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OPERASI DAN RELASI PADA HIMPUNAN FUZZY INTUISIONISTIK

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Abstrak

Himpunan fuzzy intuisionistik merupakan generalisasi dari himpunan fuzzy. Dalam himpunan fuzzy intuisionistik dapat didefinisikan beberapa operasi, seperti komplemen, irisan, gabungan, penjumlahan, dan perkalian. Selain mendefinisikan operasi, didefinisikan pula relasi pada himpunan fuzzy intuisionistik, seperti hasil kali kartesian. Misalkan S himpunan tak kosong dan F himpunan fuzzy intuisionistik di S, F disebut himpunan tautologis fuzzy intuisionistik di S, jika $\mu_F(s) \geq \nu_F(s)$ untuk setiap $s \in S$. Pada penelitian ini penulis akan mendeskripsikan sifat-sifat aljabar yang terdapat pada himpunan fuzzy intuisionistik dengan menggunakan beberapa operasi dan relasi sederhana.

Kata kunci: Himpunan Fuzzy Intuisionistik, Himpunan Tautologis Fuzzy Intuisionistik, Operasi, Relasi

Abstract

Intuitionistic fuzzy set is a generalization of fuzzy set. In the intuitionistic fuzzy set can be defined several operations, such as complement, intersection, union, addition, and multiplication. In addition to defining operations, relations are also defined on intuitionistic fuzzy set, such as cartesian product. Let S is a non-empty set and F is intuitionistic fuzzy set in S, F is called intuitionistic fuzzy tautological set in S, if $\mu_F(s) \geq \nu_F(s)$ for every $s \in S$. In this study, the writer will describe the characteristic of algebra which exist in the intuitionistic fuzzy set by using several simple operations and relations.

Keywords : Intuitionistic Fuzzy Set, Intuitionistic Fuzzy Tautological Set, Operation, Relation

1. PENDAHULUAN

Matematika merupakan disiplin ilmu yang mendasari berbagi disiplin ilmu lainnya. Matematika juga memegang peranan penting dalam perkembangan sains dan teknologi. Oleh karena itu, matematika seringkali digunakan untuk menyelesaikan permasalahan dalam kehidupan, baik permasalahan sederhana maupun permasalahan yang kompleks. Matematika sendiri terbagi menjadi beberapa cabang ilmu, salah satunya teori himpunan fuzzy.

Teori himpunan fuzzy pertama kali dikemukakan oleh Prof. Lotfi A. Zadeh dari California University USA pada tahun 1965. Konsep himpunan fuzzy merupakan perluasan dari konsep himpunan klasik. Pada himpunan klasik, fungsi keanggotaan suatu elemen dari himpunan A, bernilai 0 atau 1. Sedangkan, pada teori himpunan fuzzy, fungsi keanggotaan suatu elemen dari himpunan fuzzy $\tilde{F}$, nilainya berada dalam selang tertutup $[0,1]$. Teori himpunan fuzzy diusulkan untuk mengatasi ketidakpastian atau keragu-raguan yang seringkali ditemukan dalam kehidupan sehari-hari, seperti keragu-raguan dalam pengambilan keputusan, penalaran, pembelajaran, dan sebagainya (Zimmermann, 1996). Seiring berjalannya waktu, teori himpunan fuzzy mengalami banyak perkembangan, salah satunya teori himpunan fuzzy intuisionistik.

Teori himpunan fuzzy intuisionistik diperkenalkan oleh Krassimir T. Atanassov pada tahun 1983 (Atanassov, 2012). Himpunan fuzzy intuisionistik memperluas himpunan fuzzy dengan menambahkan satu komponen, yaitu derajat kenon-anggotaan. Pada himpunan fuzzy intuisionistik, fungsi keangggotaan dan kenon-anggotaan suatu elemen dari himpunan fuzzy intuisionistik F, nilainya berada dalam selang tertutup $[0,1]$. Adanya penambahan komponen baru pada himpunan fuzzy intuisionistik berfungsi untuk mengukur tingkat keragu-raguan dalam membuat keputusan (Atanassov, 1999). Berkaitan dengan teori himpunan fuzzy intuisionistik, tentunya tidak terlepas dari operasi dan relasi yang terdapat pada himpunan fuzzy intuisionistik. Pada penelitian ini penulis akan mendeskripsikan tentang operasi dan relasi pada himpunan fuzzy intuisionistik dan sifat-sifatnya. Operasi dan relasi yang dikaji pada penelitian ini adalah operasi dan relasi sederhana pada himpunan fuzzy intuisionistik.

2. KAJIAN TEORI

Berikut beberapa konsep dasar yang akan digunakan sebagai landasan pada pembahasan berikutnya.

Definisi 2.1

(Zimmermann, 1996) Jika S adalah kumpulan objek yang anggotanya dilambangkan secara umum oleh s, maka himpunan fuzzy $\tilde{F}$ di S didefinisikan sebagai

$$\tilde{F} = \{\langle s, \mu_{\tilde{F}}(s)\rangle | s \in S\}$$

dimana $\mu_{\tilde{F}}(s): S \to [0,1]$ menunjukkan derajat keanggotaan s di $\tilde{F}$.

Definisi 2.2

(Zimmermann, 1996) Misalkan S himpunan tak kosong. $\tilde{F}$ dan $\tilde{H}$ himpunan fuzzy di S. Didefinisikan operasi dan relasi sebagai berikut.

(1) $\tilde{F} \cap \tilde{H} = \{\langle s, \min\{\mu_{\tilde{F}}(s), \mu_{\tilde{H}}(s)\}\rangle | s \in S\}$

(2) $\tilde{F} \cup \tilde{H} = \{\langle s, \max\{\mu_{\tilde{F}}(s), \mu_{\tilde{H}}(s)\}\rangle | s \in S\}$

(3) $\tilde{F}^c = \{\langle s, 1 - \mu_{\tilde{F}}(s)\rangle | s \in S\}$

(4) $\tilde{F} + \tilde{H} = \{\langle s, \mu_{\tilde{F}}(s) + \mu_{\tilde{H}}(s) - \mu_{\tilde{F}}(s)\mu_{\tilde{H}}(s)\rangle | s \in S\}$

(5) $\tilde{F}.\tilde{H} = \{\langle s, \mu_{\tilde{F}}(s)\mu_{\tilde{H}}(s)\rangle | s \in S\}$

