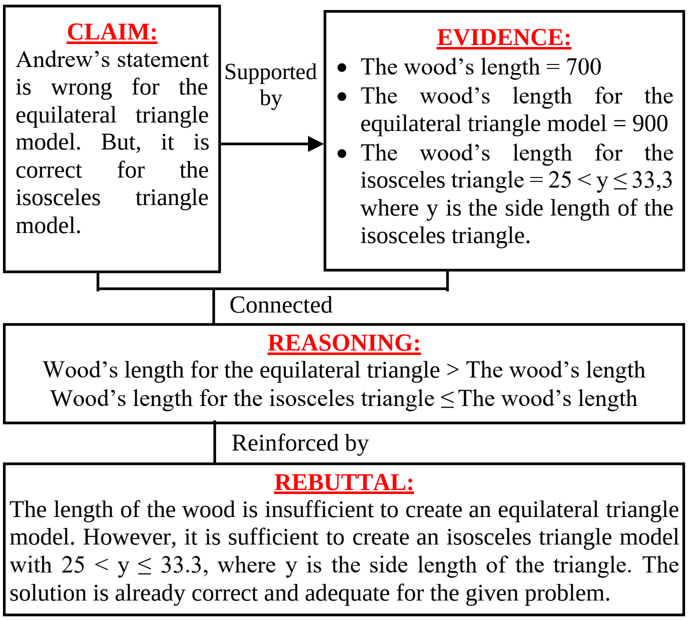

Figure 4. S1's Argumentation Diagrams for Number 1

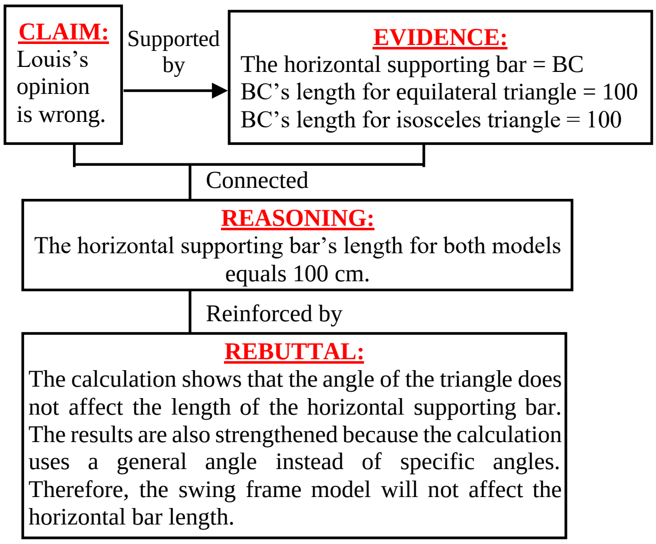

Figure 5. S1's Argumentation Diagrams for Number 2

Argumentation of student with moderate mathematics abilities (S2)

The answer sheets of S2 for both questions can be seen below.

