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The Process of System of Linear Equations in Three Variables Solving Procedure's Construction Using Analogy: Individual VS Paired

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Abstract: The process of knowledge construction can provide meaningful learning experiences for students. This is because students build new knowledge themselves by connecting one knowledge to another. The purpose of this qualitative research is to describe the process of new procedure's construction using analogy. The subjects of the research consisted of three students of grade X high school (one student took the test individually, two students took the test in pair). Data analysis based on the APOS theory's stage (Action, Process, Object, and Schema). At the action stage, both individual and paired students determine what is known and asked about the system of linear equations (SLE) in three variables problem based on analogy with the known things and asked about the SLE in two variables problem. They correctly determine the solution set of SLE in three variables. They also checked the correctness of the solution set of SLE in three variables correctly. At the process stage, they outline the steps of defining the solution set of SLE in three variables clearly. At the object stage, individual student cannot explain other methods of solving SLE in three variables, while paired students explain four other methods of solving SLE in three variables, that is the method of elimination, substitution, graphing, and matrix. At the schema stage, individual student cannot generalize some methods of solving SLE in three variables, whereas paired student generalize some methods of solving SLE in three variables. They also concluded the most effective method of solving SLE in three variables, that is the combined method. Individual student also explains that there is a SLE in three variables that has no solution, whereas paired students cannot explain it. They can construct new procedure well, despite errors in their process. In the process of new knowledge construction, the student's prior knowledge determines the quality of its construction process.

INTRODUCTION

Meaningful learning experiences for students are gained when they actively construct their own knowledge (Sharan, 2014). The knowledge construction process relies on constructivism theory which requires students to actively construct knowledge continuously (Liu et al., 2022; Permata et al., 2018). Thus, there is always a change in knowledge towards a more complete and in accordance with the scientific concepts (Anggraini et al., 2018; Husamah et al., 2015; Sugrah, 2020).

Not many teachers believe that students are able to construct knowledge. As a consequence, based on the results of the research of Indrasari et al. (2022), mathematics learning in the classroom until now is still often centered on the teacher as the material presenter. Based on observations by researcher in August 2022 at one of the high schools in

Sidoarjo, many mathematics teachers still teach by providing knowledge, not constructing knowledge. Thus, all this time mathematical knowledge is not the construction of the students themselves, it is still given by the mathematics teacher.

The process of knowledge construction is a stage in building new knowledge (Setyawan & Rahman, 2013; Zabolotna et al., 2023). Constructing knowledge means building knowledge by relating from one knowledge to another (Anggraini et al., 2018). Based on these two definitions, the process of knowledge construction is a stage in forming new knowledge guided by knowledge that has been previously possessed. Thus, the process of knowledge construction occurs when a someone learns new knowledge.

The understanding of a person's knowledge as a result of reconstruction or construction of the object he/she is studying (Mumu et al., 2018). It is called the result of construction if the knowledge learned has never been learned before (new knowledge), while the result of reconstruction if it has been learned before. That way, the construction process experienced by a person is expected to lead to an understanding of knowledge.

The mathematical knowledge that students learn is not only related to concepts, but also procedures (Hurrell, 2021). Knowledge related to procedures is often learned using direct instruction (Sulistiani, 2017). Procedures taught by direct instruction result in students only being able to carry out work procedures to the extent taught without knowing the concepts in depth. This is reinforced by the result of research from Haryandika et al. (2017) which showed that it was found that some students did not know how and when to use a procedure, students could only do questions of the same type as the sample questions given by the teacher on the blackboard. This is as a result of students not being involved in constructing procedures.

The focus of teaching material in this research is the SLE in three variables material. In this material, students are not only required to understand the concepts of equations and SLE in three variables, but also required to be able to apply the procedure of solving SLE in three variables. In addition, SLE in three variables is one of the mathematics materials that is considered challenging for students. The challenge lies in performing algebraic calculation operations with fairly long steps (Devi et al., 2020). SLE in three variables is more complex than SLE in two variables because SLE in three variables is an extension of SLE in two variables.

With regard to the process of knowledge construction, there are Dubinsky's APOS theory and Roger Bybee's Five E's. The APOS and Five E theories both reveal stages in building knowledge. APOS theory states that a person will go through four stages in building knowledge, that is the action, process, object, and schema stages (Dubinsky & McDonald, 2001). The stages in the knowledge construction process through the Five E's are engage, explore, explain, elaborate, and evaluate (Bybee, 2009).

The difference between APOS and Five E theories lies in the timing of the implementation of the process of knowledge construction. Syamsuri & Santosa (2021) stated that APOS theory studies individuals in constructing knowledge outside the learning process and it can be implemented to assist the learning process. The Five E's are applied to students during learning (Setyawan & Rahman, 2013). Therefore, researchers chose to use APOS theory because this research aims to reveal the process of new procedure’s construction for students outside the learning process (before students get SLE in three variables material).

Previous research related to APOS theory is more widely used to describe the process of reconstructing mathematical concepts (Anam et al., 2020; Israhayu et al., 2021; Kurniawan et al., 2018; Listiawati & Juniati, 2021; Safitri et al., 2021; Tatira, 2021; Zahid et al., 2014), to describe the construction process of mathematical concepts (Rosyidi & Hasanah, 2022), and used to underlie the development of worksheet (Arnawa et al., 2019; Fatimah et al., 2017). In addition, research from Ummah & Azmi (2020) focuses on concept construction through learning media. Imamuddin et al. (2019) focuses on understanding concepts, while Anggraini et al. (2018), Inganah et al. (2021), and Ni’mah et al. (2018) focus on errors in the construction of mathematical concepts. This research focuses on the process of new procedure’s construction that have never been studied before. This research is important because it can be a reference for mathematics teachers to design learning by constructing new procedures using analogy.

Previous research related to SLE in three variables material focused on concept construction (Amelia et al., 2021; Wahyuningsih et al., 2019), analysis of student errors when completing SLE in three variables (Dewi & Kartini, 2021; Habibah et al., 2020; Hariati & Septiadi, 2019; Kuswanti et al., 2018), and analysis of students' difficulties when completing SLE in three variables (Cardo A.P. et al., 2020; Wahab & Sunarti, 2022). This research focuses on the process of procedure’s construction for solving SLE in three variables using analogy.