Definisi 2.3

(Zimmermann, 1996) Misalkan $\tilde{F}$ dan $\tilde{J}$ himpunan fuzzy di S dan Q. Hasil kali kartesian himpunan fuzzy $\tilde{A}$ dan $\tilde{B}$ didefinisikan sebagai

$$\tilde{F} \times \tilde{J} = \left\{\langle (s,q), \min\left(\mu_{\tilde{F}}(s), \mu_J(q)\right)\rangle \,|\, (s,q) \in S \times Q\right\}$$

Himpunan fuzzy intuisionistik merupakan perluasan dari himpunan fuzzy, karena himpunan fuzzy dapat dipandang sebagai himpunan fuzzy intuisionistik dengan derajat ketidakpastian sama dengan 0. Misalkan $\tilde{F}$ adalah himpunan fuzzy di S, $\tilde{F}$ dapat dinyatakan sebagai himpunan fuzzy intuisionistik di S yang didefinisikan sebagai berikut.

$$\tilde{F} = \{\langle s, \mu_{\tilde{F}}(s), 1 - \mu_{\tilde{F}}(s)\rangle | s \in S\}$$

Himpunan fuzzy intuisionistik juga perluasan dari himpunan klasik, karena himpunan klasik dapat dipandang sebagai himpunan fuzzy intuisionistik dengan derajat keanggoataan sama dengan 1 dan derajat kenon-anggotaan sama dengan 0. Misalkan S himpunan klasik, S dapat dinyatakan sebagai himpunan fuzzy intuisionistik yang didefiniskan sebagai berikut.

$$S = \{\langle s, 1, 0\rangle | s \in S\}$$

3. PEMBAHASAN

Definisi 3.1

(Atanassov, 1999) Misalkan S himpunan tak kosong. Himpunan fuzzy intuisionistik F di S didefinisikan sebagai

$$F = \{\langle s, \mu_F(s), \nu_F(s)\rangle | s \in S\}$$

dimana $\mu_F(s): S \to [0,1]$ dan $\nu_F(s): S \to [0,1]$ berturut-turut menunjukkan derajat keanggotaan dan derajat kenon-anggotaan s di F dan memenuhi syarat $0 \le \mu_F(s) + \nu_F(s) \le 1$ untuk semua $s \in S$.

Definisi 3.2

(Atanassov, 1999) Misalkan F himpunan fuzzy intuisionistik di S, derajat ketidakpastian dari s di F didefinisikan sebagai

$$\pi_F(s) = 1 - \mu_F(s) - \nu_F(s)$$

Teorema 3.1

Jika F himpunan fuzzy intuisionistik di S, maka $0 \le \pi_F(s) \le 1$.

Bukti :

Berdasarkan definisi 3.1 dan definisi 3.2 jelaslah $0 \le \pi_F(s) \le 1$. ■

Definisi 3.3

(Atanassov, 1999) Misalkan S himpunan tak kosong. F dan H himpunan fuzzy intuisionistik di S. Didefinisikan operasi dan relasi sebagai berikut

(1) $F \subset H \Leftrightarrow \forall s \in S, \mu_F(s) \le \mu_H(s), \nu_F(s) \ge \nu_H(s)$

(2) $F \supset H \Leftrightarrow H \subset F$

(3) $F = H \Leftrightarrow \mu_F(s) = \mu_H(s), \nu_F(s) = \nu_H(s)$

(4) $\bar{F} = \{\langle s, \nu_F(s), \mu_F(s)\rangle | s \in S\}$

(5) $F \cap H = \{\langle s, \min(\mu_F(s), \mu_H(s)), \max(\nu_F(s), \nu_H(s))\rangle | s \in S\}$

(6) $F \cup H = \{\langle s, \max(\mu_F(s), \mu_H(s)), \min(\nu_F(s), \nu_H(s))\rangle | s \in S\}$

(7) $F + H = \{\langle s, \mu_F(s) + \mu_H(s) - \mu_F(s)\mu_H(s), \nu_F(s)\nu_H(s)\rangle | s \in S\}$

(8) $F.H = \{\langle s, \mu_F(s)\mu_H(s), \nu_F(s) + \nu_H(s) - \nu_F(s)\nu_H(s)\rangle | s \in S\}$

(9) $F@H = \left\{\langle s, \frac{\mu_F(s)+\mu_H(s)}{2}, \frac{\nu_F(s)+\nu_H(s)}{2}\rangle \,|\, s \in S\right\}$

Akan ditunjukkan $0 \le \mu_F(s) + \mu_H(s) - \mu_F(s)\mu_H(s) \le 1$

$\mu_F(s) + \mu_H(s) - \mu_F(s)\mu_H(s)$

$= \mu_F(s)(1 - \mu_H(s)) + \mu_H(s) \ge 0$

Andaikan $\mu_F(s) + \mu_H(s) - \mu_F(s)\mu_H(s) > 1$

$\mu_F(s)(1 - \mu_H(s)) > 1 - \mu_H(s)$

Jika $\mu_H(s) \ne 1$, maka $\mu_F(s) > 1$ (Kontradiksi)

Jika $\mu_H(s) = 1$, maka $1 > 1$ (Kontradiksi)

Pengandaian salah, jadi $0 \le \mu_F(s) + \mu_H(s) - \mu_F(s)\mu_H(s) \le 1$. ■

Proposisi 3.1

Jika F, H, dan J himpunan fuzzy intuisionistik di S, maka

(1) $F + H = H + F$

(2) $F.H = H.F$

(3) $(F+H)+J=F+(H+J)$
(4) $(F.H).J=F.(H.J)$
(5) $(F\cap H)+J=(F+J)\cap(H+J)$
(6) $(F\cap H).J=(F.J)\cap(H.J)$
(7) $(F\cap H)@J=(F@J)\cap(H@J)$
(8) $(F\cup H)+J=(F+J)\cup(H+J)$
(9) $(F\cup H).J=(F.J)\cup(H.J)$
(10) $(F\cup H)@J=(F@J)\cup(H@J)$
(11) $\overline{(\bar{F}+\bar{J})}=F.H$
(12) $\overline{(\bar{F}.\bar{J})}=F+H$
(13) $\overline{(\bar{F}@\bar{J})}=F@H$
(14) $F\cap F=F$
(15) $F\cup F=F$
(16) $F@F=F$

Bukti :

(1) $F+H$

$=\{\langle s,\mu_F(s)+\mu_H(s)-\mu_F(s)\mu_H(s), \nu_F(s)\nu_H(s)\rangle|s\in S\}$

$=\{\langle s,\mu_H(s)+\mu_F(s)-\mu_H(s)\mu_F(s), \nu_H(s)\nu_F(s)\rangle|s\in S\}$

$=H+F$ ■

Bukti (2) analog dengan (1).

