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The Process of Relations between Quadrilaterals' Construction Based on APOS Theory Assisted by GeoGebra Software

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Abstract: Students' low understanding of relations between quadrilaterals indicates a problem in the construction process of quadrilateral concepts. The construction process of students can be assisted by the application of APOS theory in geometry, besides that it can develop abstract geometry knowledge through the use of appropriate geometry representations. Some studies about quadrilaterals using APOS Theory don't involve technology in the construction process, which makes researchers interested in implementing GeoGebra software. This research is qualitative research that aims to reveal the students' construction process about relations between quadrilateral assisted by GeoGebra. The subjects of this study were students who already learn about quadrilaterals. To reveal the students' construction process regarding the relations between quadrilateral, the researcher made a knowledge test, construction test, and conducted interviews for students with poor, intermediate, and high knowledge improvement. The results of this study revealed that with the help of GeoGebra: (1) the subject is able to determine the characteristics of each quadrilateral in a more complex manner and specializes the quadrilaterals, (2) the subject is able to determine several quadrilateral relations but the subject has not been able to use the ability to classify quadrilaterals. There are obstacles faced by the subject such as lack of subject ability and limited time when processing GeoGebra.

INTRODUCTION

Geometry is a branch of mathematics that is taught with the aim that students can understand the properties and relations between geometry elements and can make them good problem solvers (Safrina et al., 2014). However, many students still need to gain a better understanding of the relations between quadrilateral. Syamsuddin (2019) revealed that students at the formal operation stage can only assemble 7 out of 15 quadrilateral relations and tends to use 4 attributes (position, size, shape of quadrilateral and rotational symmetry). Kusuma et al. (2021), Students who successfully know the properties and characteristics of quadrilaterals have yet to be able to mention the relation between quadrilaterals. Students' low understanding of the relations between quadrilaterals indicates a problem in the quadrilateral concept construction process.

Sinuraya (Sari et al., 2021) state that construction skills are needed by students in order to gain a new understanding of the concepts and theories learned. According to Damayanti et al. (2017), to get to the next understanding, students need to construct an understanding from the basics as the foundation. If the foundation has been well constructed, individuals will be able to construct further understanding.

In the process of understanding construction, there are stages that must be passed. APOS (Action-Process-Object-Schema) theory is a theory developed by Dubinsky. APOS theory is a model to describe how mathematical concepts can be learned. In other words, this theory is a framework used to explain how individuals mentally construct an understanding of mathematical concepts (Arnon et al., 2014). APOS theory provides a detailed account of the cognitive processes involved in mathematical learning. While another theory that is like Van Hiele theory explained that a person will go through five hierarchical levels of understanding in learning geometry (Sunardi, 2012). It is only concerned with describing the levels of geometric thought and the characteristics of each level. APOS Theory and Van Hiele's theory are both valuable frameworks for understanding the development of mathematical thinking, especially in the context of geometry.

According to Arnon et al. (2014), action is an object transformation performed by individuals that occur externally. When an Action is performed repeatedly, individuals will convert external information into internal constructions where the process occurs. The Object stage occurs when individuals can realize the unity of many actions and processes that have been carried out. Furthermore, when individuals can combine a concept's action, process, and object, a Schema is formed.

Geometry representations such as pictures, diagrams, and geometry models help students understand geometry concepts and understand geometry in a more abstract way (Ross, 2000). Thus, applying APOS theory in geometry materials can help students construct and develop more abstract geometry knowledge through appropriate geometry representations.

Anam et al. (2019) and Rusfiana et al. (2020) describe that most junior high school students can fulfill the indicators at the action, process, and object stages but have yet to succeed in fulfilling the indicators at the schema stage. Only high-ability students can meet the indicators at the schema stage (Nurrohmah et al., 2022). Students who build the concept of quadrilateral by involving characteristics at the process stage will find it easier to form relations between quadrilateral, which is the schema stage (Anam et al., 2019). If students cannot construct schemes well, then there could be a problem at the previous stage.

Several studies have revealed students' difficulties understanding the relations between quadrilateral. Forsythe (2015) stated that students' view of a geometric shape affects how they classify quadrilaterals. For example, students have difficulty understanding if a rhombus is a variation of a kite. Another example of research conducted by Forsythe (2018) found that students perceived parallelograms and rhombus as different geometric shapes. The statement still indicates a need for understanding the concept of relations between quadrilaterals in students.

Technology can be used to improve understanding of mathematical concepts. Burrill et al. (2002) covey that there are 43 studies on the use of graphic technology. They concluded that it is one of the essential factors in helping students better understand mathematical concepts. These results attracted researchers' attention to try technology's role in constructing relations between quadrilaterals.

Students' mental construction can be generated based on successfully manipulating objects using computer or paper-based activities. In this case, GeoGebra can be considered a powerful tool to connect students' computer and paper-based activities with mental operations better than other math software (Baye, 2021). Wassie and Zergaw (2019) reveal what make GeoGebra a powerful and preferred tool for student learning is its dynamic, cross-platform, user-friendly, diverse and open-access nature. In sum, GeoGebra is an accessible graphic technology capable of various mathematical representations and explorations that teachers and students can use.

Several studies on analyzing student understanding based on APOS theory, such as those conducted by Wahyuningsih et al. (2019) which describe students' ability to understand the concept of SPLTV (Three-Variable Linear Equation System), Nailopo et al. (2022) who analyzed students' concept understanding in chance material, to Sari et al. (2021) who analyzed the construction of Pythagoras material in ICARE model learning. There are still few studies that involve technology in the construction process, which may be one of the factors that make it difficult for students to form their schemes. So that researchers are encouraged to include *GeoGebra* in the construction process and conduct research "**The Process of Relations between Quadrilaterals' Construction Based on APOS Theory Assisted by GeoGebra Software**".

METHOD

The approach used in this research is qualitative research. Qualitative research means to analyze or describe social situations that occur in individuals and groups. It is also emphasized that this research involves essential efforts such as asking questions and procedures and collecting and analyzing participant data from general to specific topics (Creswell, 2009). The researcher selected the research subjects using a purposive sampling technique. A test of students' knowledge about the relations between quadrilaterals was conducted first to get subjects with knowledge less than or equal to 50%. The test intended to ensure that the students' construction process is clearly and fully described within the framework of this study.

The test technique was conducted twice. The first test was a knowledge test of the relations between quadrilaterals. It will reveal students' basic understanding of the relations between quadrilaterals, the indicators shown in Table 1. The second test was conducted to reveal students' process of relations between quadrilaterals' construction arranged based on each step of APOS theory assisted by GeoGebra, the indicators shown in Table 2. Furthermore, the researcher selected three subjects with the following three categories: (1) subjects with poor knowledge improvement, (2) subjects with moderate knowledge improvement, and (3) subjects with high knowledge improvement. The process of those three subjects' construction will be revealed here. Interviews were conducted to explore information and reveal the construction process of students cognitively. The type of interview chosen by the researchers is a semi-structured interview with guidelines based on the stages of APOS theory.