In the process of knowledge construction, the use of analogy can help students build conceptual bridges between knowledge already known and new knowledge (Uyen, 2021). Holyoak & Morrison (2005) define analogy as the similarity of two different things with respect to a goal and state that the mapping process is oriented towards achieving the goal. Mofidi et al. (2012) stated that analogy is a mapping between elements, that is between the source domain (old objects that have been known before) and the target domain (new objects). Based on both definitions, analogy is the similarity between something new and something that has been known before.

In analogy there are two terms, that is source problem and target problem (Kristayulita et al., 2020). The source problem is a basic problem that has been studied before and is useful as a provision for solving more complex problems, while the target problem is a more complex problem and solving it by finding similarities with the basic problem (Assmus et al., 2014; Purwanti et al., 2016). Saleh et al. (2017) and Siswono & Suwidiyanti (2009) stated that to find solution to source problem, student will use the knowledge they already have, while to find solution to target problem, student will use source problem. Thus, when students learn the procedure of solving a SLE in three variables, students can use analogy with the procedure of solving SLE in two variables. This is because the procedure for solving SLE in three variables has similarities with the procedure for solving SLE in two variables.

In Vygotsky's constructivist theory, it is argued that the development of a person's abilities is divided into two levels, that is the level of actual and potential development (Slavin, 2006). The actual level of development is seen from a person's ability to solve problems or complete tasks with their own efforts, while the level of potential development is seen from a person's ability to solve more complex problems or complete tasks when collaborating with peers (Slavin, 2006; Suardipa, 2020). By collaborating with peers, a person is expected to solve more complex problems so that they can achieve their potential development. Therefore, researchers want to see the process of new knowledge construction of students individually and in pair.

This research is expected to give a description related to the process of SLE in three variables solving procedure’s construction using analogy. So, the purpose of this research is to describe the process of SLE in three variables solving procedure’s construction between individual student and paired students using analogy.

METHOD

This research uses a qualitative descriptive method that aims to provide an overview of the process of SLE in three variables solving procedure’s construction for high school students using analogy based on APOS theory. The selection of research subjects was determined by purposive sampling technique. The selection of research subjects with the consideration that the student has not studied the material of the SLE in three variables. All selected research subjects were indicated to use analogies when solving SLE in three variables and being able to solve SLE in two variables correctly. The prospective research subjects consisted of 137 grade X students in one of the senior high schools in Sidoarjo in the even semester of the 2022/2023 academic year. Prospective research subjects consisted of 68 class X students who took the test individually and 35 pairs of class X students who took the test in pairs. The research subjects in this research consisted of a student who took the procedure construction test individually and paired students who took the procedure construction test in pair. Therefore, the subjects in this research were two subjects.

The instruments in this research are procedure construction test and task-based interview (test). The procedure construction test consists of two kinds of problems, that is the source and the target problems. The question on the source problem is the SLE in two variables problem, while the question on the target problem is the SLE in three variables problem. The problems on the source problem and the target problem have been adjusted to the indicators of the process of SLE in three variables solving procedure’s construction using analogy based on APOS theory. The procedure construction test questions can be seen in figure 1.

Source Problem

Given a system of linear equations in two variables as follows.

$$\begin{cases} x + 2y = 10 \\ 3x - y = 9 \end{cases}$$

1) Solve the system of linear equations in two variables above!
2) Write down the steps you used to solve the system of linear equations in two variables above!

Target Source

Given a system of linear equations in three variables as follows.

$$\begin{cases} x + y + z = 6 \\ 3x + 2y + z = 10 \\ x - 2y + 3z = 6 \end{cases}$$

1) Solve the system of linear equations in three variables above just like you solved the system of linear equations in two variables in the source problem!
2) Write down the steps you used to solve the system of linear equations in three variables above based on your steps in solving the system of linear equations in two variables!
3) If there is a system of the following equations:

$$\begin{cases} ax + by + cz = d \\ ex + fy + gz = h \\ px + qy + rz = s \end{cases}$$

and based on your experience learning system of linear equations in two variables, how do you solve those system of linear equations in three variables?

Figure 1. Procedure Construction Test

The results of the written test were analyzed, then 2 subjects were selected (one subject worked individually and two subjects worked in pair) to be interviewed based on the information written on the answer sheet. Task-based interview (test) are conducted to confirm the subjects' answers and explore the construction process of solving SLE in three variables using analogy based on APOS theory. The process of SLE in three variables solving procedure's construction using analogy is analyzed based on APOS theory as presented in table 1.

Table 1. Indicators of the Process of SLE in Three Variables Solving Procedure's Construction Using Analogy Based on APOS Theory

APOS Theory Stage	Indicator	Code
Action	1. Determine what is known and asked about a SLE in three variables problem based on analogy with what is known and asked about a SLE in two variables problem.	A_1
	2. Determine the solution set of SLE in three variables by analogy with how to determine the solution set of SLE in two variables.	A_2
	3. Checking the correctness of the solution set of SLE in three variables that has been obtained using an analogy by checking the correctness of the solution set of SLE in two variables.	A_3
Process	Outlines the steps of determining the solution set of SLE in three variables based on analogy with the steps of determining the solution set of SLE in two variables.	P
Object	Describes other methods of solving SLE in three variables by analogy with methods of solving SLE in two variables.	O
Schema	1. Generalizes several methods of solving SLE in three variables after analogizing with methods of solving SLE in two variables.	S_1
	2. Conclude the most effective method of solving SLE in three variables based on analogy with the method of solving SLE in two variables.	S_2

Advanced Table 1

APOS Theory Stage	Indicator	Code
	3. Describe possible solutions of SLE in three variables based on analogy with possible solutions of SLE in two variables.	S_3