(3) $(F+H)+J$

$=\{\langle s,\mu_F(s)+\mu_H(s)-\mu_F(s)\mu_H(s), \nu_F(s)\nu_H(s)\rangle|s\in S\}+J$

$=\{\langle s,\mu_F(s)+\mu_H(s)+\mu_J(s)-\mu_F(s)\mu_H(s)-\big(\mu_F(s)+\mu_H(s)-\mu_F(s)\mu_H(s)\big)\mu_J(s), \nu_F(s)\nu_H(s)\nu_J(s)\rangle|s\in S\}$

$=\Big\{\langle s,\mu_F(s)+\mu_H(s)+\mu_J(s)-\mu_H(s)\mu_J(s)-\Big(\mu_H(s)+\mu_J(s)-\mu_H(s)\mu_J(s)\Big)\mu_F(s), \nu_F(s)\nu_H(s)\nu_J(s)\rangle\,|s\in S\Big\}$

$=F+\{\langle s,\mu_H(s)+\mu_J(s)-\mu_H(s)\mu_J(s), \nu_H(s)\nu_J(s)\rangle|s\in S\}$

$=F+(H+J)$ ■

Bukti (4) analog dengan (3).

(5) $(F\cap H)+J$

$=\{\langle s,\min\big(\mu_F(s),\mu_H(s)\big),\max\big(\nu_F(s),\ \nu_H(s)\big)\rangle|s\in S\}+J$

$=\{\langle s,\min\big(\mu_F(s),\mu_H(s)\big)+\mu_J(s)-\min\big(\mu_F(s),\mu_H(s)\big)\,\mu_J(s),\max\big(\nu_F(s),\nu_H(s)\big)\nu_J(s)\rangle|s\in S\}$

$(F+J)\cap(H+J)$

$=\{\langle s,\mu_F(s)+\mu_J(s)-\mu_F(s)\mu_J(s), \nu_F(s)\nu_J(s)\rangle|s\in S\}\cap\{\langle s,\mu_H(s)+\mu_J(s)-\mu_H(s)\mu_J(s),\nu_H(s)\nu_J(s)\rangle|s\in S\}$

$=\Big\{\langle s,\min\Big(\mu_F(s)+\mu_J(s)-\mu_F(s)\mu_J(s),\mu_H(s)+\mu_J(s)-\mu_H(s)\mu_J(s)\Big),\max\Big(\nu_F(s)\nu_J(s), \nu_H(s)\nu_J(s)\Big)\rangle\,|s\in S\Big\}$

Jika $\mu_F(s)\geq\mu_H(s)$ dan $\nu_F(s)\geq\nu_H(s)$, maka $(F\cap H)+J=(F+J)\cap(H+J)$

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Jika $\mu_F(s)\geq\mu_H(s)$ dan $\nu_H(s)\geq\nu_F(s)$, maka $(F\cap H)+J=(F+J)\cap(H+J)$

Jadi, $(F\cap H)+J=(F+J)\cap(H+J)$ ■

Bukti (6)-(10) analog dengan (5).

(13) $\overline{(\bar{F}@\bar{J})}$

$=\overline{\Big\{\langle s,\frac{\nu_F(s)+\nu_H(s)}{2},\frac{\mu_F(s)+\mu_H(s)}{2}\rangle\,|s\in S\Big\}}$

$=\Big\{\langle s,\frac{\mu_F(s)+\mu_H(s)}{2},\frac{\nu_F(s)+\nu_H(s)}{2}\rangle\,|s\in S\Big\}$

$=F@H$ ■

Bukti (11) dan (12) analog dengan (13).

(14) $F\cap F$

$=\{\langle s,\min\big(\mu_F(s),\mu_F(s)\big),\max\big(\nu_F(s),\nu_F(s)\big)\rangle|s\in S\}$

$=\{\langle s,\mu_F(s),\nu_F(s)\rangle|s\in S\}$

$=F$ ■

Bukti (15) dan (16) analog dengan (14).

Proposisi 3.2

Jika F dan H himpunan fuzzy intuisionistik di S, maka

(1) $(F\cap H)+(F\cup H)=F+H$
(2) $(F\cap H).(F\cup H)=F.H$
(3) $(F\cap H)@(F\cup H)=F@H$
(4) $(F+H)@(F.H)=F@H$

Bukti :

(1) $(F\cap H)+(F\cup H)$

$=\{\langle s,\min\big(\mu_F(s),\mu_H(s)\big),\max\big(\nu_F(s),\ \nu_H(s)\big)\rangle|s\in S\}+\{\langle s,\max\big(\mu_F(s),\mu_H(s)\big),\min\big(\nu_F(s),\nu_H(s)\big)\rangle|s\in S\}$

$=\{\langle s,\min\big(\mu_F(s),\mu_H(s)\big)+\max\big(\mu_F(s),\mu_H(s)\big)-\min\big(\mu_F(s),\mu_H(s)\big)\max\big(\mu_F(s),\mu_H(s)\big),\max\big(\nu_F(s),\nu_H(s)\big)\min\big(\nu_F(s),\nu_H(s)\big)\rangle|s\in S\}$

Karena $\forall a,b\in\mathbb{R}$, berlaku $\max(a,b)+\min(a,b)=a+b$ dan $\max(a,b)\,.\min(a,b)=ab$, sehingga $(F\cap H)+(F\cup H)$

$= \{\langle s, \mu_F(s) + \mu_H(s) - \mu_F(s)\,\mu_H(s),$
$\nu_F(s)\nu_H(s)\rangle | s \in S\}$
$= F + H$ ■

Bukti (2) dan (3) analog dengan (1).