Table 1. Knowledge Test Indicators

Indicators	Question Form	Num	Score
1. Given a table, students can determine the relations between trapezoid to square, rectangle, parallelogram, rhombus, and kite.	Classification	1	5
2. Given a table, students can determine the relations between parallelogram to square, rectangle, rhombus, trapezoid, and kite.		2	5
3. Given a table, students can determine the relations between rectangle to square, parallelogram, rhombus and kite.		3	5
4. Given a table, students can determine the relationship between rhombus to trapezoid, parallelogram, rectangle, square and kite.		4	5
5. Given a table, students can determine the relations between square to trapezoid, parallelogram, rectangle, rhombus and kite.		5	5
6. Given a table, students can determine the relationship between kite to trapezoid, parallelogram, rectangle, rhombus and square.		6	5

Table 2. Construction Test Indicators

APOS Theory Stage	Indicators	Question Form	Num	Score
Action	Given 7 kinds of characteristics that quadrilateral may have, students classify them into the table provided.	Classification	1	-
Process	Given a table, students can determine the relations between trapezoid to square, rectangle, parallelogram, rhombus, and kite using GeoGebra.	Classification	2	5
	Given a table, students can determine the relations between parallelogram to square, rectangle, rhombus, trapezoid, and kite.			5
	Given a table, students can determine the relations between rectangle to square, parallelogram, rhombus and kite.			5
	Given a table, students can determine the relationship between rhombus to trapezoid, parallelogram, rectangle, square and kite.			5
	Given a table, students can determine the relations between square to trapezoid, parallelogram, rectangle, rhombus and kite.			5
	Given a table, students can determine the relationship between kite to trapezoid, parallelogram, rectangle, rhombus and square.			5
Object	Presented 4 quadrilaterals, students determine which are the trapezoid.	Multiple choices	3a	-
	Presented 4 quadrilaterals, students determine which are the parallelogram		3b	
	Presented 4 quadrilaterals, students determine which are the square		3c	
	Presented 4 quadrilaterals, students determine which are the rhombus		3d	
	Presented 4 quadrilaterals, students determine which one is square		3e	
	Presented 4 quadrilaterals, students determine which are the kite		3f	
Schema	Students diagram the relationship between quadrilaterals.	Connecting diagrams	4	-

The process of relations between quadrilaterals' relations that was shown in second test is analyzed based on APOS theory presented in the table below.

Table 3. Indicator of the Process of Relations between Quadrilaterals' Construction Assisted by GeoGebra Software Based on APOS Theory

APOS Theory Stage	Indicator	Code
Action	With external assistance, students identify each quadrilateral.	A
Process	Students determine the relations between quadrilaterals through external interaction (Assisted by GeoGebra).	P_1
	Students repeatedly reflect and interact with the environment (perform the action stage) to identify the relations between quadrilateral.	P_2
Object	Students see mathematical objects from different perspectives by comparing two or more quadrilateral.	O
Schema	Students combine action, process, and object by determining the relations structure between quadrilateral.	S

RESULT AND DISCUSSION

There are only five students who can determine the relations above 50%. In addition, only 20 out of 53 students with knowledge below 50% gave complete answers to the test, so these 20 students were chosen by the researcher to be treated with GeoGebra-assisted tests.

Table 4. Results of Both Test

No.	Name	Gender	Score	
			First Test	Second Test
1.	AZA	F	9	8
2.	FRL	M	7	11
3.	NAC	F	8	11
4.	RPP	F	8	11
5.	ANU	F	6	11
6.	ARP	F	8	12
7.	RN	M	11	12
8.	NAR	F	11	12
9.	FF	F	11	13
10.	SAO	F	14	14
11.	CRF	M	14	14
12.	AY	F	14	14
13.	NKN	F	13	14
14.	NCA	F	11	15
15.	ARS	F	12	16
16.	BR	F	8	16
17.	SMI	F	14	17
18.	NTS	F	11	18
19.	GDP	M	15	20
20.	KAJ	F	8	21

From these results, the researcher selected three subjects with the following three categories: (1) subjects with poor knowledge improvement, (2) subjects with moderate knowledge improvement, and (3) subjects with high knowledge improvement.

Results and Data Analysis of Subject R1 (Subject with Poor Knowledge Improvement)

Knowledge Test on Relations between Quadrilateral

Based on the results of the knowledge test of relations between quadrilaterals, subject R1 determined 11 out of 30 relations between quadrilaterals. Three things are considered by subject R1 when determining the relations between quadrilaterals: a pair of parallel sides of equal length, a sloping line, and a combination of several shapes.

Action Stage

The answers to construction test of relations between quadrilateral at the action stage by subject R1 are shown in Table 5 below.

Table 5. Answer of R1 at Action Stage

Figure			Code
No.	Segiempat	Karakteristik	A
1.	Trapesium	c, g	
2.	Jajargenjang	b, d, [illegible]	
3.	Persegipanjang	b, d, [illegible], g	
4.	Belahketupat	a, b, [illegible], e, f	
5.	Persegi	a, b, g, e	
6.	Layang-layang	d, e, f	

The following is a transcript of the interview conducted with R1 at the action stage.

M_{01} : *Based on the characteristics of quadrilaterals that you already know, what do you think a trapezoid is?*
$R1_{01}$: *A trapezoid is a quadrilateral that has diagonals and one pair of parallel sides*
M_{02} : *In your work at the action stage, does a trapezoid always have right angles?*
$R1_{02}$: *No.*
M_{03} : *You think a rhombus has similar characteristics to a square. When a square has right angles, can a rhombus have right angles?*
$R1_{03}$: *A rhombus cannot have right angles.*
M_{04} : *Does a square have diagonals?*
$R1_{04}$: *Yes, it does.*

Subject R1 still did not understand the quadrilateral concept and could not properly determine the relations between quadrilateral. At the action stage, the subject was asked to determine the characteristics of each quadrilateral. Seen from the answer of subject R1, he could determine the characteristics of the quadrilateral even though there were some mistakes. So, this indicates that subject R1 is already at the action stage.

Process Stage

The answers to the construction test of relations between quadrilateral at the process stage by subject R1 are shown in Table 6 below.

Table 6. Answer of R1 at Process Stage

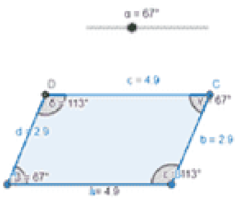
Figure	**Code**
	P_1 and P_2

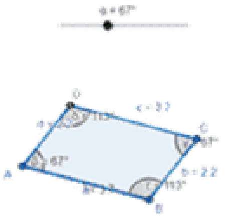
The following is a transcript of the interview conducted with R1 at the process stage

M_{05} : *Is it possible for you to identify the relations between quadrilaterals by answering the first question alone?*
$R1_{05}$: *I am confused, Ms. It's easier if there are shapes in GeoGebra that can be changed.*
M_{06} : *Please show me how you operate the parallelogram on GeoGebra.*
$R1_{06}$: *(Subject R1 showed how he operated the parallelogram on GeoGebra but did not use the slider provided, only shifting the sides)*
M_{07} : *Yesterday, did you adjust the angle of the parallelogram using the slider?*
$R1_{07}$: *No Ms.*
M_{08} : *What about the sides?*
$R1_{08}$: *I forgot, but yesterday I didn't slide the sides also.*
M_{09} : *Now, tell me how you operate the kite?*
$R1_{09}$: *Yesterday I found a rhombus and a parallelogram (showing the kite operation on GeoGebra)*

From the interviews conducted, subject R1 does not use GeoGebra optimally at the process stage ($R1_{06}$ dan $R1_{07}$). In the process, the R1 subject does not use the slider or pull the sides to make them the same length. The GeoGebra manipulation done by R1 on the parallelogram only rotates the building.