RESULT AND DISCUSSION

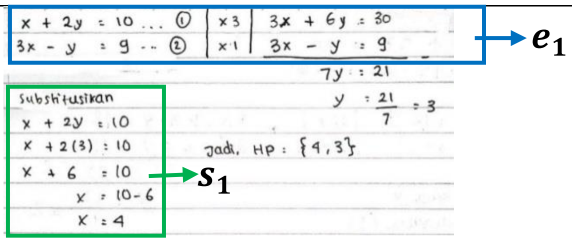
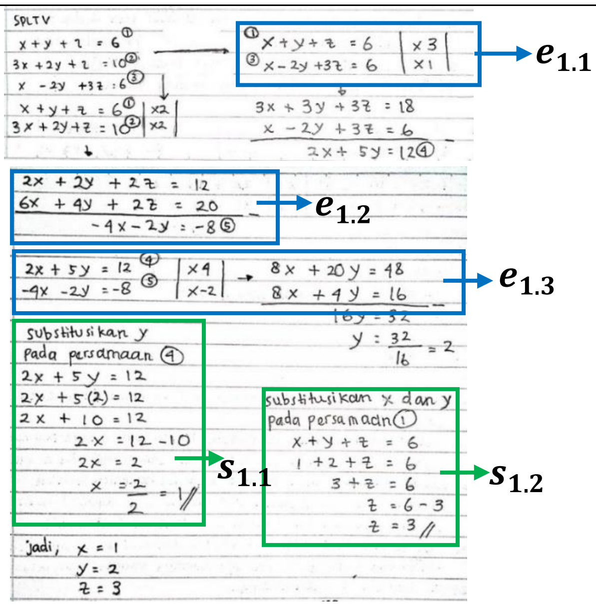
The following are presented the results and analysis of research data on the process of SLE in three variables solving procedure's construction of high school students using analogy.

1. **Results and Analysis of the Process of SLE in Three Variables Solving Procedure's Construction of Individual Student Using Analogy (Individual Subject Correctly Solves Problems Involving SLE in Two and Three Variables / I-CC Subject)**

   a. Action Stage

   The answer of the I-CC subject in the procedure construction test at the action stage are presented in table 2.

Table 2. Answer of I-CC Subject in the Action Stage

Figure	Code
 (Answer to the source problem / problem of SLE in two variables)	A_2
 (Answer to the target problem/problem of SLE in three variables)	A_2

The following is an excerpt of an interview with the I-CC subject in the action stage.

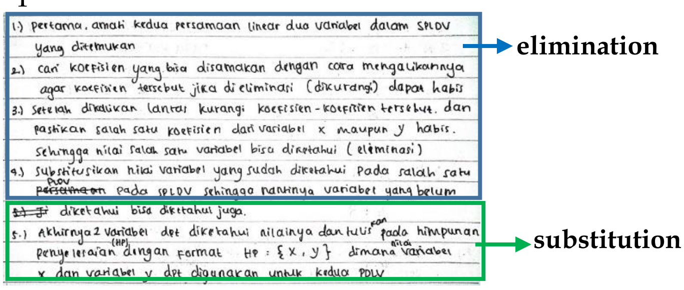
RI_1-01: *What is known in the problem of SLE in three variables according to your experience learning SLE in two variables?*
IS_1-01: *There are three variables and three equations, Ma'am. If in the SLE in two variables, the equation is only two and the variable is also two.* ***(A_1)***
RI_1-02: *What is asked in the problem of a SLE in three variables?*
IS_1-02: *In a SLE in two variables, usually look for x and y values, then what is asked in a SLE in three variables is the value of the variables x, y, and z.* ***(A_1)***
RI_1-03: *How do you prove that the set of solutions to the SLE in three variables you have obtained is correct?*
IS_1-03: *By substituting the values of x, y, and z into 1st equation just as in a SLE in two variables, how to check it by substituting the values of x and y into the equation in the problem.* ***(A_3)***
RI_1-04: *Is it enough to check in one equation only?*
IS_1-04: *No, in three equations must correct because as I remember in the SLE in two variables were checked in both equations.* ***(A_3)***

Based on the work of individual subject in table 2 and the results of interview, I-CC subject determines what is known and asked about a SLE in three variables problem based on analogy with what is known and asked about a SLE in two variables problem (A_1; IS_1-01; IS_1-02). She also correctly determines the solution set of SLE in three variables (A_2). The analogy is seen in the way she solves problems of SLE in two and three variables, that is using the combined method by method of elimination (e_1 ke $e_{1.1}$; $e_{1.2}$; $e_{1.3}$) continued substitution (s_1 ke $s_{1.1}$; $s_{1.2}$). In addition, she also checks the correctness of the solution set of SLE in three variables that have been obtained correctly (A_3; IS_1-03; IS_1-04).

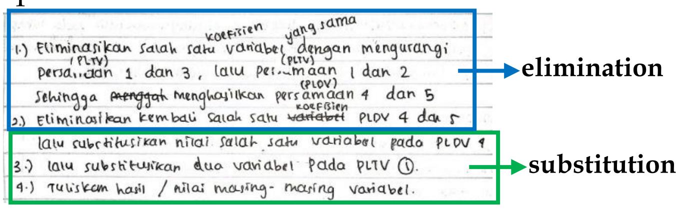
b. Process Stage

The answer of the I-CC subject in the procedure construction test at the process stage are presented in table 3.

Table 3. Answer of I-CC Subject at the Process Stage

Figure	Code
Steps to Solve a SLE in Two Variables  *Translation:* *1. Observe both linear equations of two variables in the SLE in two variables found;* *2. find a coefficient that can be equated by multiplying it so that the coefficient if eliminated can run out;*	P

Advanced Table 3

<table>
<tr><th>Figure</th><th>Code</th></tr>
<tr><td>
3. after multiplying, then subtract the coefficients and make sure one of the variables can be known (elimination);
4. substitute the value of a known variable in one of the linear equation of two variables in the SLE in two variables so that later unknown variables can also be known;
5. finally, two variables can be known for their values and write them on the solution set with the format HP = {x, y} where the values of variable x and variable y can be used for both linear equations of two variables.
</td><td>P</td></tr>
<tr><td>
Steps to Solve a SLE in Three Variables


Translation:
1. eliminate one of the variable coefficients of the same in linear equation of three variables by subtracting 1st and 3rd equations, then 1st and 2nd equations;
2. eliminate one of the coefficients of the linear equation of two variables, that is 4th and 5th equations and substitute the value of one of the variables in the 4th linear equation of two variables;
3. then substitute two variables in the first linear equation of three variables;
4. write down the result/value of each variable.
</td><td>P</td></tr>
</table>

The following is an excerpt of an interview with the I-CC subject at the process stage.