(4) $(F + H)@(F.H)$
$= \{\langle s, \mu_F(s) + \mu_H(s) - \mu_F(s)\mu_H(s),$
$\nu_F(s)\nu_H(s)\rangle | s \in S\}@\{\langle s, \mu_F(s)\mu_H(s),\ \nu_F(s) +$
$\nu_H(s) - \nu_F(s)\nu_H(s)\rangle | s \in S\}$
$= \left\{\langle s, \frac{\mu_F(s)+\mu_H(s)}{2}, \frac{\nu_F(s)+\nu_H(s)}{2}\rangle \,|\, s \in S\right\}$
$= F@H$ ■

Teorema 3.2

Jika F dan H himpunan fuzzy intuisionistik di S, maka

$$\big((F \cap H) + (F \cup H)\big)@\big((F \cap H).(F \cup H)\big) = F@H$$

Bukti :

Berdasarkan (1), (2), dan (4) pada Proposisi 3.2, jelaslah pernyataan Teorema 3.2 bernilai benar. ■

Definisi 3.4

(Danchev, 1996) Misalkan F dan H himpunan fuzzy intuisionistik di S, didefinisikan

$$F \underset{m,n}{\otimes} H = \{\langle s, f(m, \mu_F(s), \mu_H(s)), f(n, \nu_F(s), \nu_H(s))\rangle | s \in S \ \wedge m, n \in \mathbb{R}\}$$

dimana

$$f(k,a,b) = \begin{cases} \left(\frac{a^k+b^k}{2}\right)^{1/k} & ;\ k > 0 \vee (k < 0 \wedge ab > 0) \\ \sqrt{ab} & ;\ k = 0 \\ 0 & ;\ k < 0 \wedge ab = 0 \end{cases}$$

Proposisi 3.3

Jika F dan H himpunan fuzzy intuisionistik di S, maka

$$F \underset{1,1}{\otimes} H = F@H$$

Bukti :

$F \underset{1,1}{\otimes} H$
$= \{\langle s, f(1, \mu_F(s), \mu_H(s)), f(1, \nu_F(s), \nu_H(s))\rangle | s \in S\}$

Karena $m, n > 0$, maka

$F \underset{1,1}{\otimes} H$
$= \left\{\langle s, \left(\frac{\mu_F(s)^1+\mu_H(s)^1}{2}\right)^1, \left(\frac{\nu_F(s)^1+\nu_H(s)^1}{2}\right)^1 \rangle \,|\, s \in S\right\}$
$= \left\{\langle s, \frac{\mu_F(s)+\mu_H(s)}{2}, \frac{\nu_F(s)+\nu_H(s)}{2}\rangle \,|\, s \in S\right\}$
$= F@H$ ■

Definisi 3.5

(Atanassov, 1999) Misalkan F dan H himpunan fuzzy intuisionistik di S, didefinisikan operasi berikut

$F \mapsto H =$
$\{\langle s, \max(\nu_F(s), \mu_H(s)), \min(\mu_F(s), \nu_H(s))\rangle | s \in S\}$

Proposisi 3.4

Jika F, H, dan J himpunan fuzzy intuisionistik di S, maka

(1) $(F \cap H) \mapsto J \supset (F \mapsto J) \cap (H \mapsto J)$
(2) $(F \cup H) \mapsto J \subset (F \mapsto J) \cup (H \mapsto J)$
(3) $(F \cap H) \mapsto J = (F \mapsto J) \cup (H \mapsto J)$
(4) $(F \cup H) \mapsto J = (F \mapsto J) \cap (H \mapsto J)$

Bukti :

(1) $(F \cap H) \mapsto J$
$= \{\langle s, \min(\mu_F(s), \mu_H(s)), \max(\nu_F(s),\ \nu_H(s))\rangle | s \in S\} \mapsto J$
$= \left\{\langle s, \max\left(\nu_F(s), \nu_H(s), \mu_J(s)\right), \min\left(\mu_F(s), \mu_H(s), \nu_J(s)\right)\rangle \,|\, s \in S\right\}$

$(F \mapsto J) \cap (H \mapsto J)$
$= \left\{\langle s, \max\left(\nu_F(s), \mu_J(s)\right), \min\left(\mu_F(s), \nu_J(s)\right)\rangle \,|\, s \in S\right\} \cap \left\{\langle s, \max\left(\nu_H(s), \mu_J(s)\right), \min\left(\mu_H(s), \nu_J(s)\right)\rangle \,|\, s \in S\right\}$
$= \left\{\langle s, \min\left(\max\left(\nu_F(s), \mu_J(s)\right), \max\left(\nu_H(s), \mu_J(s)\right)\right), \max\left(\min\left(\mu_F(s), \nu_J(s)\right), \min\left(\mu_H(s), \nu_J(s)\right)\right)\rangle \,|\, s \in S\right\}$

Berdasarkan definisi 3.3, $(F \cap H) \mapsto J \supset (F \mapsto J) \cap (H \mapsto J)$, jika $\mu_{(F\mapsto J)\cap(H\mapsto J)} \le \mu_{(F\cap H)\mapsto J}$ dan $\nu_{(F\mapsto J)\cap(H\mapsto J)} \ge \nu_{(F\cap H)\mapsto J}$.

$\max\left(\nu_F(s), \nu_H(s), \mu_J(s)\right)$
$\ge \max\left(\nu_F(s), \mu_J(s)\right)$
$= \min\left(\max\left(\nu_F(s), \mu_J(s)\right)\right)$
$\ge \min\left(\max\left(\nu_F(s), \mu_J(s)\right), \max\left(\nu_H(s), \mu_J(s)\right)\right)$

$\max\left(\min\left(\mu_F(s), \nu_J(s)\right), \min\left(\mu_H(s), \nu_J(s)\right)\right)$
$\ge \max\left(\min\left(\mu_F(s), \nu_J(s)\right)\right)$
$= \min\left(\mu_F(s), \nu_J(s)\right)$
$\ge \min\left(\mu_F(s), \mu_H(s), \nu_J(s)\right)$

Karena $\mu_{(F\mapsto J)\cap(H\mapsto J)} \le \mu_{(F\cap H)\mapsto J}$ dan $\nu_{(F\mapsto J)\cap(H\mapsto J)} \ge \nu_{(F\cap H)\mapsto J}$, maka $(F \cap H) \mapsto J \supset (F \mapsto J) \cap (H \mapsto J)$. ■

Bukti (2) analog dengan (1).