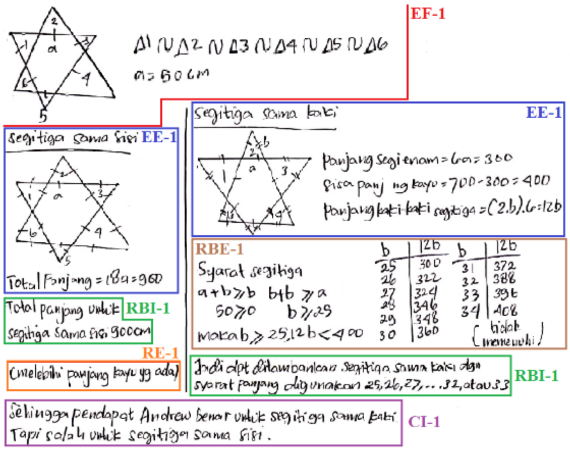

Figure 6. S2's Answer Sheets for Number 1

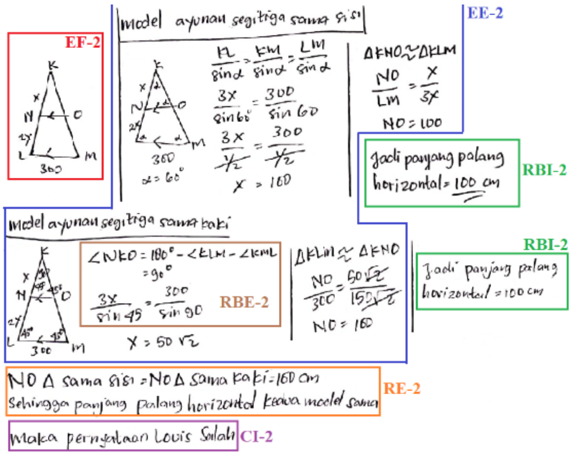

Figure 7. S2's Answer Sheets for Number 2

Based on the student's answer sheets in Figure 6 and Figure 7, it can be seen that the student's process of solving mathematical literacy problems in forming their arguments. In order to get in-depth descriptions of students' arguments, interviews were conducted as follows.

Table 4. Interviews Transcript of S2

R	:	Try to explain the steps to solve the problems you have written for question number 1
S2	:	First, I made a geometric illustration and wrote down the information known **(EF-1)**. In order to find the model of the equilateral triangle, I assume the length of hexagonal sides as a, so the total length of wood is 18a**(EE-1)**, so the required length of wood is 900 **(RBI-1)** more than the available wood **(RE-1)**. Meanwhile, for the isosceles triangle model, since the side length of the hexagon is a, the total wood for the hexagon's perimeter is 300, so the remaining wood that can be used is 700-300=400 **(EE-1)**. Suppose the side length of the triangle legs is b; then the total length of the legs is 12b with 12b≤400. In order to ensure that the conditions for forming a triangle are also fulfilled, I looked for the value of b using the triangle inequality. Since the triangle is isosceles, we obtain two inequalities, namely a+b≥b and b+b≥a, then b≥25 and 12b≤400 **(RBE-1)**. After that, I looked for the natural number values which satisfy b≥25 and 12b≤400 and found that the value of b is the range from 25 to 33, such as 26, 27, 28, etc. Thus, the isosceles triangle model can be made if the length of the wood chosen is in the range of 25 to 33 **(RBI-1)**. So, Andrew's statement is true for isosceles triangles but wrong for equilateral triangles. **(CI-1).**
R	:	Try to explain the steps to solve the problems you have written for question number 2
S2	:	First, I made a geometric illustration and wrote down the information known **(EF-2)**. In order to find the model of the equilateral triangle, I use the sine rule with an angle of 60 degrees. With the similarity of two triangles, I obtained that the length of NO is 100 cm **(EE-2)**. So, the length of the supporting bar for the equilateral triangle model is 100 cm **(RBI-2)**. Meanwhile, for the isosceles triangle model, I use the sine rule. Since the model is an isosceles triangle, the angles are 45 degrees and 90 degrees **(RBE-2)**. After obtaining the value of x, using triangular similarities, I looked for the length of NO, which is equal to 100 cm **(EE-2)**. So, the length of the supporting bar for the isosceles triangle model is 100 cm **(RBI-2)**. Thus, whatever the model that will be used, the length of the horizontal bar is always equal to 100 cm **(RE-2)**. Therefore, Louis' opinion is wrong **(CI-2)**.
R	:	Do you think the answer you found is correct and sufficient to answer the problem?

S2	:	Yeah. I think it's already correct and sufficient to answer the problem because the answer is not in decimal form. In addition, for the second problem, as long as the length ratio is maintained, no matter how much angle is applied, it will have no effect because those will eliminate each other.

Based on S2's answers and interviews, the components of the mathematical argument can be seen as follows. In boxes EF-1 and EF-2, it can be seen that S2 can identify the information provided and formulate a mathematical model of the two contextual problems. S2 can convert contextual information into mathematical symbols and provide geometrical representations in mathematical models, such as triangles and parallels. However, even though S2 can express contextual information on mathematical symbols, S2 has not been able to use the correct symbols as a whole. It was proven when S2 still had difficulty distinguishing symbols of similarities and congruence and how S2 stated a triangle using numbering in solving the first problem, as seen in the EF-1. Even so, S2 can explain the design of the model very well.

In examining whether Andrew and Louis's statements are valid, S2 separates the main problem into two minor cases: an equilateral triangle and an isosceles triangle. Then, S2 employs mathematical principles according to the existing problems to obtain the solution for each problem. However, some of the concepts used are still not completely accurate. This may be seen in applying the triangle inequality theorem while solving the first problem, where S2 utilizes the symbol more than or equal (≥) instead of greater than(>), as seen in EE-1. In addition, S2 also made several calculation errors which can be seen in the second question, namely calculating the value of sin 60 equal to half, as seen in EE-2.