RI_1-05: *How do you solve this SLE in three variables based on your experience solving a SLE in two variables?*

IS_1-05: *In a SLE in two variables, I eliminate both equations and then get the value of one of the variables and then substitute it into one of the equations so that the value of the two variables is obtained. In the SLE in three variables, I do the same thing, I eliminate two equations first and then two other equations, first that 1st equationis reduced by 2nd equation and the result is that later there are only two variables, then 1st and 3rd equations are reduced and then there is another result in the form of a linear equation of two variables. Well, the result just now was reduced. After that, I can get the x value, then the x value is substituted into one of the eliminated equations and then I get the y value then find the z value by means of x and y values are substituted into the first equation so that the z value is obtained.*

Based on the results of the work of individual subject in table 3 and the results of interview, at the process stage, I-CC subject describes the steps of determining the solution set of SLE in three variables clearly based on analogy with the steps of determining the solution set of SLE in two variables (P; IS_1-08). The analogy can be seen from the pattern she describes the steps of determining the solution set of SLE in two and three variables, that is using elimination followed by substitution (mixed method).

c. Object Stage

The following is an excerpt of an interview with the I-CC subject at the object stage.

RI_1-06: *Is there a method of solving a SLE in three variables other than the one you used earlier?*

IS_1-06: *It can be used like the method of a SLE in two variables, there are elimination and substitution (combined), graph, and I forgot, but if it is a graph method, I don't understand.*

RI_1-07: *Can a SLE in three variables be solved by elimination only or substitution only from beginning to end?*

IS_1-07: *No, Ma'am, because if I use elimination, it will still end up finding the value of the third variable using susbtitution. If I use substitution only, it cannot be because all variables do not have one known value and must be found first using the elimination method.*

In the object stage, I-CC subject cannot explain other methods of solving SLE in three variables using analogy with methods of solving SLE in two variables (IS_1-06; IS_1-07). She only mentioned one other method, that the graphing method, but she could not elaborate further on the graphing method. She made a little mistake because she thought the substitution method could not be used to solve a SLE in three variables because the condition for using the substitution method was that the value of one of the variables must be known (IS_1-07).

d. Schema Stage

The following is an excerpt of an interview with the I-CC subject at the schema stage.

RI_1-08: *What is the most effective method of solving SLE in three variables?*

IS_1-08: *The most effective is elimination and substitution (combined) because SLE in two variables are also the easiest to use the combined method.* ***(S_2)***

RI_1-09: *If suppose there is any SLE in three variables, how do you solve the problem?*

IS_1-09: *Eliminated then substituted (combined), in the SLE in two variables I also always use elimination and substitution.* ***(S_1)***

RI_1-10: *Can the SLE in two variables be solved by combined method by substitution method first and then elimination?*

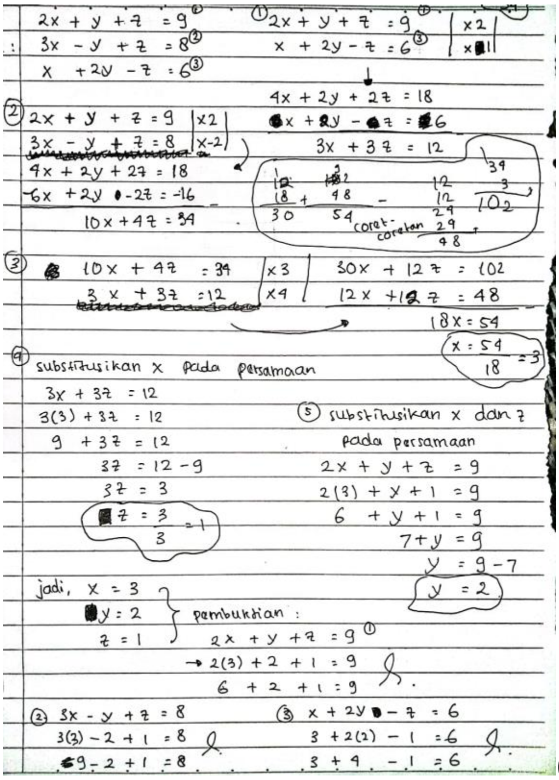
IS_1-10: *It can't because one of the variables must be known. If in the problem of a SLE in two variables there are only 2 equations, then we are told to find the x and y values, then use elimination first and then substitution. If we know the x or y value, we can use the substitution first. In the problem of a SLE in three variables, if given 3 equations and there is information that the value of x is equal to 1 for example, then we can use substitution first and then elimination. If the problem is only given a SLE in three variables, so we use elimination and then substitution.* ***(S_1)***

RI_1-11: *If the following system of equations is known:*

$$\begin{cases} 2x + y + z = 9 \\ 3x - y + z = 8 \\ x + 2y - z = 6 \end{cases}$$

Try to solve the SLE in three variables!

IS_1-11:



(S_1)

RI_1-12: *Do you think a SLE in two variables always has a solution?*

IS_1-12: *No.*

RI_1-13: *Can you give an example of a SLE in two variables that has no solution?*

IS_1-13:

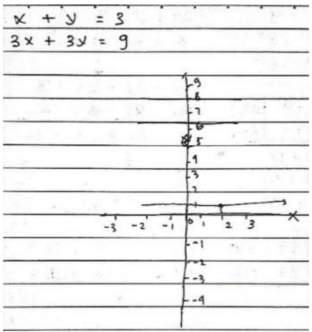
$$\begin{cases} x + y = 3 \\ 3x + 3y = 9 \end{cases}$$

RI_1-13: *Why doesn't the SLE in two variables have a solution?*

IS_1-14: *Because this is a multiple of 3.*

RI_1-15: *If the SLE in two variables is drawn in the cartesian plane, then what does it look like?*

IS_1-15: *The lines will be parallel.*



RI_1-16: *Do you think there are SLE in two variables that have no solution, how about SLE in three variables? Is there a SLE in three variables that has no solution?*