(3) $(F \cap H) \mapsto J$
$= \{\langle s, \min(\mu_F(s), \mu_H(s)), \max(\nu_F(s),\ \nu_H(s))\rangle | s \in S\} \mapsto J$

$$= \left\{\langle s, \max\left(\nu_F(s), \nu_H(s), \mu_J(s)\right), \min\left(\mu_F(s), \mu_H(s), \nu_J(s)\right)\rangle \mid s \in S\right\}$$

$$= \left\{\langle s, \max\left(\max\left(\nu_F(s), \mu_J(s)\right), \max\left(\nu_H(s), \mu_J(s)\right)\right), \min\left(\min\left(\mu_F(s), \nu_J(s)\right), \min\left(\mu_H(s), \nu_J(s)\right)\right)\rangle \mid s \in S\right\}$$

$$= \left\{\langle s, \max\left(\nu_F(s), \mu_J(s)\right), \min\left(\mu_F(s), \nu_J(s)\right)\rangle \mid s \in S\right\} \cup \left\{\langle s, \max\left(\nu_H(s), \mu_J(s)\right), \min\left(\mu_H(s), \nu_J(s)\right)\rangle \mid s \in S\right\}$$

$$= (F \mapsto J) \cup (H \mapsto J)$$ ■

Bukti (4) analog dengan (3).

Definisi 3.5

(Atanassov, 2012) Himpunan fuzzy intuisionistik F di S disebut himpunan tautologis fuzzy intuisionistik jika untuk setiap s, berlaku $\mu_F(s) \geq \nu_F(s)$.

Teorema 3.3

Jika F, H, dan J himpunan fuzzy intuisionistik di S, maka

(1) $F \mapsto F$

(2) $F \mapsto (H \mapsto F)$

(3) $F \cap H \mapsto F$

(4) $H \mapsto (F \cup H)$

(5) $(\bar{F} \mapsto \bar{H}) \mapsto \left((\bar{F} \mapsto H) \mapsto F\right)$

(6) $F \mapsto \left(H \mapsto (F \cap H)\right)$

(7) $(F \mapsto J) \mapsto \left((F \mapsto J) \mapsto \left((F \cup H) \mapsto J\right)\right)$

adalah himpunan tautologis fuzzy intuisionistik.

Bukti :

(6) $F \mapsto \left(H \mapsto (F \cap H)\right)$

$$= F \mapsto \left(H \mapsto \{\langle s, min\left(\mu_F(s), \mu_H(s)\right), mas\left(\nu_F(s), \nu_H(s)\right)\rangle | s \in S\}\right)$$

$$= F \mapsto \{\langle s, \max\left(\nu_H(s), \min\left(\mu_F(s), \mu_H(s)\right)\right), \min\left(\mu_H(s), \max\left(\nu_F(s), \nu_H(s)\right)\right)\rangle | s \in S\}$$

$$= \{\langle s, \max\left(\nu_F(s), \max\left(\nu_H(s), \min\left(\mu_F(s), \mu_H(s)\right)\right)\right), \min\left(\mu_F(s), \min\left(\mu_H(s), \max\left(\nu_F(s), \nu_H(s)\right)\right)\right)\rangle | s \in S\}$$

$$= \{\langle s, \max\left(\nu_F(s), \nu_H(s), \min\left(\mu_F(s), \mu_H(s)\right)\right), \min\left(\mu_F(s), \mu_H(s), \max\left(\nu_F(s), \nu_H(s)\right)\right)\rangle | s \in S\}$$

Akan ditunjukkan $\mu_{F \mapsto (H \mapsto (F \cap H))} \geq \nu_{F \mapsto (H \mapsto (F \cap H))}$

$$\max\left(\nu_F(s), \nu_H(s), \min\left(\mu_F(s), \mu_H(s)\right)\right)$$

$$\geq \max\left(\min\left(\mu_F(s), \mu_H(s)\right)\right)$$

$$= \min\left(\mu_F(s), \mu_H(s)\right)$$

$$\geq \min\left(\mu_F(s), \mu_H(s), \max\left(\nu_F(s), \nu_H(s)\right)\right)$$

Karena $\mu_{F \mapsto (H \mapsto (F \cap H))} \geq \nu_{F \mapsto (H \mapsto (F \cap H))}$, maka $F \mapsto \left(H \mapsto (F \cap H)\right)$ adalah himpunan tautologis fuzzy intuisionistik. ■

(7) $(F \mapsto J) \mapsto \left((F \mapsto J) \mapsto \left((F \cup H) \mapsto J\right)\right)$

$$= \left\{\langle s, \max\left(\nu_F(s), \mu_J(s)\right), \min\left(\mu_F(s), \nu_J(s)\right)\rangle \mid s \in S\right\} \mapsto \left(\left\{\langle s, \max\left(\nu_H(s), \mu_J(s)\right), \min\left(\mu_H(s), \nu_J(s)\right)\rangle \mid s \in S\right\} \mapsto \left(\{\langle s, \max\left(\mu_F(s), \mu_H(s)\right), \min\left(\nu_F(s), \nu_H(s)\right)\rangle | s \in S\} \mapsto J\right)\right)$$

$$= \left\{\langle s, \max\left(\min\left(\mu_F(s), \nu_J(s)\right), \min\left(\mu_H(s), \nu_J(s)\right), \min\left(\nu_F(s), \nu_H(s)\right), \mu_J(s)\right), \min\left(\max\left(\nu_F(s), \mu_J(s)\right), \max\left(\nu_H(s), \mu_J(s)\right), \max\left(\mu_F(s), \mu_H(s)\right), \nu_J\right)\rangle \mid s \in S\right\}$$

Akan ditunjukkan $\mu_{(F \mapsto J) \mapsto \left((H \mapsto J) \mapsto \left((F \cup H) \mapsto J\right)\right)} \geq \nu_{(F \mapsto J) \mapsto \left((H \mapsto J) \mapsto \left((F \cup H) \mapsto J\right)\right)}$

$$\max\left(\min\left(\mu_F(s), \nu_J(s)\right), \min\left(\mu_H(s), \nu_J(s)\right), \min\left(\nu_F(s), \nu_H(s)\right), \mu_J(s)\right)$$

$$\geq \max\left(\min\left(\mu_F(s), \nu_J(s)\right), \min\left(\mu_H(s), \nu_J(s)\right)\right)$$

$$= \min\left(\max\left(\mu_F(s), \mu_H(s)\right), \nu_J(s)\right)$$

$$\geq \min\left(\max\left(\nu_F(s), \mu_J(s)\right), \max\left(\nu_H(s), \mu_J(s)\right), \max\left(\mu_F(s), \mu_H(s)\right), \nu_J\right)$$

Karena $\mu_{(F \mapsto J) \mapsto \left((H \mapsto J) \mapsto \left((F \cup H) \mapsto J\right)\right)} \geq \nu_{(F \mapsto J) \mapsto \left((H \mapsto J) \mapsto \left((F \cup H) \mapsto J\right)\right)}$, maka $(F \mapsto J) \mapsto \left((H \mapsto J) \mapsto \left((F \cup H) \mapsto J\right)\right)$ adalah himpunan tautologis fuzzy intuisionistik. ■

Bukti (1)-(5) analog dengan (6) dan (7).