As seen in boxes RE-1 and RE-2, S2 can connect the calculation result information with existing information to obtain a mathematical solution. In question number 1, S2 states that the length of the lumber for the equilateral triangle exceeds the length of the available lumber based on the calculation results. In addition, S2 can also relate information from the terms of the length of the legs of an isosceles triangle, which must be more than or equal to 25, and the length of twelve legs must not exceed 400 cm, as seen in RE-1. While in question number 2, S2 can connect information from the two calculations for each model. It can be seen when S2 states that the length of NO in an equilateral triangle equals the length of NO in an isosceles triangle, which is 100 cm, as seen in RE-2.

S2 can provide alternate solutions to mathematical model problems that have been constructed, as seen by the RBE-1 in Figure 6. S2 can provide numerous alternate answers for length b, representing the length of an isosceles triangle's sides in the first problem. Despite being able to propose alternate answers for the sides length of an isosceles triangle using the triangle inequality theorem, S2 is still committing calculation errors and applying mathematics concepts wrongly. Misapplying the mathematics principle can be demonstrated in the symbol used in the triangle inequality theorem. Meanwhile, the inaccuracy of calculation may be noticed in the value of 12b for b = 26, 28, and 32. While answering the second problem, S2 still employs a certain angle, which equals 45 degrees, to obtain the proper solution, as seen in RBE-2. It shows that S2 has not been able to solve mathematical model problems using the general solution form.

After obtaining the solution from the mathematical model, S2 transforms the mathematical solution obtained into a contextual solution. The RBI-1 and RBI-2 boxes show that S2 interprets the mathematical solutions for both questions correctly, even though there are some error calculations in finding the mathematical solution.

Furthermore, based on the interview results, S2 can also explain and provide an argument that supports or rejects the mathematical solution for both questions. The interview script in Table 4 indicates that S2 can state whether the solution interpretation found fits the given problem or not. S2 can provide the supporting argument for the solution found. However, even though S2 has been able to provide arguments that support the results of the interpretation solution, S2 does not recheck the results of the existing solutions. Therefore, S2 does not realize that the alternative solution is not completely correct.

S2 can make conclusions based on the data that has been collected and the contextual solutions found. In the first problem, as seen in CI-1, S2 finds that Andrew's claim is right for isosceles triangles but wrong for equilateral triangles. While on the second question, as seen in CI-2, S2 concluded that Louis' opinion was wrong.

Based on student answer sheets and interviews, an S2 argumentation diagram can be made as follows.