Table 7. GeoGebra Manipulation by R1

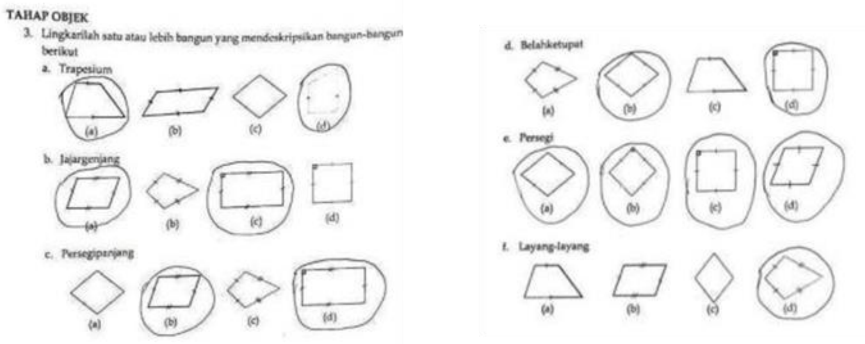
GeoGebra Manipulations	**Notes**
	The parallelogram in GeoGebra

GeoGebra Manipulations	Notes
	The subject didn't use slider and shift the side ($R1_{07}$ and $R1_{08}$)

Object Stage

The answers to the construction test of relations between quadrilateral at the object stage by subject R1 are shown in Table 8 below.

Table 8. Answer of R1 at Object Stage

Figure	Code
	O

The following is a transcript of the interview conducted with R1 at the object stage.

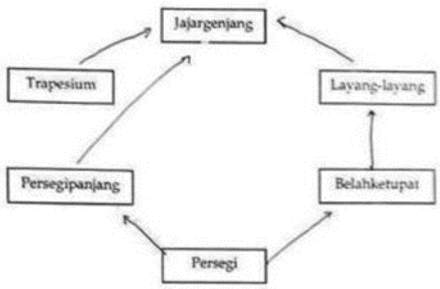
M_{10} : *When identifying trapezoidal, what are the initial characteristics that you observe?*
$R1_{10}$: *The one that has diagonals and the opposite sides have different lengths.*
M_{11} : *What about a parallelogram?*
$R1_{11}$: *It has two pairs of sides that are equal in length.*
M_{12} : *What about square and kite? Do they have two pairs of equal lengths?*
$R1_{12}$: *Because the square has all the same length sides, while the kite only the adjacent*
M_{13} : *You determined that a rectangle is also a parallelogram. Can you do it if I ask you to find the area of a rectangle using the area formula of a parallelogram?*
$R1_{13}$: *I cannot*

If subject R1 has yet to go beyond the process stage well, subject R1 will be challenging to step into the next stage, which is the object stage (O and $R1_{11}$). It can also be seen that subject R1 could not show the relations structure between quadrilateral at the scheme stage (S).

Schema Stage

The answers to the construction test of relations between quadrilateral at the process stage by subject R1 are shown in Table 9 below.

Table 9. Answer of R1 at Schema Stage

Figure	Code
	S

The following is a transcript of the interview conducted with R1 at the schema stage.

M_{14} : *Can you explain how you linked the six quadrilaterals during the scheme stage?*
$R1_{14}$: *At first, I didn't understand what does it means to link six of them but*
(1) *Square is related to the rectangle and rhombus. A square and a rhombus all have the same side length.*
(2) A rectangle can be formed from two squares.
(3) A rhombus and a kite both have diagonals.
(4) Trapezoids and parallelogram both also have diagonals
(5) Parallelogram and kite have sloping sides
(6) A kite can be formed, shifted into a rhombus

As we can interpret from the interview answers ($R1_{14}$), subject R1 has not been able to explain the relations between quadrilateral. From the results of the researcher's analysis of the subject's knowledge at the process stage and the subject's scheme regarding the relations between quadrilateral, the subject tends to generalize a quadrilateral shape by changing one of its properties.

Results and Data Analysis of Subject R2 (Subject with Moderate Knowledge Improvement)

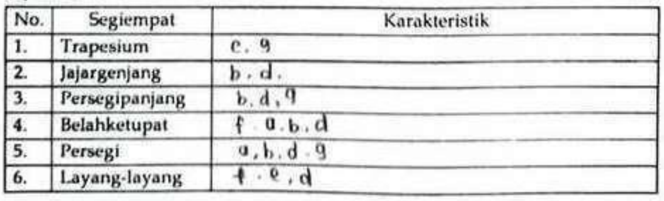
Knowledge Test on Relations between Quadrilateral

Based on the results of the knowledge test of the relations between quadrilaterals, subject R2 determined 11 out of 30 relations between quadrilaterals. There are three things that subject R2 pays attention to when determining the relations between quadrilaterals; a pair of same-length parallel sides, slanted lines, and diagonals.

Action Stage

The answers to construction test of relations between quadrilateral at the action stage by subject R2 are shown in Table 10 below.

Table 10. Answer of R2 at Action Stage

Figure	Code
	A

No.	Segiempat	Karakteristik
1.	Trapesium	e, g
2.	Jajargenjang	b, d.
3.	Persegipanjang	b, d, g
4.	Belahketupat	f, a, b, d
5.	Persegi	a, b, d, g
6.	Layang-layang	f, e, d

The following is a transcript of the interview conducted with R2 at the action stage.