Teorema 3.4

Jika F dan H himpunan fuzzy intuisionistik di S, maka

(1) Jika $F \subset H$, maka $F \mapsto H$ adalah himpunan tautologis fuzzy intuisionistik.

(2) $\left(F \cap (F \mapsto H)\right) \mapsto H$ adalah himpunan tautologis fuzzy intuisionistik.

(3) $\left((F \mapsto H) \cap \bar{H}\right) \mapsto \bar{F}$ adalah himpunan tautologis fuzzy intuisionistik.

Bukti :

(1) $F \mapsto H = \{\langle s, \max\left(\nu_F(s), \mu_H(s)\right), \min\left(\mu_F(s), \nu_H(s)\right)\rangle | s \in S\}$

Karena $F \subset H$, maka $\mu_F(s) \le \mu_H(s)$ dan $\nu_F(s) \ge \nu_H(s)$, sehingga $\mu_{F \mapsto H} \ge \nu_{F \mapsto H}$. Jelaslah $F \mapsto H$ adalah himpunan tautologis fuzzy intuisionistik. ■

(3) $\big((F \mapsto H) \cap \bar{H}\big) \mapsto \bar{F}$

$= \big(\{\langle s, \max(\nu_F(s), \mu_H(s)), \min(\mu_F(s), \nu_H(s))\rangle \mid s \in S\} \cap \bar{H}\big) \mapsto \bar{F}$

$= \{\langle s, \min(\nu_H(s), \max(\nu_F(s), \mu_H(s))), \max(\mu_H(s), \min(\mu_F(s), \nu_H(s)))\rangle \mid s \in S\} \mapsto F^c$

$= \{\langle s, \max(\nu_F(s), \mu_H(s), \min(\mu_F(s), \nu_H(s))), \min(\mu_F(s), \nu_H(s), \max(\nu_F(s), \mu_H(s)))\rangle \mid s \in S\}$

Akan ditunjukkan $\mu_{((F \mapsto H) \cap H^c) \mapsto F^c} \ge \nu_{((F \mapsto H) \cap H^c) \mapsto F^c}$

$\max(\nu_F(s), \mu_H(s), \min(\mu_F(s), \nu_H(s)))$

$\ge \max(\min(\mu_F(s), \nu_H(s)))$

$= \min(\mu_F(s), \nu_H(s))$

$\ge \min(\nu_H(s), \mu_F(s), \max(\nu_F(s), \mu_H(s)))$

Karena $\mu_{((F \mapsto H) \cap H^c) \mapsto F^c} \ge \nu_{((F \mapsto H) \cap H^c) \mapsto F^c}$, maka $\big((F \mapsto H) \cap H^c\big) \mapsto F^c$ adalah himpunan tautologis fuzzy intuisionistik. ■

Bukti (2) analog dengan (3).

Definisi 3.7

(Atanassov, 2012) Misalkan S dan Q himpunan tak kosong, dan misalkan

$$F = \{\langle s, \mu_F(s), \nu_F(s)\rangle \mid s \in S\}$$
$$H = \{\langle q, \mu_H(q), \nu_H(q)\rangle \mid q \in Q\}$$

adalah himpunan fuzzy intuisionistik, F di S dan H di Q. Hasil kali kartesian dari F dan H didefinisikan sebagai berikut.

(1) $F \times_1 H = \{\langle (s,q), \mu_F(s)\mu_H(q), \nu_F(s)\nu_H(q)\rangle \mid s \in S, q \in Q\}$

(2) $F \times_2 H = \{\langle (s,q), \mu_F(s) + \mu_H(q) - \mu_F(s)\mu_H(q), \nu_F(s)\nu_H(q)\rangle \mid s \in S, q \in Q\}$

(3) $F \times_3 H = \{\langle (s,q), \mu_F(s)\mu_H(q), \nu_F(s) + \nu_H(q) - \nu_F(s)\nu_H(q)\rangle \mid s \in S, q \in Q\}$

(4) $F \times_4 H = \{\langle (s,q), \min(\mu_F(s), \mu_H(q)), \max(\nu_F(s), \nu_H(q))\rangle \mid s \in S, q \in Q\}$

(5) $F \times_5 H = \{\langle (s,q), \max(\mu_F(s), \mu_H(q)), \min(\nu_F(s), \nu_H(q))\rangle \mid s \in S, q \in Q\}$

(6) $F \times_6 H = \left\{\langle (s,q), \frac{\mu_F(s)+\mu_H(q)}{2}, \frac{\nu_F(s)+\nu_H(q)}{2}\rangle \mid s \in S, q \in Q\right\}$

Proposisi 3.5

Jika F, H, J, dan L himpunan fuzzy intuisionistik, dimana F, H di S, J di Q, dan L di P, maka

(1) $(F \times J) \times L = F \times (J \times L)$; untuk $\times \in \{\times_1, \times_2, \times_3, \times_4, \times_5\}$

(2) $(F \cup H) \times J = (F \times J) \cup (H \times J)$; untuk $\times \in \{\times_1, \times_2, \times_3, \times_4, \times_5, \times_6\}$

(3) $(F \cap H) \times J = (F \times J) \cap (H \times J)$; untuk $\times \in \{\times_1, \times_2, \times_3, \times_4, \times_5, \times_6\}$

(4) $J \times (F \cup H) = (J \times F) \cup (J \times H)$; untuk $\times \in \{\times_1, \times_2, \times_3, \times_4, \times_5, \times_6\}$

(5) $J \times (F \cap H) = (J \times F) \cap (J \times H)$; untuk $\times \in \{\times_1, \times_2, \times_3, \times_4, \times_5, \times_6\}$

Bukti :

(1) Akan ditunjukkan $(F \times_1 J) \times_1 L = F \times_1 (J \times_1 L)$

$(F \times_1 J) \times_1 L$

$= \{\langle (s,q), \mu_F(s)\mu_J(q), \nu_F(s)\nu_J(q)\rangle \mid s \in S, q \in Q\} \times_1 L$