IS_1-16: *Just like the SLE in two variables, there is a SLE in three variables that has no solution because there must be a form of SLE in three variables whose equation if eliminated, then the coefficients of the variables x, y, and z will run out or 0 but the constant is still left. So, later it cannot be eliminated anymore because the variable has been exhausted so the result will be 0 = ...* ***(S_3)***

In the schema stage, I-CC subject cannot generalize some methods of solving SLE in three variables after analogizing with methods of solving SLE in two variables (S_1;

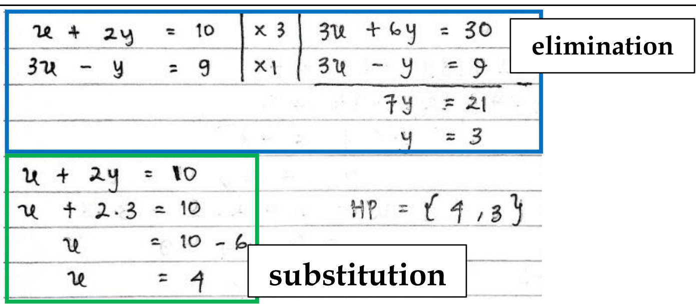
IS_1-09). To solve a SLE in three variables, she always uses the combined method (elimination followed by substitution) just as she solved the problem of a SLE in two variables (IS_1-09; IS_1-11). In addition, she also concluded the most effective method of solving SLE in three variables, that is the combined method (elimination followed by substitution) (S_2; IS_1-08). According to the I-CC subject, this combined method cannot be done by substitution followed by elimination because to make a substitution in the first step must know the value of one of the variables first (IS_1-10). She also explained that there is SLE in three variable that has no solution (S_3; IS_1-16) although there was a slight error in her prior knowledge, that is, she was wrong when giving example of SLE in two variables that had no solution (S_3; IS_1-13 - IS_1-15). When asked to give an example of a SLE in two variables that has no solution, she instead gives an example of a SLE in two variables that has infinite solutions (IS_1-15).

2. **Results and Analysis of the Process of SLE in Three Variables Solving Procedure's Construction of Paired Students Using Analogy (Paired Subjects Correctly Solve Problems Involving SLE in Two and Three Variables / P-CC Subjek)**

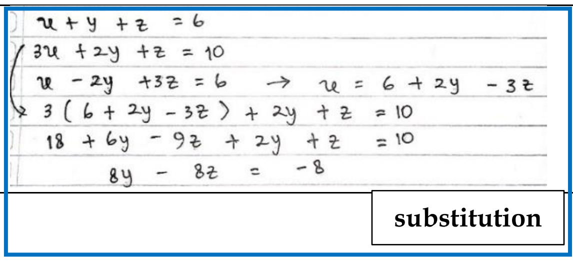
   a. **Action Stage**

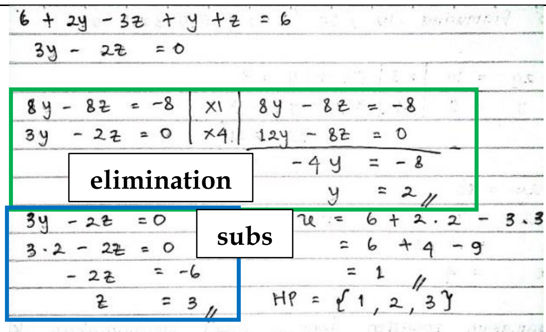
The answer of the P-CC subject in the procedure construction test at the action stage are presented in table 4.

Table 4. Answer of P-CC Subject in the Action Stage

Figure	Code
 (Answer to the source problem / problem of SLE in two variables)	A_2

Advanced Table 4

Figure	Code
	A_2



(Answer to the target problem/problem of SLE in three variables)

The following is an excerpt of an interview with the P-CC subject in the action stage.

RP_1-01: *Based on your experience when learning SLE in two variables, what is known in this SLE in three variables?*

PS_1-01: *If in SLE in two variables look for the value of two variables, if in a SLE in three variables look for the value of three variables.* ***(A_1)***

RP_1-02: *What else is known?*

PS_1-02: *If in a SLE in two variables, there are 2 equations, in a SLE in three variables, there are 3 equations.* ***(A_1)***

RP_1-03: *What is asked in SLE in three variables?*

PS_1-03: *Usually in a SLE in two variables look for the values of x and y and then in a SLE in three variables look for the values of the variables x, y, z.* ***(A_1)***

RP_1-04: *How do you convince yourself that the solution set of SLE in three variables you obtained is true?*

PS_1-04: *How to check it is the same as checking the solution set of SLE in two variables, in a SLE in three variables, the solution set is obtained 1, 2, 3, with x = 1, y = 2, z = 3, then checked by entering into 1st, 2nd, and 3rd equations.* ***(A_3)***

Based on the work of P-CC subject in table 4 and the results of interview, P-CC subject determines what is known and asked about SLE in three variables based on analogy with what is known and asked about SLE in two variables (A_1; PS_1-01; PS_1-02; PS_1-03). They also correctly determine the solution set of SLE in three variables (A_2). The analogy is seen in the way they solve problems of SLE in two and three variables, that is using the combined method (elimination and substitution). In addition, they also check the correctness of the solution set of SLE in three variables that have been obtained correctly (A_3; PS_1-04).

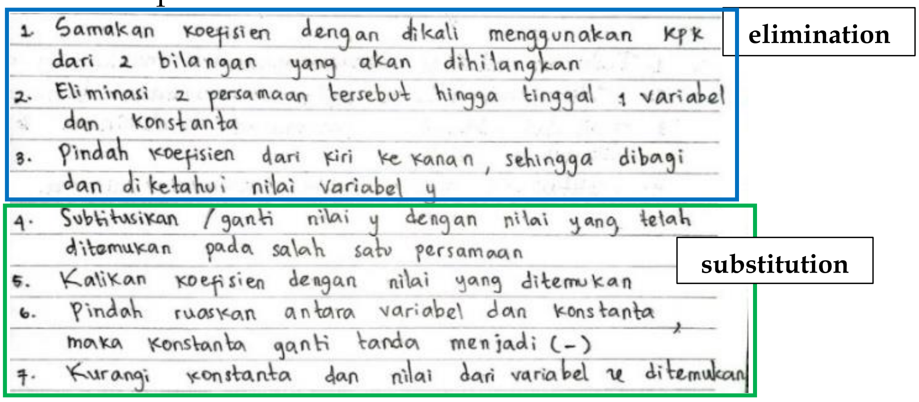
b. Process Stage

Answer to P-CC subject in the procedure construction test at the process stage are presented in table 5.