M_{01} : *Based on the characteristics of quadrilaterals that you already know, what do you think a trapezoid is?*
$R2_{01}$: *A quadrilateral has two parallel sides with different lengths, and the other two sides have the same length.*

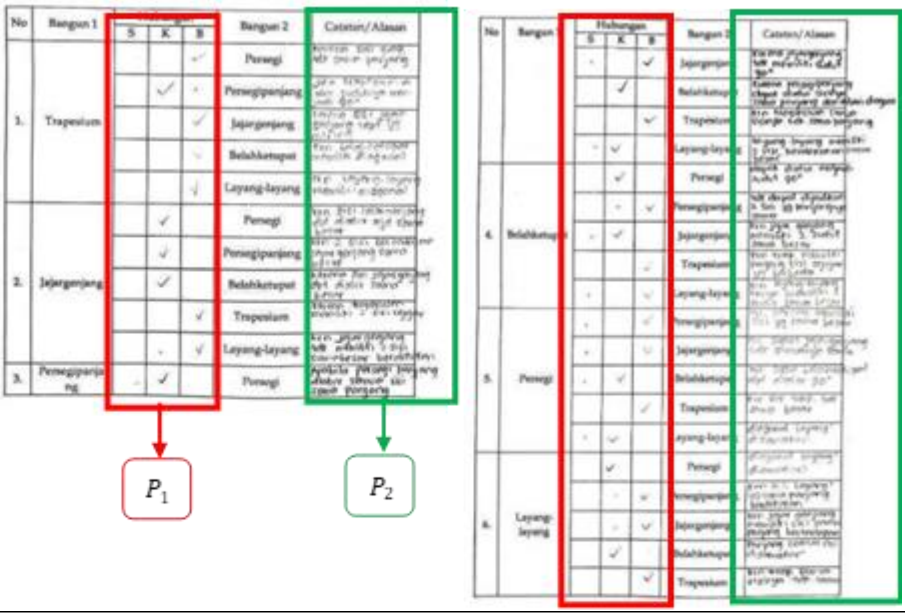
M_{02} : *What if the non-parallel sides are not the same length? Is it still a trapezoid?*
$R2_{02}$: *If it's not the same, it can be a right-angled trapezoid.*
M_{03} : *Based on your response, it seems that a rhombus and a square have many similarities. However, can you please clarify what sets them apart from each other?*
$R2_{03}$: *If the rhombus is right-angled, it can be a square.*
M_{04} : *Then, what do you think is a kite?*
$R2_{04}$: *A kite is a quadrilateral whose adjacent sides are equal in length, then it has perpendicular diagonals, each of which has a different length.*

In the action stage, students are able to define quadrilateral well. This is shown in the interview answers from $R2_{01}$to $R2_{04}$ although there are some mistakes in expressing their ideas.

Process Stage

The answers to the construction test of relations between quadrilateral at the process stage by subject R2 are shown in Table 11 below.

Table 11. Answer of R2 at Process Stage

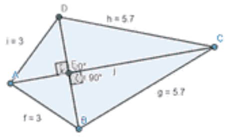
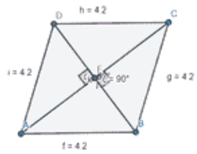
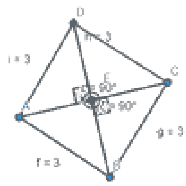
Figure	Code
	P_1 and P_2

The following is a transcript of the interview conducted with R2 at the process stage.

M_{05} : *Can you determine the relations between quadrilaterals by doing question number one, without GeoGebra?*
$R2_{05}$: *I don't think so...*
M_{06} : *Is it easier with GeoGebra?*
$R2_{06}$: *Yes…*
M_{07} : *Can you show me how you operate the kite in GeoGebra?*
$R2_{07}$: *I shifted some of the points to make a parallelogram and shifted the same side again to make a rhombus and a square at the same time.*
M_{08} : *What about the square?*
$R2_{08}$: *It can only be a rhombus. It can't be anything else. It can be a kite and a rhombus if the diagonals and angles are the same.*
M_{09} : *What do you mean the angles are the same?*
$R2_{09}$: *If the kite has the same angle, the right and left are the same, the top and bottom are the same, it can be a rhombus if they are all the same, it can be a square.*
M_{10} : *Then can a square be said to be a kite?*
$R2_{10}$: *I think so... just give it a diagonal.*

The subject interacted with GeoGebra well ($R2_{06}$ and $R2_{07}$) and the subject identified the relations between the quadrilateral that had been formed ($R2_{08}$, $R2_{09}$ and $R2_{10}$). When examining the relations between quadrilaterals, the diagram created does not reveal any clear patterns. Some relations are not only affected by GeoGebra but also by its definition. The following is a GeoGebra manipulation performed by subject R2, especially the kite.

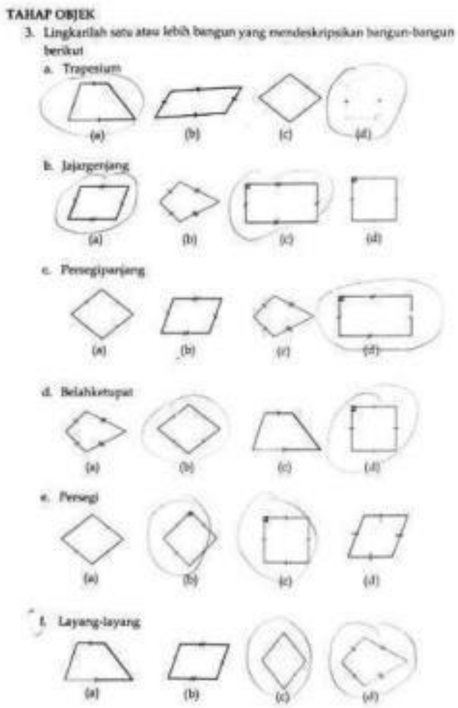
Table 12. GeoGebra Manipulation by R2

GeoGebra Manipulation	Notes
	The kite in GeoGebra
	"Some of the points (point A, B, and C) are shifted to be parallelogram" ($R2_{07}$)
	"The sides are shifted again to be rhombus and square." ($R2_{07}$)

Object Stage

The answers to the construction test of relations between quadrilateral at the object stage by subject R2 are shown in Table 13 below.

Table 13. Answer of R2 at Object Stage

Figure	Code
	O

The following is a transcript of the interview conducted with R2 at the object stage.

M_{11} : *Can you calculate the area of a rectangle using the formula for the area of a parallelogram if I provide you with its length and width?*

$R2_{11}$: *If* $\frac{(a+b)t}{2}$ *is rhombus, then the parallelogram formula is… is it also* $\frac{(a+b)t}{2}$ *?*

M_{12} : *Suppose the length of the rectangle is 5cm and the width is 3cm. How do you find the area of the rectangle using the area of a parallelogram formula?*

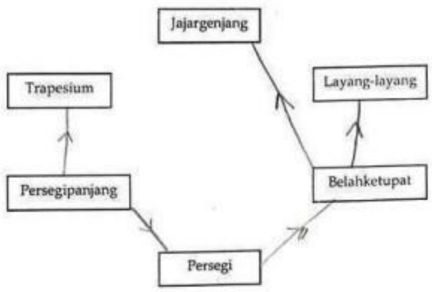
$R2_{12}$: $\frac{(a+b)t}{2} = \frac{(5+5)3}{2} = \frac{30}{2} = 15cm^2$. *It is same with* $5 \times 3 = 15cm^2$

Subject R2 is not yet entirely at the object stage, and it can be seen in the student's interview answer that can infer the relations between two quadrilaterals from another perspective ($R2_{12}$) although it is less precise at the time of the interview.

Schema Stage

The answers to the construction test of relations between quadrilateral at the process stage by subject R2 are shown in Table 14 below.

Table 14. Answer of R2 at Schema Stage

Figure	Code
	S

The following is a transcript of the interview conducted with R2 at the schema stage.