$= \{\langle (s,q,p), \mu_F(s)\mu_J(q)\mu_L(p), \nu_F(s)\nu_J(q)\nu_L(p)\rangle \mid s \in S, q \in Q, p \in P\}$

$= F \times_1 \{\langle (q,p), \mu_J(q)\mu_L(p), \nu_J(q)\nu_L(p)\rangle \mid q \in Q, p \in P\}$

$= F \times_1 (J \times_1 L)$ ■

Bukti $(F \times J) \times L = F \times (J \times L)$ untuk $\times \in \{\times_2, \times_3, \times_4, \times_5\}$ analog dengan $(F \times_1 J) \times_1 L = F \times_1 (J \times_1 L)$. Sehingga $(F \times J) \times L = F \times (J \times L)$ untuk $\times \in \{\times_1, \times_2, \times_3, \times_4, \times_5\}$. ■

(2) Akan ditunjukkan $(F \cup H) \times_1 J = (F \times_1 J) \cup (H \times_1 J)$

$(F \cup H) \times_1 J$

$= \{\langle s, \max(\mu_F(s), \mu_H(s)), \min(\nu_F(s), \nu_H(s))\rangle \mid s \in S\} \times_1 J$

$= \{\langle (s,q), \mu_J(q)\max(\mu_F(s), \mu_H(s)), \nu_J(q)\min(\nu_F(s), \nu_H(s))\rangle \mid s \in S, q \in Q\}$

$= \left\{\langle (s,q), \max\left(\mu_F(s)\mu_J(q), \mu_H(s)\mu_J(q)\right), \min\left(\nu_F(s)\nu_J(q), \nu_H(s)\nu_J(q)\right)\rangle \mid s \in S, q \in Q\right\}$

$= \{\langle (s,q), \mu_F(s)\mu_J(q), \nu_F(s)\nu_J(q)\rangle \mid s \in S, q \in Q\}$
$\cup \{\langle (s,q), \mu_H(s)\mu_J(q), \nu_H(s)\nu_J(q)\rangle \mid s \in S, q \in Q\}$

$= (F \times_1 J) \cup (H \times_1 J)$ ■

Akan ditunjukkan $(F \cup H) \times_2 J = (F \times_2 J) \cup (H \times_2 J)$

$(F \cup H) \times_2 J$

$= \{\langle s, \max(\mu_F(s), \mu_H(s)), \min(\nu_F(s), \nu_H(s))\rangle \mid s \in S\} \times_2 J$

$= \{\langle (s,q), \max(\mu_F(s), \mu_H(s)) + \mu_J(q) - \mu_J(q)\max(\mu_F(s), \mu_H(s)), \nu_J(q)\min(\nu_F(s), \nu_H(s))\rangle \mid s \in S, q \in Q\}$

$(F \times_2 J) \cup (H \times_2 J)$

$= \{\langle (s,q), \mu_F(s) + \mu_J(q) - \mu_F(s)\mu_J(q), \nu_F(s)\nu_J(q)\rangle | s \in S, q \in Q\} \cup \{\langle (s,q), \mu_H(s) + \mu_J(q) - \mu_H(s)\mu_J(q), \nu_H(s)\nu_J(q)\rangle | s \in S, q \in Q\}$

$= \{\langle (s,q), \max\left(\mu_F(s) + \mu_J(q) - \mu_F(s)\mu_J(q), \mu_H(s) + \mu_J(q) - \mu_H(s)\mu_J(q)\right), \min\left(\nu_F(s)\nu_J(q), \nu_H(s)\nu_J(q)\right)\rangle | s \in S, q \in Q\}$

Jika $\mu_F(s) \geq \mu_H(s)$ dan $\nu_F(s) \geq \nu_H(s)$, maka $(F \cup H) \times_2 J = (F \times_2 J) \cup (H \times_2 J)$.
Jika $\mu_F(s) \geq \mu_H(s)$ dan $\nu_H(s) \geq \nu_F(s)$, maka $(F \cup H) \times_2 J = (F \times_2 J) \cup (H \times_2 J)$.
Jika $\mu_H(s) \geq \mu_F(s)$ dan $\nu_F(s) \geq \nu_H(s)$, maka $(F \cup H) \times_2 J = (F \times_2 J) \cup (H \times_2 J)$.
Jika $\mu_H(s) \geq \mu_F(s)$ dan $\nu_H(s) \geq \nu_F(s)$, maka $(F \cup H) \times_2 J = (F \times_2 J) \cup (H \times_2 J)$.
$\therefore (F \cup H) \times_2 J = (F \times_2 J) \cup (H \times_2 J)$ ■

Akan ditunjukkan $(F \cup H) \times_3 J = (F \times_3 J) \cup (H \times_3 J)$

$(F \cup H) \times_3 J$
$= \{\langle s, \max(\mu_F(s), \mu_H(s)), \min(\nu_F(s), \nu_H(s))\rangle | s \in S\} \times_3 J$
$= \{\langle (s,q), \mu_J(q)\max(\mu_F(s), \mu_H(s)), \nu_J(q) + \min(\nu_F(s), \nu_H(s)) - \nu_J(q)\min(\nu_F(s), \nu_H(s))\rangle | s \in S, q \in Q\}$

$(F \times_3 J) \cup (H \times_3 J)$
$= \{\langle (s,q), \mu_F(s)\mu_J(q), \nu_F(s) + \nu_J(q) - \nu_F(s)\nu_J(q)\rangle | s \in S, q \in Q\} \cup \{\langle (s,q), \mu_H(s)\mu_J(q), \nu_H(s) + \nu_J(q) - \nu_H(s)\nu_J(q)\rangle | s \in S, q \in Q\}$
$= \{\langle (s,q), \max\left(\mu_F(s)\mu_J(q), \mu_H(s)\mu_J(q)\right), \min\left(\nu_F(s) + \nu_J(q) - \nu_F(s)\nu_J(q), \nu_H(s) + \nu_J(q) - \nu_H(s)\nu_J(q)\right)\rangle | s \in S, q \in Q\}$