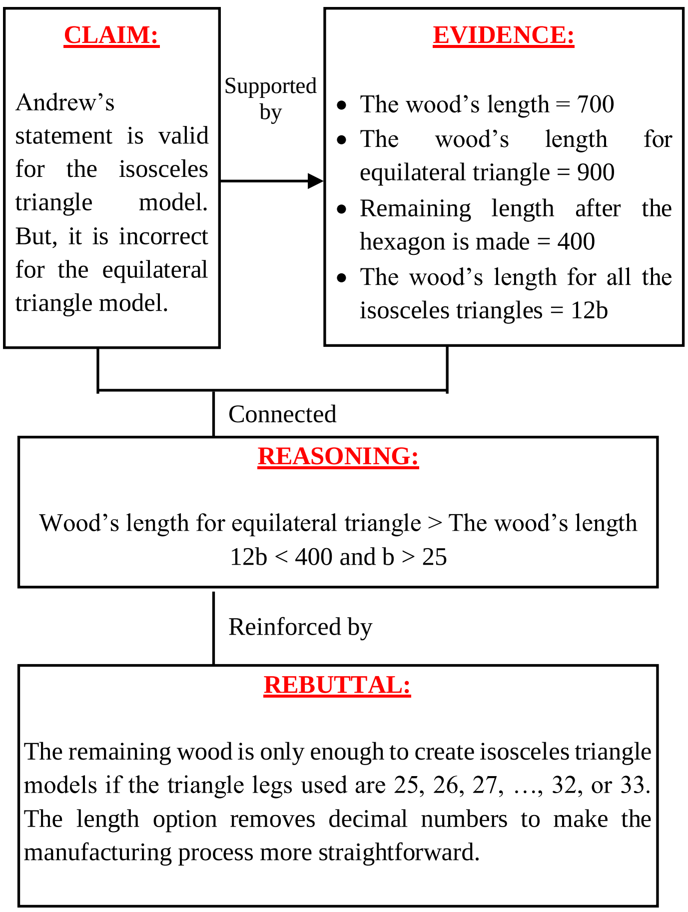

Figure 8. S2's Argumentation Diagrams for Number 1

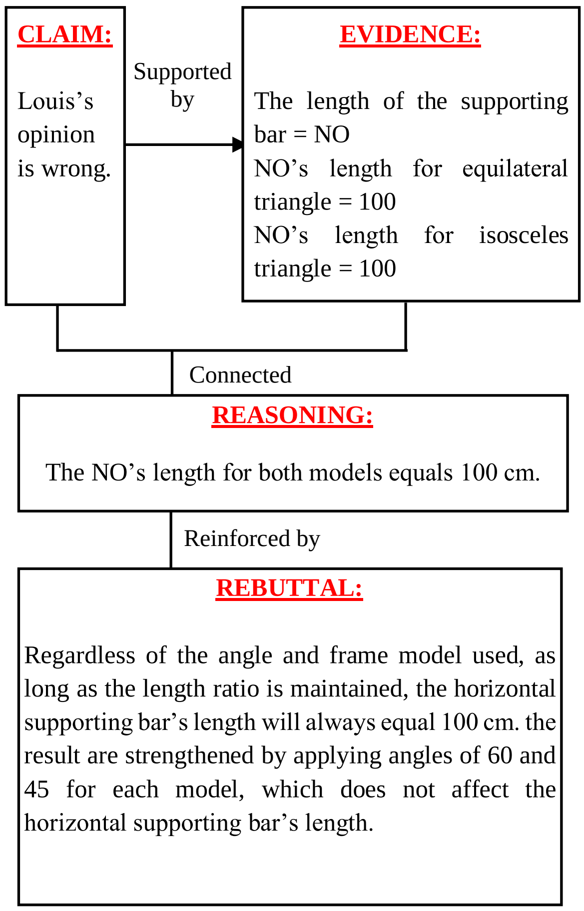

Figure 9. S2's Argumentation Diagrams for Number 2

Argumentation of student with low mathematics abilities (S3)

The answer sheets of S3 for both questions can be seen below.

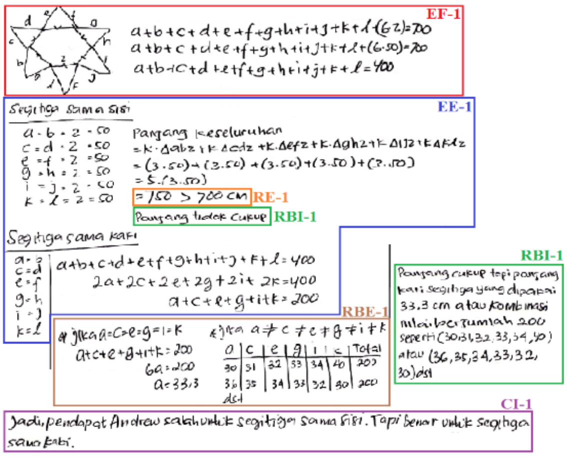

Figure 10. S3's Answer Sheets for Number 1

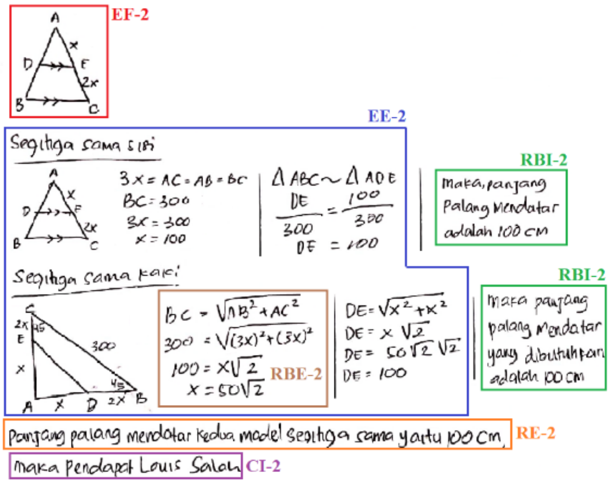

Figure 11. S3's Answer Sheets for Number 2

Based on the student's answer sheets in Figure 10 and Figure 11, it can be seen that the student's process of solving mathematical literacy problems in forming their arguments. In order to get in-depth descriptions of students' arguments, interviews were conducted as follows.