M_{13} : *How did you connect the six quadrilaterals in the schema stage?*

$R2_{13}$: *A square becomes a rhombus because the four sides are the same, and if you give it a diagonal, it can also become a kite.*
A rhombus can be a kite because if you give a rhombus a different diagonal and position it differently, it can be a kite.
A rhombus can become a parallelogram when you rotate the rhombus, the sides are parallel, just like a parallelogram.
Then why does a rectangle become a square if the width and length of the rectangle have the same length to make a square
But a square can't be a rectangle because the characteristics of a square are that all sides must be the same.
A rectangle becomes a trapezoid because ... a rectangle, if the sides are all the same, becomes a trapezoid.

M_{14} : *If I were to state that a square is a rectangle, would that statement be accurate?*

$R2_{14}$: *No*

M_{15} : *Is a rectangle a square?*

$R2_{15}$: *It can be*

M_{16} : *Why didn't you connect the parallelogram with the rectangle and trapezoid?*

$R2_{16}$: *Because the trapezoid has two parallel sides that are not the same length, while the parallel sides of the parallelogram are the same length. The difference between a parallelogram and a rectangle is the angle.*

It can be seen that subject R2 still has difficulty in classifying which quadrilateral is part of another quadrilateral.

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Results and Data Analysis of Subject R3 (Subject with High Knowledge Improvement)

Knowledge Test on Relations between Quadrilateral

Based on the results of the knowledge test of the relations between quadrilaterals, subject R3 determined 8 out of 30 relations between quadrilaterals. The thing that subject R3 pays attention to when determining the relations between quadrilateral is only the length of the side and the pair of parallel sides. Therefore, subject R3 still needs to gain more initial knowledge of the relations between quadrilateral.

Action Stage

The answers to the construction test of relations between quadrilateral at the action stage by subject R3 are shown in Table 15 below.

Table 15. Answer of R3 at Action Stage

Figure			Code
No.	Segiempat	Karakteristik	A
1.	Trapesium	c, b (kadang)	
2.	Jajargenjang	d, b	
3.	Persegipanjang	g, d, b	
4.	Belahketupat	b, a, f, e, d, g (kadang)	
5.	Persegi	a, d, g, e, f, b	
6.	Layang-layang	f, d, e	

The following is a transcript of the interview conducted with R3 at the schema stage.

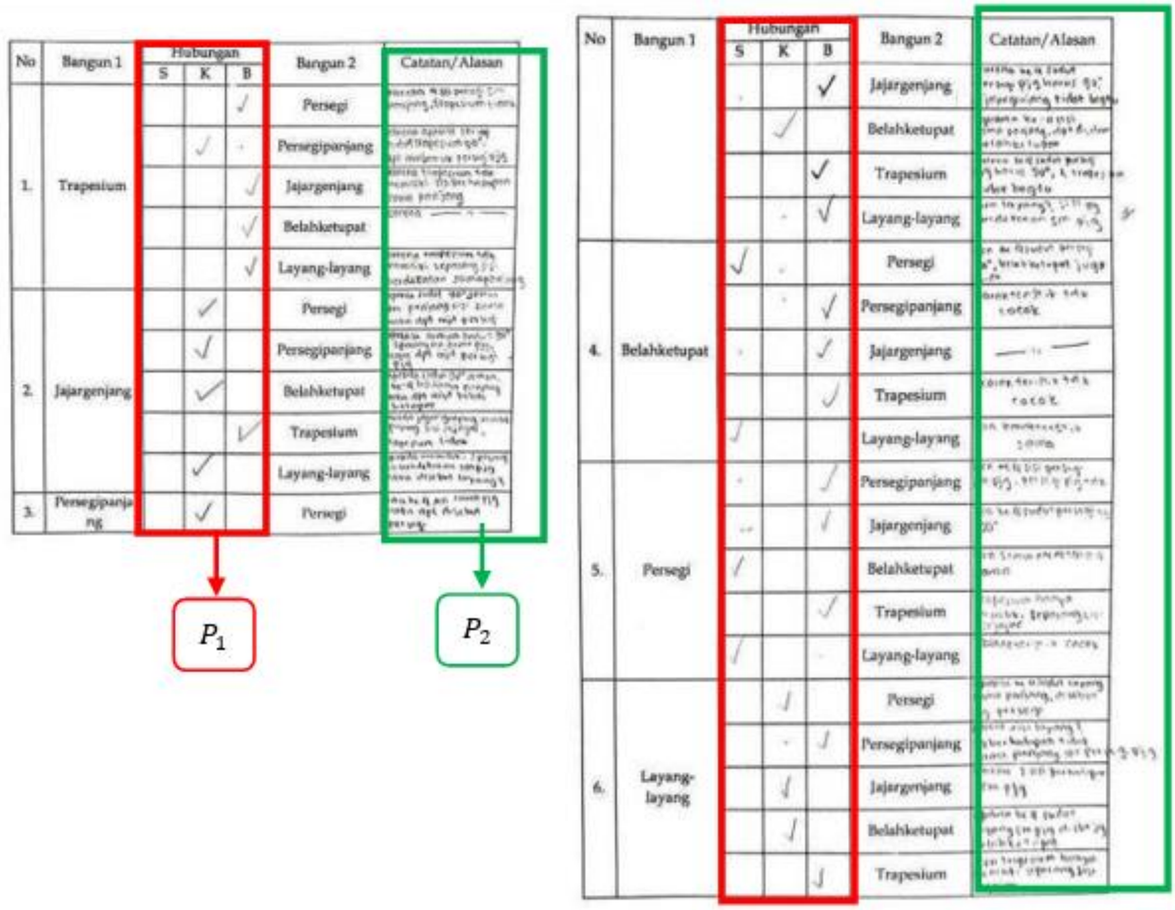
M_{01} : *Based on the characteristics of quadrilaterals you already know, what do you think a trapezoid is?*
$R3_{01}$: *It has one pair of parallel sides, and the other is not.*
M_{02} : *If a quadrilateral has two pairs of parallel sides, is it a trapezoid?*
$R3_{02}$: *No, there is only one parallel side of a trapezoid; the rest are slanted lines.*
M_{03} : *You think a rhombus has the same characteristics as a square? What is the difference?*
$R3_{03}$: *Sometimes, a rhombus has an angle of 90^o, the name is still a rhombus, but it can be called a square too.*

According to the results of the tests and interviews, subject R3 understood the concept of quadrilateral and experienced a significant improvement in understanding the relations between quadrilateral. Subject R3 was able to determine the characteristics of quadrilaterals at the action stage very well. (Code A supported by $R3_{01}$, $R3_{02}$, and $R3_{03}$)

Process Stage

The answers to the construction test of relations between quadrilateral at the process stage by subject R3 are shown in Table 16 below.

Table 16. Answer of R3 at Process Stage

Figure	Code
	P_1 and P_2

The following is a transcript of the interview conducted with R3 at the schema stage.