Table 5. Answer of P-CC Subject at the Process Stage

Figure	Code

Steps to Solve a SLE in Two Variables

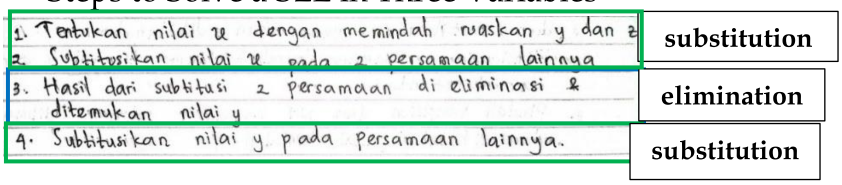


P

Translation:

1. *Match the coefficient by multiplied using the smallest common multiple of the two numbers to be omitted;*
2. *eliminate the two equations until there is only one variable and constant;*
3. *move the coefficient from left to right so that it is divided and known the value of the variable y;*
4. *substitute the value of y with the value found in one of the equations;*
5. *multiply the coefficient by the found value;*
6. *move the field between the variable and the constant, then the constant replaces the negative sign;*
7. *subtract the constant and the value of the variable x is found.*

Steps to Solve a SLE in Three Variables



P

Translation:

1. *Determine the value of x by moving the y and z fields;*
2. *substitute the value of x in the other two equations;*
3. *the result of the substitution of two equations is eliminated and the value of y is found;*
4. *substitute the value of y in another equation.*

The following is an excerpt of an interview with P-CC subject at the process stage.

RP_1-05: *How do you solve a SLE in three variables based on your experience learning a SLE in two variables?*

PS_1-05: *Yesterday we substituted the value of x first, so on the left we moved to the right, the left is only x. So, 3rd equation we change gets x = 6+2y-3z. Then, this x value is substituted into two other equations, 1st and 2nd equations, so we get 4th and 5th equations. After obtaining the results and then eliminated so that the value of y is obtained, then the value of y can be entered into 4th or 5th equation so that the value of z is obtained. Then the value of z can be substituted into the initial equation x to find the value of x.*

RP_1-06: *In a SLE in two variables, you solve them by eliminating them first and then substituting. In a SLE in three variables, you solve them by substitution and then elimination. How can you think of using the substitution method first and then elimination to solve a SLE in three variables?*

PS_1-06: *Because since in the SLE in two variables, we always solve by the combined method with elimination followed by substitution, while when we work on a SLE in three variables, initially we have tried with elimination followed by substitution, but we cannot, confused in the middle, finally I thought of substituting first then elimination because in the SLE in two variables can use substitution, changed to x = ... or y = ... then substituted to another equation then obtained the value of x or y then substituted again to x = ... or y = ... So, we can get x and y values. Then, as we discussed, we agreed to try that substitution in a SLE in three variables, when we substituted x = ... to both equations obtained a SLE in two variables, finally we used elimination and substitution to solve a SLE in two variables, it turned out to be successfully obtained ...*

Based on the results of the work of the P-CC subject in table 5 and the results of the interview, at the process stage, the P-CC subject describes the steps of determining the the solution set of SLE in three variables clearly based on analogy with the steps of determining the the solution set of SLE in two variables (P; PS_1-05). The analogy is seen when they use substitution method as in SLE in two variables to solve SLE in three variables. First, it converts one of the equations in a SLE in three variables to x = ... Then, they substitute x = ... to the other two equations so that a SLE in two variables is obtained. Next, they solve the SLE in two variables using the combined method (elimination followed by substitution) so that the values of the three variables are obtained.

c. Object Stage

The following is an excerpt of an interview with P-CC subject at the object stage.

RP_1-07: *According to your experience learning SLE in two variables, is there any method of solving SLE in three variables other than the one you used yesterday?*

PS_1-07: *There is elimination first and then substitution (combined method) as in a SLE in two variables.*

RP_1-08: *Are there other methods of solving SLE in three variables besides the combined method?*

PS_1-08: *We can use the matrix method as far as I know when learning the SLE in two variables, there is a matrix method, but I don't know how if it is applied.*

PS_1-09: *It can be just elimination or substitution from beginning to end but it is complicated, and it will be longer later.*

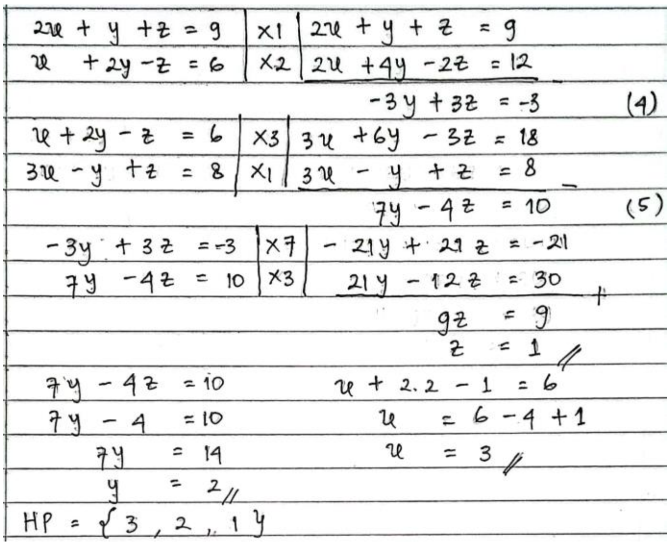
RP_1-09: *Besides the matrix, what else is there?*

PS_1-10: *We can also use the graph method because the SLE in two variables can also be solved using the graph method, but we don't know how.*

In the object stage, the P-CC subject describes other methods of solving SLE in three variables using analogy with methods of solving SLE in two variables, that is the methods of elimination, substitution, combined method (with elimination followed by substitution), matrix, and graph (PS_1-07; PS_1-08; PS_1-09; PS_1-10), but they could not explain further regarding the graphing and matrix methods (PS_1-08; PS_1-10).

d. Schema Stage

The following is an excerpt of an interview with the P-CC subject at the schema stage.