Table 5. Interviews Transcript of S3

R	:	Try to explain the steps to solve the problems you have written for question number 1
S3	:	First, I made a geometric illustration and wrote down the information known **(EF-1)**. For the equilateral triangle model, I assume that the side length of the hexagon is z, then the side length of the triangle is also z. So, the total length of wood is the perimeter of 6 equilateral triangles that are equal to 750 cm **(EE-1)**. Since 750 cm is greater than the available wood **(RE-1)**, the equilateral triangle model can't be made because of the insufficiency of wood **(RBI-1)**. Meanwhile, for the isosceles triangle model, the remaining length that can be used for the legs of the triangle is 700-6z=400. Because the triangle is isosceles, two sides have the same side length; solved algebraically, I obtained 200 **(EE-1)**. Next, I determined the side lengths of each triangle. If each side length between the triangles is the same, the value is 6a = 200, so a = 33.3. But, if the side lengths between the triangles differ, the length of the triangles is the combination of numbers totaling 200, such as 30, 32, 32, 33, 34, and 40 **(RBE-1)**. Thus, the isosceles triangle model can be made if the length of the wood chosen for the sides of the triangle is 33.3 or a combined number that equals 400 **(RBI-1)**. Therefore, Andrew's statement is wrong for equilateral triangles and true for isosceles triangles **(CI-1).**
R	:	Try to explain the steps to solve the problems you have written for question number 2
S3	:	First, I made a geometric illustration and wrote down the information known **(EF-2)**. In order to find the model of the equilateral triangle, I find the value of x using the perimeter of the triangle. Then, using the similarity of two triangles, I obtained the length of DE equal to 100 **(EE-2)**. Thus, the length of the supporting bar for the equilateral triangle model is 100 cm **(RBI-2)**. Meanwhile, for the isosceles triangle model, I look for the value of x with the Pythagorean theorem **(EE-2)** with angles of 45 degrees and 90 degrees **(RBE-2)**. With triangular similarities, I obtained the length of DE equal to 100. So, the length of the supporting bar for the isosceles triangle model is 100 cm **(RBI-2)**. Thus, whatever the model that will be used, the length of the horizontal bar is always equal to 100 cm **(RE-2)**. Therefore, Louis' opinion is wrong **(CI-2)**.

R	:	Do you think the answer you found is correct and sufficient to answer the problem?
S3	:	Yeah, it seems so. I just matched the questions asked with my answers. Since my answer is in accordance with the question asked, then I think the answer is already done.
R	:	Did you double-check it to make sure that your answer matched with the question asked?
S3	:	No, I didn't

Based on S3's answers and interviews, the components of the mathematical argument are shown as follows. In boxes EF-1 and EF-2, it can be seen that S3 can formulate mathematical models from both contextual problems. In the first problem, as shown in EF-1, S3 formulated a mathematics model using geomaterial illustration and algebraic expression. Whereas in the second problem, S3 can write mathematics symbols such as parallelism. Although S3 can successfully create mathematical models, S3 has not been able to write down all of the information given, as seen in EF-1, S3 missed the triangle congruence concept in the first problem.

In considering the validity of Andrew and Louis's statement, S3 split the main issue into two separate cases: equivalent and isosceles triangles. In the first problem, as shown in EE-1, S3 makes calculation errors in the length of an equilateral triangle. In addition, students also missed an important concept used to solve the first problem, which is the congruence of triangles and triangle inequality theorem. Nevertheless, S3 can apply some mathematical concepts correctly in the second question, such as similarity, parallelism, and the Pythagorean theorem.

As seen in boxes RE-1 and RE-2, S3 can connect the existing information with the calculated data. In the first problem, as shown in RE-1, S3 can connect the length for the equilateral triangle model is greater than the available lumber's length. Meanwhile, in the second problem, S3 connects information from the calculation of both models. It can be seen when S3 states that the length of DE in the equilateral triangle equals the length of DE in the isosceles triangle, which is 100 cm.

In solving the mathematical model problem generated, S3 has provided alternative solutions, which may be seen in the RBE-1 indicator in Figure 10. In the first problem, S3 can propose alternative solutions in the form of a combination of the isosceles triangle's sides length that can be used. However, even though S3 has been able to provide alternative solutions, the solutions provided are still not adequate since S3 missed the crucial concept in formulating the mathematics model.