M_{04} : *Can you determine the relations between quadrilaterals just by doing question number one and without GeoGebra?*

$R3_{04}$: *It's easier because it is visualized, but I also can determine the relations only using their characteristics.* M_{05}: *What did you do with GeoGebra to help you determine the relations between quadrilaterals?*

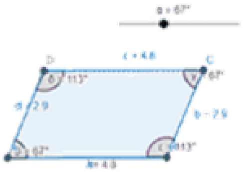
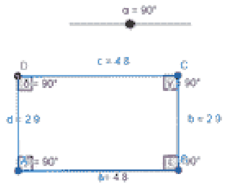
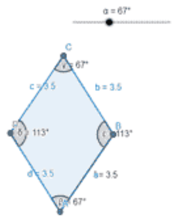
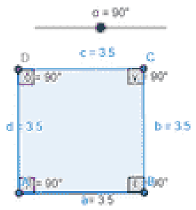
$R3_{05}$: *As I remember, yesterday I changed the angle, then the length can be changed, and I also rotated the shapes.*

M_{06} : *Explain more specifically the GeoGebra manipulations you did on the rhombus*

$R3_{06}$: *Yes, if all of the rhombus' angles are the same, it becomes a square.*

In accordance with the interviews that have been conducted, at the process stage, subject R3 can determine the relations between quadrilateral with the characteristics he knows ($M_{04}, R3_{04}$) because subject R3 was able to interact using GeoGebra well and could combine the interactions with the knowledge he had (Code P_2 supported $R3_{04}$, and $R3_{05}$). Subject R3 can determine 21 out of 30 relations between quadrilateral (P_1). The following are the GeoGebra manipulations performed by R3 on the parallelogram and rhombus.

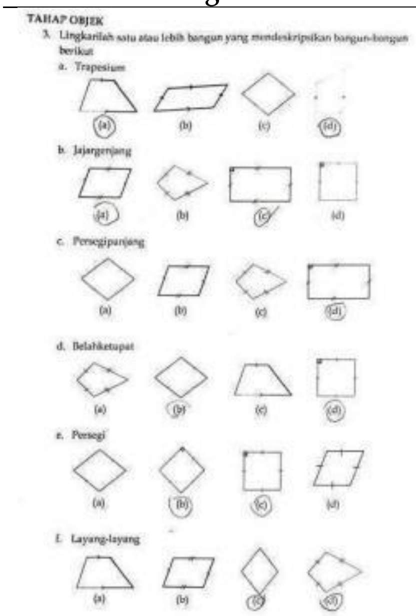
Table 17. GeoGebra Manipulation by R3

GeoGebra Manipulation	Notes
	The parallelogram in GeoGebra
	*Slider of the parallelogram is shifted to* 90^o to be rectangle
	Rhombus is a parallelogram that the four sides are equalized and the shape is rotated ($R3_{05}$). It can also called a kite.
	If a rhombus has its angles equalized (using sliders) and the shape is rotated, it will become a square. ($R3_{06}$)

Object Stage

The answers to the construction test of relations between quadrilateral at the stage by subject R3 are shown in Table 18 below.

Table 18. Answer of R3 at Object Stage

Figure	Code
	O

The following is a transcript of the interview conducted with R3 at the schema stage.

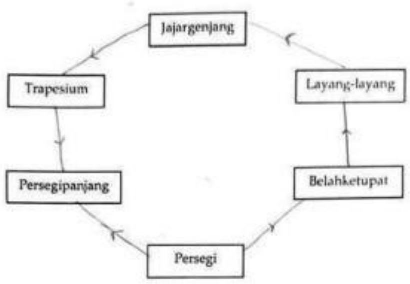
M_{07} : *As you compare the quadrilaterals at the object stage, what things did you notice?*
$R3_{07}$: *The characteristics from action stage and the help of GeoGebra in process stage, also the table helps me to compare these quadrilaterals.*
M_{08} : *You determined that a square is also a rhombus. Can you do it if I ask you to find the area of a square using the area of a rhombus?*
$R3_{08}$: *I think so*
M_{09} : *Let's say the square has a side length of 3cm. Find the area of the square using the rhombus area formula.*
$R3_{09}$: *As what I remember it is* $\frac{1}{2} \cdot d_1 \cdot d_2$*, if the length of the side is 3cm and the diagonal* $3\sqrt{2}$*. Then* $L = \frac{1}{2} \cdot d_1 \cdot d_2 = \frac{1}{2} \cdot 3\sqrt{2} \cdot 3\sqrt{2} = \frac{1}{2} \cdot 18 = 9$.

Although there were some mistakes, subject R3 was already at the object stage, where the subject was able to see the quadrilateral object from different perspectives by making comparisons between two or more quadrilaterals (Code O supported by $R3_{07}$ and $R3_{08}$).

Schema Stage

The answers to the construction test of relations between quadrilateral at the process stage by subject R3 are shown in Table 19 below.

Table 19. Answer of R3 at Schema Stage

Figure	Code
	S

The following is a transcript of the interview conducted with R3 at the schema stage.

M_{10} : *Okay. How did you connect the six quadrilaterals that you did in the schema stage?*
$R3_{10}$: *(1) "Both square and rhombus both have four sides of equal length, but the angles of rhombus does not have to be* 90^o*, and the angles of square has to be* 90^o*."*
(2) "Rhombus to the kite, the adjacent sides are the same length."
(3) "From a kite to a parallelogram, a parallelogram has two pairs of parallel sides, the same as a kite, so I connected them."
(4) "For parallelogram to trapezoid, I forgot why I connected it, but after thinking about it, this one is wrong."
(5) "I am confused why did I connect the rectangle to trapezoid hehehe"
(6) "Both square and rectangle has two pairs of parallel sides of the same length, the only difference is that the rectangle has different lengths"
M_{11} : *Why are you confused?*
$R3_{11}$: *Yesterday it was rushed, there wasn't enough time...*
M_{12} : *Then, how do you determine the direction of the arrow that you made in determining the quadrilateral relations?*
$R3_{12}$: *A square can be called a rectangle but a rectangle cannot be called a square. It's like that with everything.*

Furthermore, at the schema stage, it can be seen that subject R3 has an understanding of the differences and relations between quadrilaterals. In $R3_{11}$, subject R3 can understand that a quadrilateral fulfills the definition of another quadrilateral. In addition, subject R3 still felt confused when presenting the relations between quadrilaterals using the relations structure. R3 expressed feeling rushed because of time limitations.

Discussion

There were several obstacles, such as the subject's lack of skills and limited time when operating GeoGebra. This is in line with the findings of Baye et al. (2021). Two factors hindering subjects in developing their mathematical concepts in using GeoGebra are their limited time and the lack of skills to manipulate GeoGebra. Both factors occurred again in this study.

Of the six quadrilaterals used in this study, the parallelogram relations formed by the three subjects had significant changes. It can be seen that three of the six quadrilaterals (rectangle, rhombus, and square) are specializations of the parallelogram. In GeoGebra, if we change one of the angles of the parallelogram to 90^o, the parallelogram will become a rectangle. The rectangle is further specialized by pulling the sides and making them equal, and the rectangle will become a square.