RP_1-10:	*In the future, will you use substitution first to elimination or elimination first to substitution to solve the problem of a SLE in three variables?*
PS_1-11:	*Elimination first then substitution as when we solve a SLE in two variables.* ***(S_1)***
RP_1-11:	*If I had any SLE in three variables, how would you solve it?*
PS_1-12:	*We will use the method of elimination first followed by substitution as when we solve a SLE in two variables. Maybe the two equations were eliminated first, finally formed 4^{th} equation, then look for two more equations and then form the 5^{th} equation. Then, 4^{th} equation and 5^{th} equation is eliminated later obtained the value of one of the variables, if for example we will find the value of x first, then the value of x is obtained, then the value of x is substituted to 4^{th} or 5^{th} equation, later the value of the second variable is obtained for example y, then the value of x and y is substituted again to 1^{st}, 2^{nd}, or 3^{th} equation to get third variable.* ***(S_1)***
PS_1-13:	*It can also be used like yesterday when we both worked on using substitution followed by elimination.* ***(S_1)***
PS_1-14:	*We can also use elimination or substitution, but the process will be more complicated later.* ***(S_1)***
RP_1-12:	*If the following system of equations is known:* $\begin{cases} 2x + y + z = 9 \\ 3x - y + z = 8 \\ x + 2y - z = 6 \end{cases}$ *Try to solve the SLE in three variables!*
PS_1-15:	 ***(S_1)***
RP_1-13:	*Is the way you solved the SLE in three variables problem just now similar to the steps you used in yesterday's test?*
PS_1-16:	*This is similar to using the method of elimination and substitution, but the difference yesterday in the SLE in three variables was substitution first and then elimination, which was just eliminated first and then substitution was exactly the same as when solving a SLE in two variables.*
RP_1-14:	*Why have you now decided to do elimination first and then substitution?*
PS_1-17:	*Because yesterday when we did it using substitution first and then elimination, it was more complicated, then we know that the mixed method in a SLE in two variables can be eliminated first and then substitution so we try to use this now, it turns out that the method is simpler.*
RP_1-15:	*What is the most effective method of solving SLE in three variables?*

PS_1-18:	*Elimination first then substitution like the combined method in a SLE in two variables.* ***(S_2)***
RP_1-16:	*Do you think a SLE in two variables always has a solution?*
PS_1-19:	*Some are not. If the question is wrong.*
RP_1-17:	*No, what if the problem is true?*
PS_1-20:	*It means we are doing it wrong. The time to calculate it is not thorough.*
RP_1-18:	*Is there a SLE in two variables that has no solution?*
PS_1-21:	*(They are discussing) Maybe there is.*
RP_1-19:	*What does an example look like?*
PS_1-22:	*For example, the value of the variable will be obtained fractionally. We think so, Ma'am, but we don't know anymore.*
RP_1-20:	*Then, does a SLE in three variables always have a solution?*
PS_1-23:	*No, but my friend initially did.*
PS_1-24:	*If I initially did, Ma'am, there is always a solution because all this time I have always worked on a SLE in two variables and always found the answer. So, the possibility of a SLE in three variables also always has a solution.* ***(S_3)***
RP_1-21:	*What is your reason for deciding that there is a SLE in three variables that has no solution?*
PS_1-25:	*The value found, the value of the variable is not appropriate when entered into the equation as if in a SLE in two variables, when the values of x and y are entered into the equation in the problem does not match the result. For example, $2x + y + z = 9$ then when searched, it turns out that for example $x = 2$, $y = 1$, $z = 5$, then when added $2(2) + 1 + 5$ is not equal to 9.* ***(S_3)***

At the schema stage, the P-CC subject generalizes several methods of solving SLE in three variables after analogizing with the method of solving SLE in two variables, that is to solve a SLE in three variables can use a combined method with elimination and then substitution, a combined method with substitution and then elimination, elimination only, or substitution only (S_1; PS_1-11; PS_1-12; PS_1-13; PS_1-14; PS_1-15). They also concluded the most effective method of solving SLE in three variables, that is the combined method with elimination and then substitution (S_2; PS_1-18). Moreover, they cannot explain that SLE in three variables exist that have no solution (S_3; PS_1-24; PS_1-25), because according to them, a SLE in three variables is called having no solution if the values of the three variables that have been obtained are then substituted into one of the equations in the SLE in three variables, the results in the left segment and the right segment are different (PS_1-25).

Based on the data analysis that has been done, a summary of the process of SLE in three variables solving procedure's construction using analogy between individual and paired students are presented in table 6.

Table 6. Comparative Recapitulation of the Process of SLE in Three Variables Solving Procedure's Construction Between Individual Student and Paired Students

Stage	Similarities	Differences
Action	1. Equally determine what is known and asked about	Individual student determines the solution set of SLE in three variables using the combined

	the problem of a SLE in three variables based on analogy correctly; and 2. equally check the correctness of the solution set of SLE in three variables precisely.	method (elimination then substitution), while paired students using the combined method (substitution then elimination).
Process	Both describe the steps of determining the solution set of SLE in three variables clearly.	The steps outlined by individual student with the elimination method continued substitution, while paired students with the substitution method continued elimination.
Object	-	Individual student cannot explain other methods of solving SLE in three variables. She only mentioned one other method, the graphing method, but she could not elaborate further on the method. On the other hand, paired students explained four other methods of solving SLE in three variables using analogy, that is the method of elimination, susbitution, the graph method, and the matrix, but they could not explain further about the graph and matrix methods.
Schema	Both conclude the most effective method of solving a SLE in three variables, that is the combined method.	1. Individual student cannot generalize some methods of solving a SLE in three variables, whereas paired students generalize some methods of solving a SLE in three variables correctly; 2. when concluding the most effective method of solving a SLE in three variables, individual student explains that this combined method cannot be done by the method of substitution then elimination, while paired students explain that this combined method can be done by the method of substitution then elimination or

Advanced Table 6.

Stage	Similarities	Differences
		opposite; and 3. when explaining that there is a SLE in three variables that has no solution, individual student experiences little error in their prior knowledge, whereas paired students cannot explain that there is SLE in three variables that has no solution.