Unlike the other subject's method, S3 has a different approach to solving the second problem. S3 uses the concept of the Pythagorean theorem instead of trigonometry, which can be seen in the RBE-2 indicator in Figure 11. Although the Pythagorean theorem applied by S3 is correct, the solution is not generally applicable since S3 uses particular angles in applying the Pythagorean theorem.

After obtaining the mathematical solutions, S3 transforms the mathematical solutions into real-world contexts. In the RBI-1 and RBI-2 boxes, it can be seen that S3 can reflect the mathematical solution for both questions well, even though there are some mistakes made by S3 when finding the mathematical solution.

Furthermore, based on the interview results, S3 has not been able to provide an argument for whether the mathematical solutions found to fit the problems. The interview script in Table 5 indicates that S3 cannot state whether the solution interpretation found fits the given problem or not. S3 has difficulty in providing a supporting argument for the solution found. Besides, S3 does not recheck the solutions found. Therefore, S3 does not realize that the alternative solution is not completely correct.

S3 can make valid claims utilizing the data calculations for both problems. In the first problem, as seen in CI-1, S3 states that Andrew's opinion is wrong for an equilateral triangle but is correct for an isosceles triangle. Meanwhile, in question number 2, as seen in CI-2, S3 concluded that Louis's opinion was inaccurate.

Based on student answer sheets and interviews, an S3 argumentation diagram can be made as follows.

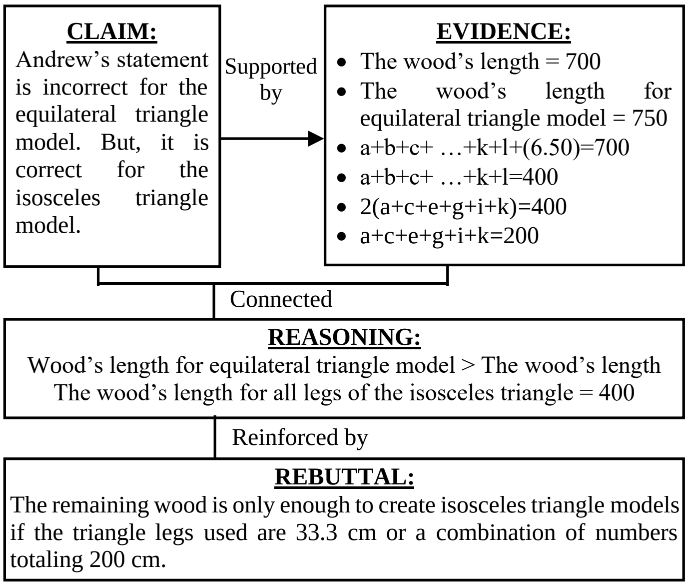

Figure 12. S3's Argumentation Diagrams for Number 1

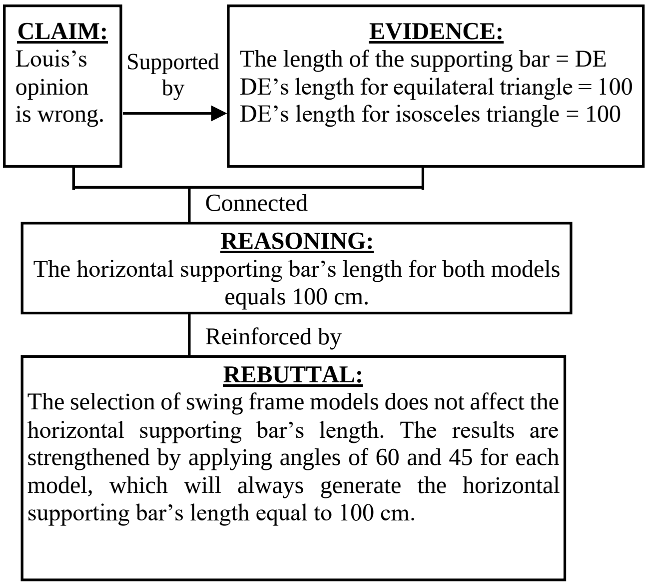

Figure 13. S3's Argumentation Diagrams for Number 2

Disscusion

The results of the research above show that each level of mathematical ability provides different arguments. The argumentation components provided by each mathematical ability when solving literacy problems have different characteristics. It is due to differences in understanding, identifying, formulating, analyzing, determining mathematical approaches, and interpreting in finding solutions.