Of the three subjects studied, during the interview, the researcher found two types of traits possessed by: (1) subjects who did not use GeoGebra well and (2) subjects who used GeoGebra well. Subjects who did not use GeoGebra well were only able to determine how to connect a parallelogram with a rectangle. Subjects with good GeoGebra manipulation skills could determine the relationship of the parallelogram with the other five shapes, so they had sufficient and high improvement in understanding.

Subjects who did not use GeoGebra well did not manipulate many shapes in GeoGebra. In this study, the subject only manipulated the kite into rhombus, parallelogram, and square. For the other five shapes, the subject only uses his knowledge at the action stage. For example, a trapezoid is defined as a quadrilateral with a diagonal, and a rectangle is a quadrilateral formed from two squares. Although the skill of manipulating GeoGebra is lacking, the subject has improved in understanding the definition. From the beginning of the knowledge test, the subject was only able to determine the characteristics of a quadrilateral based on the slanted lines, but after getting external treatment, and GeoGebra assistance, the subject was able to reflect and be more complex in determining the characteristics of quadrilaterals such as angles, diagonals, and side lengths.

On the other hand, the other two subjects used GeoGebra well. From the research results, GeoGebra was able to help specialize the characteristics of a quadrilateral into another quadrilateral. For example, the subject changes the angle to 90^o in a rhombus to form a square or changes the angle in a rectangle to form a square. However, students are still lacking in seeing a quadrilateral characteristic in another quadrilateral. Particularly in the square shape, the subject has not been able to realize that the square has the characteristics of the same side length, which is also owned by the square, parallelogram, and rhombus. The difference between the two subjects after manipulating quadrilaterals on GeoGebra is that subjects with high knowledge improvement can determine the relations between quadrilaterals by determining their analytic definitions.

There are two possible areas for improvement in this study: (1) the GeoGebra that has been created has unclear instructions, and (2) the subjects need better interpretation skills. One example of an error that can be found in the three subjects is when connecting a trapezoid with a rectangle. In GeoGebra, when the three corners of the trapezoid are changed to right angles, the three subjects assume that the trapezoid can turn into a rectangle even though, in that condition, the two sides of the trapezoid are removed. This means that the subject failed to interpret GeoGebra's intention.

The following is a summary of GeoGebra's role in changing knowledge of the relations between quadrilaterals by the three subjects.

Table 20. Comparison Table Between Three Subjects

Num	Relationship		Subject			Code
	Quadrilateral 1	Quadrilateral 2	R1	R2	R3	
1.	Trapezoid	Square	3	3	3	
		Rectangle	3	1	1	
		Parallelogram	1	3	3	
		Rhombus	2	2	3	
		Kite	2	2	3	
2.	Parallelogram	Square	1	3	3	
		Rectangle	2	3	3	
		Rhombus	1	3	3	
		Trapezoid	1	3	3	
		Kite	1	1	3	
3.	Rectangle	Square	3	3	3	
		Parallelogram	1	1	1	
		Rhombus	1	3	3	
		Trapezoid	2	3	3	**1**: Fail
		Kite	1	3	1	**2**: Thrived without the help of GeoGebra
4.	Rhombus	Square	3	3	1	**3** : Thrived with the help of GeoGebra
		Rectangle	1	1	1	
		Parallelogram	1	1	1	
		Trapezoid	3	3	2	
		Kite	1	1	3	
5.	Square	Rectangle	1	1	1	
		Parallelogram	1	1	1	
		Rhombus	1	1	3	
		Trapezoid	3	3	3	
		Kite	1	1	3	
6.	Kite	Square	1	3	3	
		Rectangle	1	1	1	
		Parallelogram	1	1	3	
		Rhombus	3	3	3	
		Trapezoid	2	3	3	

From the results of this study, in general, understanding the three subjects has increased the knowledge of quadrilateral relationships well with the help of GeoGebra. Two of the three grade 8 junior high school subjects, subjects with low and moderate improvement in understanding, were able to determine some quadrilateral relationships, but the subjects were not yet able to use the ability to classify quadrilateral. This differs from Syamsuddin (2019) research that students can solve problems by using classification, proportion or ratio, probability, and correlation skills.

CONCLUSION AND SUGGESTIONS

Conclusions are obtained based on the results of data analysis and discussion of the construction process of relationships between quadrilateral based on APOS theory assisted by GeoGebra Software. With the help of GeoGebra, students are able to determine the characteristics of each quadrilateral with more complexity and specialize the quadrilateral. That way, students can more easily determine the relationship between quadrilateral. In determining the relationship between quadrilaterals, students tend to use the intersection of the characteristics of the two quadrilaterals, "having four equal sides", and "having opposite sides equal in length". But students still cannot understand that the two characteristics have a relationship. It is because there are obstacles faced, such as the lack of subject skills and limited time when operating GeoGebra. Besides, the understanding of the three subjects increased the knowledge of quadrilateral relationships well with the help of GeoGebra. Two of the three 8th-grade junior high school subjects were able to determine some quadrilateral relationships, but the subjects were not yet able to use the ability to classify quadrilateral.

Based on the results of the research that has been conducted, several suggestions are expected to improve the quality of research on similar topics in the future. Other researchers are expected to implement research on APOS Theory assisted by GeoGebra Software on other materials. They can also continue this research by focusing on the time used in building students' quadrilateral concepts and providing more explicit GeoGebra operating instructions. Using GeoGebra Classroom may help to get a picture of student GeoGebra manipulation and more accurate data analysis. Lastly, teachers who implement GeoGebra in geometry learning, especially quadrilateral material, ensure that students already recognize what GeoGebra is and have clear operating guidelines. This is so that students easily combine their concepts with what is done on GeoGebra.

REFERENCES

Aisyah, P. N., Khasanah, S. umi N., Yuliani, A., & Rohaeti, E. E. (2018). Analisis Kemampuan Pemecahan Masalah Matematis Siswa SMP Pada Materi Segiempat dan Segitiga. JPMI (Jurnal Pembelajaran Matematika Inovatif), 1(5), 1025.

Alexander, D. C., & Koeberlin, G. M. (2011). Elementary Geometry for College Students (5th Edition). BROOKS/COLE CENGAGE Learning.

Anam, A. C., Juniati, D., & Wijayanti, P. (2019). Understanding the Quadrilateral Concept of Junior High School Students Based on APOS Theory in Terms of Differences in Cognitive Styles. Atlantis Press.

Arikunto, S. (2010). Prosedur Penelitian Suatu Pendekatan Praktik. Jakarta: Rineka Cipta.

Arnon, I., Cottrill, J., Dubinsky, E., Oktaç, A., Solange, Fuentes, R., Trigueros, M., & Weller, K. (2014). APOS Theory A Framework for Research and Curriculum Development in Mathematics Education. Springer Science+Business Media.