In the action stage, both individual student and paired students succeeded in determining the solution set of SLE in three variables using analogy by determining the solution set of SLE in two variables. Individual student uses the combined method with the method of elimination then substitution to solve a SLE in three variables, while paired students use the combined method with the method of substitution then elimination. This result is in accordance with Zahid (2016) opinion that when someone faces a problem, he/she will try to connect the problem with the knowledge he/she already has.

In the process stage, both individual student and paired students outline the steps of determining the solution set of SLE in three variables using analogy clearly. The steps of determining the solution set of SLE in three variables using analogy performed by individual student and paired students are presented in table 7.

Table 7. Comparison of Steps to Determine the Solution Set of SLE in Three Variables Using Analogy Performed by Individual Student and Paired Students

Individual Student

System of Linear Equations in Two Variables
1. Use 1st and 2nd equations to eliminate x so that she obtained the value of y.
2. Substitute the value of y that has been obtained to 1st equation so that the value of x is obtained.

System of Linear Equations in Three Variables
1. Use 1st and 3rd equations to eliminate x so that she obtained 4th equation.
2. Use 1st and 2nd equations to eliminate x so that she obtained 5th equation.
3. Use 4th and 5th equations to eliminate x so that she obtained the value of y.
4. Substitute the value of y into 4th equation so that she obtained the value of x.
5. Substitute the value of x and y to 1st equation so that she obtained the value of z.

Paired Students

System of Linear Equations in Two Variables
1. Use 1st and 2nd equations to eliminate x so that they obtained the value of y.
2. Substitute the value of y that has been obtained to 1st equation so that the value of x is obtained.

System of Linear Equations in Three Variables
1. Change 3rd equation in the form x = ...
2. Substitute the equation in step 1 to 1st and 2nd equations so that they obtained 4th and 5th equations in the form of a system of linear equations in two variables.
3. Use 4th and 5th equations to eliminate z so that they obtained the value of y.
4. Substitute the value of y into 5th equation so that they obtained the value of z.
5. Substitute the value of y and z to 3rd equation so that they obtained the value of x.

The use of analogy by individual student can be seen from the way she solves problems of SLE in two and three variables, that is using a combined method by method of elimination followed by substitution. On the other hand, the use of analogy by paired students can be seen from the way they solve problems of SLE in two and three variables, that is using the combined method. These results reinforce the finding of Zahid (2016) and Umbara (2017) that the knowledge already possessed by students greatly determines the success of the new knowledge construction process.

In the object stage, individual student only mentioned one other method of solving the SLE in three variables, that is the graph method, while the paired students explained four other methods in solving the SLE in three variables, that is the elimination, substitution, graph, and matrix methods, but they could not explain further related to the graph and matrix methods. This depends on the prior knowledge they have in the material of SLE in two variables. These results reinforce the finding of Zahid (2016) and Umbara (2017) that the knowledge already possessed by students greatly determines the success of the new knowledge construction process.

In the schema stage, individual student explains that to solve a SLE in three variables using only the combined method (elimination followed by substitution). She also concluded the most effective method of solving SLE in three variables, that is the combined method (elimination followed by substitution). According to individual student, this combined method cannot be done by substitution first and then elimination. According to her, the combined method with substitution followed by elimination can only be done if in a problem a SLE in three variables is known one of the variable values. Thus, individual student encounter true pseudo construction error. She was able to conclude the most effective method of solving SLE in three variables, but she was wrong in giving her explanation. This result reinforces the finding of Inganah et al. (2021) that when constructing knowledge, students can experience true pseudo construction. True pseudo construction is when students can give the right answer, but when traced it turns out that students make mistakes in justifying the answer (Inganah et al., 2021).

On the other hand, paired students generalize several methods of solving SLE in three variables using analogy. They concluded the most effective method of solving SLE in three variables, that is the combined method. Paired students explain that this combined method can be done by substitution then elimination or elimination then substitution. This result is in accordance with the finding of Ahdiyat & Sarjaya (2014) that by collaborating with peer, a person gains broader knowledge than he/she thinks alone.

In the schema stage, individual student also explains that there is a SLE in three variables that has no solution based on its analogy with the possibility of solving a SLE in two variables. She experienced a slight error in her early knowledge, she answered correctly that there is a SLE in two variables that has no solution. When she was asked to give an example of a SLE in two variables that had no solution, she instead gave an example of a SLE in two variables whose solution was infinite. However, when explaining there is a SLE in three variables that has no solution, she can give the right conclusion. Thus, this individual student experiences construction hole. This reinforces the finding of Ni'mah et al. (2018) that students can experience construction holes when constructing knowledge. Construction holes are when students can give the right answer, but there is an improper process of student knowledge construction (Ni'mah et al., 2018).

On the other hand, paired students cannot explain that there is a SLE in three variables that has no solution. This can be because students' prior knowledge related to various solutions to SLE in two variables is not complete. The result of this research is in accordance with the explanation of Husamah et al. (2015) that students' prior knowledge also determines success when they construct knowledge with their peers.

CONCLUSION AND SUGGESTIONS

Based on the results of data analysis and discussion, it can be concluded that individual student and paired students can go through the four stages of the procedure construction process well. Paired students can meet all indicators at the action, process, object, and two indicators at the schema stage, while individual student can meet all indicators at the action, process, and two indicators at the schema stage. The difference lies in the flexibility of the method. Individual student looks monotonous in the process of procedure's construction using analogy. She only took advantage of the prior knowledge she had. Paired students can explore and find different ways of solving SLE in three variables. This is because they collaborate with peer to solve given problems.

Based on the research that has been done, suggestions from researchers are as follows.

1. When the teacher wants to teach students about the procedure for solving a SLE in three variables, he/she can involve students to construct the procedure. In this case, the use of analogies can facilitate students in constructing procedure for solving SLE in three variables. When constructing the procedure for solving a SLE in three variables using analogy, students can work individually or in pairs, but based on the results of this research, paired students can find various ways to solve a SLE in three variables.
2. For other researchers, they can conduct further research with a quantitative approach related to the effectiveness of using analogies to construct procedure for solving SLE in three variables between individual student and paired students.

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