In the evidence argumentation component, all students at each level of mathematical ability can understand problems well. Students at each level of mathematical ability can identify the information provided and formulate a mathematical model of the two contextual problems. Students with high mathematical ability are highly good at formulating mathematical models utilizing mathematical language and geometrical representations. In addition, students have a deep understanding of mathematical concepts, so that students can apply various mathematical concepts and perform procedures well to find mathematical solutions to each problem. It can be seen when students understand and use various related concepts, such as congruence triangles, the perimeter of hexagonal shapes, the inequality triangle theorem, properties of equilateral triangles, and properties of isosceles triangles. These results align with the research of Anshori (2021), Laamena (2018), and Oktaviyanthi (2019) that states students with high mathematical abilities can formulate mathematical models with image representations and apply the mathematical principles they have learned correctly to construct their arguments and at the evidence stage. Students with moderate mathematical skills may identify and formulate all the provided contextual information into mathematical models using mathematical symbols and geometrical representations, even though some of the mathematical symbols created by students are only partially accurate. In addition, students can think critically and analyze the information presented to evaluate statements/claims by breaking down the primary issue into two minor cases. Divining the main issues into two small cases makes it easier for students to apply relevant mathematical concepts and analyze the information to support their arguments. These findings support the findings of Anshori (2021), Indrawatiningsih (2019) and Yulia (2021), which state that students with moderate mathematical abilities can analyze the information provided and describe their mathematical model in order to provide supporting evidence to evaluate an argument, even though students do miscalculating and misapplying concepts at the evidence stage. In the meantime, students with low mathematical skills might construct contextual information presented with mathematical terminology and geometrical visualizations to help in problem-solving. Nevertheless, students were unable to formulate the key concept to solve the issue, which is the similarity of triangles. Additionally, students have been unable to use the mathematics symbols correctly and still make miscalculations. These results are consistent with those of Anshori (2021) and Yulia (2021), who found that students with low mathematics skills can give evidence to evaluate an argument but only partially apply their concepts and knowledge to solve problems and provide evidence for their arguments.

In the reasoning argumentation component, students of each level of mathematical ability can connect the pieces of information found as justification for statements accompanied by evidence to determine the solution to solving a given problem. Students with high mathematical abilities can identify and relate existing information to support claims at reasoning indicators, which can be seen from the students' answers that connect the mathematical solution of each triangular model to the information on the number of available lumber in the problem. Students with moderate mathematics abilities can identify and relate existing information to support claims, even though the information is only partially correct. Meanwhile, students with low mathematics abilities can connect existing information to support claims well, even though some information is incorrect due to inaccurate evidence indicators. These findings align with Anshori (2021), which states that students with high and moderate mathematics abilities can meet reasoning indicators well but contrarily for students with low mathematics abilities. In this research, students with low mathematical abilities can relate existing information found and provide reasons as justification for statements accompanied by evidence. Therefore, in this research, students with low mathematics abilities can meet the reasoning indicators.

In the rebuttal argumentation component, students from each level of mathematical ability could provide several alternative solutions in mathematical form, even though not all levels of mathematical ability could provide general mathematical solutions. Students of each level of mathematical ability can also transform very well the mathematical solutions that have been obtained into contextual solutions, even though not all levels of mathematical ability can assess the adequacy of the contextual answers given to the questions posed. Students with high mathematics abilities can provide several alternative solutions to the given problem by applying the inequality triangle theorem. Furthermore, the alternative solutions which students provide are solutions that apply in general. These solutions are applied for general because the solutions given by students use general angles, symbolized by alpha/beta, instead of specific certain angles in problem-solving processes. In addition, students can represent the mathematical solutions in the real-world context and assess whether the mathematical solutions provided are appropriate to contextual problems by rechecking the mathematical solutions. It indicates that it has no difficulty interpreting solutions to contextual problems, which aligns with research (Oktaviyanthi, 2019). Students with moderate mathematics abilities can provide alternative solutions for the mathematical models created by applying the inequality triangle theorem. However, because students miscalculate and misapply the triangle inequality theorem concept in solving problems in the evidence component, some alternative solutions that students provide do not correct. In addition, the solutions provided by students have not been able to apply in general, which can be seen from the selection of angles that still use a specific angle that equals 45 degrees. Regardless of whether the mathematical solution is correct, students with moderate mathematics abilities can correctly interpret mathematical solutions into contextual solutions and provide explanations that support/reject whether the mathematical solutions obtained are sufficient for the problems given. This result aligns with Oktaviyanthi's (2019) and Yulia's (2021) research, which states that students with moderate abilities can interpret existing mathematical solutions into contextual solutions. Meanwhile, students with low mathematics abilities can reflect on mathematical solutions to the real-world context at the rebuttal indicators and provide alternative solutions. Nevertheless, since students miss the crucial concept of solving problems, the alternative solutions students give are only partially correct. In addition, the students' solution does not apply in general, as seen from the selection of angles in the Pythagorean theorem, which still uses a specific angle that equals 45 degrees. Regardless of whether the mathematical solution is correct, students with low mathematics abilities can correctly interpret mathematical solutions into contextual solutions well. Students also still have difficulty assessing whether the mathematical solutions provided are appropriate for contextual problems. This result aligns with Hidayati's (2020), which states that students with low abilities can interpret the solution contextually even though the solution is wrong.