Baye, M. G., Ayele, M. A., & Wondimuneh, T. E. (2021). Implementing GeoGebra integrated with multi-teaching approaches guided by the APOS theory to enhance students' conceptual understanding of limits in Ethiopian Universities. Heliyon, 7(5), e07012.

Burrill, G., Director, Allison, J., Breaux, G., Kastberg, S., Leatham, K., & Sanchez, W. (2002). Handheld Graphing Technology in Secondary Mathematics.

Creswell, J. W. (2009). Research Design: Qualitative, Quantitative, and Mixed Methods Approaches (3rd Edition). SAGE Publications, Inc.

Dubinsky, E., & Mcdonald, M. A. (2001). APOS: A Constructivist Theory of Learning in Undergraduate Mathematics Education Research. In The Teaching and Learning of Mathematics at University Level (Vol. 7, pp. 275–282). Kluwer Academic Publishers. https://doi.org/10.1007/0-306-47231-7_25

Dubinsky, E., Weller, K., McDonald, M. A., & Brown, A. (2005). Some historical issues and paradoxes regarding the concept of infinity: An APOS-based analysis: Part 1. Educational Studies in Mathematics, 58(3), 335–359. https://doi.org/10.1007/s10649-005-2531-z

Dubinsky, Ed. (2000). Using a Theory of Learning in College Mathematics Courses.

Forsythe, S. K. (2015). Dragging maintaining symmetry: can it generate the concept of inclusivity as well as a family of shapes? Research in Mathematics Education, 17(3), 198–219.

Forsythe, S. K. (2018). Working with the Dynamic Perpendicular Quadrilateral in a Whole Class Setting: Supporting Students in Developing a Hierarchical Classification of the Kites. International Journal for Mathematics Teaching and Learning, 19(2), 269–286. http://www.cimt.org.uk/ijmtl/index.php/IJMTL/article/view/148

Irmaningsih, A., Putra, R. W. Y., & Netriwati. (2021). Mengupas Materi dan Soal Bangun Datar SMP. Arjasa Pratama. www.arjasapratama.com

Keogh, B., & Naylor, S. (1999). Concept cartoons, teaching and learning in science: an evaluation. International Journal of Science Education, 21(4), 431–446. https://doi.org/10.1080/095006999290642

Kusaeri, Lailiyah, S., Arrifadah, Y., & Hidayati, N. (2018). Proses Berpikir Siswa dalam menyelesaikan Masalah Matematika Berdasarkan Teori Pemprosesan Informasi. Suska Journal of Mathematics Education, 4(2), 125–141.

Kusuma, M. A., Susanto, Yuliati, N., Maharani, P., & Hasanah, N. (2021). Thinking process of 7th-class students in understanding quadrilateral concepts based on Van Hiele theory. Journal of Physics: Conference Series, 1839(1). https://doi.org/10.1088/1742-6596/1839/1/012012

Mulyono. (2011). Teori APOS dan Implementasinya dalam Pembelajaran. JMEE, 1(1).

Nailopo, E., Fitriani, F., & Simarmata, J. E. (2022). Analisis Pemahaman Konsep Siswa pada Materi Peluang Ditinjau dari Teori APOS pada Siswa SMP Kelas VIII. Jurnal Eduscience, 9(1), 168–181. https://doi.org/10.36987/jes.v9i1.2587

Nurrohmah, S., Anwar Hadi Firdos Santosa, C., Iskandar, K., & Sultan Ageng Tirtayasa, U. (2022). Konsep Matematis Siswa SMP dari Perspektif Teori APOS pada Materi Segi Empat. LAPLACE: Jurnal Pendidikan Matematika, 395–406.

Ormrod, & Ellis, J. (2008). Psikologi Pendidikan Jilid 1. Jakarta: Penerbit. Erlangga.

Pramesti, B. T., & Mampouw, H. L. (2020). Analisis Pemahaman Konsep Peluang Siswa SMP Ditinjau Dari Teori APOS. Jurnal Cendekia: Jurnal Pendidikan Matematika, 4(2), 1054–1063.

Pulgies, et al. (2005). Mathematics for year 5 second edition. Australia: Haese,Harris Publicatons.

Rusfiana, M., Matematika, P., & Singaperbangsa Karawang, U. (2020). Deskripsi Kemampuan Pemahaman Matematis Siswa SMP Berdasarkan Teori APOS. Jurnal Ilmu-Ilmu Pendidikan Dan Sains, 8.

Safrina, K., Ikhsan, M., & Ahmad, A. (2014). Peningkatan Kemampuan Pemecahan Masalah Geometri melalui Pembelajaran Kooperatif Berbasis Teori Van Hiele. Jurnal Didaktik Matematika, 1(1), 9–20.

Sari, D. P., Sastro, G., Pamulang, U., & Al-Khairiyah, Mt. (2021). Analisis Konstruksi Pengetahuan Berdasarkan Teori APOS Materi Teorema Pythagoras pada Pembelajaran Model ICARE. Jurnal Pendidikan Matematika Raflesia, 06(03).

Setyawan, D., & Rahman, A. (2013). Eksplorasi Proses Konstruksi Pengetahuan Matematika Berdasarkan Gaya Berpikir Construction Process Exploration of Mathematics Knowledge Based on Thought Patterns. Jurnal Sainsmat, 2(2), 140–152.

Sunardi. (2012). Mathematics Teaching and Learning Strategies. University of Jember.

Siyepu, S. W. (2015). Analysis of errors in derivatives of trigonometric functions. International Journal of STEM Education, 2(1). https://doi.org/10.1186/s40594-015-0029-5

Syahbana, A. (2016). Belajar Menguasai Geogebra. Noerfikri Offset.

Syamsuddin, A. (2019). Investigating Students' Geometric Thinking in Looking for the Linkages of Quadrilaterals (A Case Study on Students in The Formal Operation Stage). International Journal of Environmental and Science Education, 14(7), 435–444. http://www.ijese.com

Tanzimah. (2019). Pemanfaatan Geogebra dalam Pembelajaran.

Tosho, G. (2021). Matematika untuk Sekolah Dasar (Vol. 1). Pusat Kurikulum dan Perbukuan.

Usiskin, Z. & Griffin, J. 2008. The Classification of Quadrilaterals: A Study of Definition. e United States of America: Information Age Publishing, Inc.

Wagiyo, A., Surati, F., & Supradiarini, I. (2008). Pegangan Belajar Matematika. Pusat Perbukuan Departemen Pendidikan Nasional.

Wahyuningsih, H., Chairun Nissa, I., & Yuntawati. (2019). Analisis Kemampuan Siswa Dalam Memahami Konsep Sistem Persamaan Linier Tiga Variabel (SPLTV) Berdasarkan Teori APOS. Jurnal Matematika Dan Pendidikan Matematika, 7(1), 36–50.

Yerizon, Arnawa, I. M., Fitriani, N., & Tajudin, N. M. (2022). Constructing Calculus Concepts through Worksheet-Based Problem-Based Learning Assisted by GeoGebra Software. HighTech and Innovation Journal, 3(3), 282–296. https://doi.org/10.28991/HIJ-2022-03-03-04