In the claim argumentation component, students of each level of mathematical ability can make relevant conclusions and claims based on the results of problem-solving that have been done. Students with high mathematical abilities can provide complete and correct claims for the claim indicators. This study's results align with the studies of Anshori (2021) and Laamena (2018), which imply that students with high mathematics abilities can make valid and correct claims. Students with moderate abilities can also provide correct claims by referring to the previous solutions, even though the answers found by the students are only partially correct. Students can generate correct conclusions because students only make minor errors in calculating and applying mathematical concepts. Therefore, the overall findings of the student calculations still lead to a proper conclusion. These results coincide with Anshori's (2021) and Indrawatiningsih's (2019) studies, which suggest that students can give the correct claim and examine an argument/claim from others. Meanwhile, students with low mathematics abilities can provide correct claims, even though the contextual solution found by students is only partially correct. The claim is correct because the contextual solution is relevant to the question asked, even though the computation results are incorrect. These findings align with Fujiwijaya's studies (2017) which state that students with low mathematics abilities have good argument abilities in writing the reasons underlying the solving process to obtain the answers. Even though the process is not entirely correct, the students can solve the questions, make conclusions and make valid arguments with the correct claims using their language (Anshori, 2021; Fujiwijaya, 2017; Laamena, 2018).

CONCLUSION AND SUGGESTIONS

Based on the results and discussion above, several conclusions can be drawn regarding students' argumentation. Students with high mathematical abilities could formulate and perform the procedures at the evidence indicator. Students are also able to connect information found at the reasoning indicator. At the rebuttal indicator, students can provide general alternative solutions and represent and assess whether the mathematical solutions are appropriate to real-world problems by rechecking the mathematical solutions. Meanwhile, at the claim indicator, students can make a correct claim.

Students with moderate mathematical ability could apply mathematical concepts, although students make a miscalculation at the evidence indicator. Students are also able to connect some information at the reasoning indicators. At the rebuttal indicators, students provide and interpret alternative solutions from the mathematical models, even if some of those are not fully correct in calculation and applied concepts. Students are also able to represent and evaluate the sufficiency of the mathematics solutions at the rebuttal indicator. Meanwhile, at the claim indicator, students are able to generate a correct claim.

Students with low mathematical ability miss a crucial concept and make miscalculations at the evidence indicator. But, students can connect the information at the reasoning indicators. At the rebuttal indicators, students can provide and interpret partially correct alternative mathematical solutions to the real-world context but cannot evaluate the sufficiency of the mathematical solutions. Meanwhile, at the claim indicator, students are able to provide a correct claim.

Based on the results of the research above, it can be concluded that students with high mathematical abilities can provide excellent arguments supported by in-depth mathematical concepts in the problem-solving process. Students with moderate mathematical abilities can give good arguments but still make calculation mistakes in solving problems. Meanwhile, students with low mathematical abilities can provide a fairly good argument by fulfilling several components of the argument but misapplying mathematical concepts in problem-solving.

Several recommendations can be put forward based on the results of this study. Teachers should develop appropsriate learning models and activities to facilitate students of every level of mathematical ability to have good argumentation. More in-depth study on learning models is needed to increase students' argumentation in solving mathematical literacy problems at each ability level.

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