Jika $\mu_F(s) \geq \mu_H(s)$ dan $\nu_F(s) \geq \nu_H(s)$, maka $(F \cup H) \times_3 J = (F \times_3 J) \cup (H \times_3 J)$.
Jika $\mu_F(s) \geq \mu_H(s)$ dan $\nu_H(s) \geq \nu_F(s)$, maka $(F \cup H) \times_3 J = (F \times_3 J) \cup (H \times_3 J)$.
Jika $\mu_H(s) \geq \mu_F(s)$ dan $\nu_F(s) \geq \nu_H(s)$, maka $(F \cup H) \times_3 J = (F \times_3 J) \cup (H \times_3 J)$.
Jika $\mu_H(s) \geq \mu_F(s)$ dan $\nu_H(s) \geq \nu_F(s)$, maka $(F \cup H) \times_3 J = (F \times_3 J) \cup (H \times_3 J)$.
$\therefore (F \cup H) \times_3 J = (F \times_3 J) \cup (H \times_3 J)$ ■

Akan ditunjukkan $(F \cup H) \times_4 J = (F \times_4 J) \cup (H \times_4 J)$

$(F \cup H) \times_4 J$
$= \{\langle s, \max(\mu_F(s), \mu_H(s)), \min(\nu_F(s), \nu_H(s))\rangle | s \in S\} \times_3 J$
$= \{\langle (s,q), \min\left(\max(\mu_F(s), \mu_H(s)), \mu_J(q)\right), \max\left(\min(\nu_F(s), \nu_H(s)), \nu_J(q)\right)\rangle | s \in S, q \in Q\}$
$= \{\langle (s,q), \max\left(\min\left(\mu_F(s), \mu_J(q)\right), \min\left(\mu_H(s), \mu_J(q)\right)\right), \min\left(\max\left(\nu_F(s), \nu_J(q)\right), \max\left(\nu_H(s), \nu_J(q)\right)\right)\rangle | s \in S, q \in Q\}$
$= \{\langle (s,q), \min\left(\mu_F(s), \mu_J(q)\right), \max\left(\nu_F(s), \nu_J(q)\right)\rangle | s \in S, q \in Q\} \cup \{\langle (s,q), \min\left(\mu_H(s), \mu_J(q)\right), \max\left(\nu_H(s), \nu_J(q)\right)\rangle | s \in S, q \in Q\}$
$= (F \times_4 J) \cup (H \times_4 J)$ ■

Akan ditunjukkan $(F \cup H) \times_5 J = (F \times_5 J) \cup (H \times_5 J)$
$= \{\langle s, \max(\mu_F(s), \mu_H(s)), \min(\nu_F(s), \nu_H(s))\rangle | s \in S\} \times_5 J$
$= \{\langle (s,q), \max\left(\mu_F(s), \mu_H(s), \mu_J(q)\right), \min\left(\nu_F(s), \nu_H(s), \nu_J(q)\right)\rangle | s \in S, q \in Q\}$
$= \{\langle (s,q), \max\left(\mu_F(s), \mu_J(q)\right), \min\left(\nu_F(s), \nu_J(q)\right)\rangle | s \in S, q \in Q\} \cup \{\langle (s,q), \max\left(\mu_H(s), \mu_J(q)\right), \min\left(\nu_H(s), \nu_J(q)\right)\rangle | s \in S, q \in Q\}$
$= (F \times_5 J) \cup (H \times_5 J)$ ■

Akan ditunjukkan $(F \cup H) \times_6 J = (F \times_6 J) \cup (H \times_6 J)$

$(F \cup H) \times_6 J$
$= \{\langle s, \max(\mu_F(s), \mu_H(s)), \min(\nu_F(s), \nu_H(s))\rangle | s \in S\} \times_6 J$
$= \{\langle (s,q), \frac{\max(\mu_F(s), \mu_H(s)) + \mu_J(q)}{2}, \frac{\min(\nu_F(s), \nu_H(s)) + \nu_J(q)}{2}\rangle | s \in S, q \in Q\}$

$(F \times_6 J) \cup (H \times_6 J)$
$= \{\langle (s,q), \frac{\mu_F(s) + \mu_J(q)}{2}, \frac{\nu_F(s) + \nu_J(q)}{2}\rangle | s \in S, q \in Q\} \cup \{\langle (s,q), \frac{\mu_H(s) + \mu_J(q)}{2}, \frac{\nu_H(s) + \nu_J(q)}{2}\rangle | s \in S, q \in Q\}$
$= \{\langle (s,q), \max\left(\frac{\mu_F(s) + \mu_J(q)}{2}, \frac{\mu_H(s) + \mu_J(q)}{2}\right), \min\left(\frac{\nu_F(s) + \nu_J(q)}{2}, \frac{\nu_H(s) + \nu_J(q)}{2}\right)\rangle | s \in S, q \in Q\}$

Jika $\mu_F(s) \geq \mu_H(s)$ dan $\nu_F(s) \geq \nu_H(s)$, maka $(F \cup H) \times_6 J = (F \times_6 J) \cup (H \times_6 J)$.
Jika $\mu_F(s) \geq \mu_H(s)$ dan $\nu_H(s) \geq \nu_F(s)$, maka $(F \cup H) \times_6 J = (F \times_6 J) \cup (H \times_6 J)$.

Jika $\mu_H(s) \geq \mu_F(s)$ dan $\nu_F(s) \geq \nu_H(s)$, maka $(F \cup H) \times_6 J = (F \times_6 J) \cup (H \times_6 J)$.
Jika $\mu_H(s) \geq \mu_F(s)$ dan $\nu_H(s) \geq \nu_F(s)$, maka $(F \cup H) \times_6 J = (F \times_6 J) \cup (H \times_6 J)$.
$\therefore (F \cup H) \times_6 J = (F \times_6 J) \cup (H \times_6 J)$ ■
Jadi, $(F \cup H) \times J = (F \times J) \cup (H \times J)$ untuk $\times \in \{\times_1, \times_2, \times_3, \times_4, \times_5, \times_6\}$. ■
Bukti (3)-(5) analog dengan (2).

Proposisi 3.6
Jika F, H, dan J himpunan fuzzy intuisionistik, dimana F, H di S dan J di Q, maka
(1) $(F + H) \times J \subset (F \times J) + (H \times J)$
(2) $(F.H) \times J \supset (F \times J).(H \times J)$
(3) $(F@H) \times J = (F \times J)@(H \times J)$
(4) $J \times (F + H) \subset (J \times F) + (J \times H)$
(5) $J \times (F.H) \supset (J \times F).(J \times H)$
(6) $J \times (F@H) = (J \times F)@(J \times H)$
untuk $\times \in \{\times_1, \times_2, \times_3, \times_6\}$.
Bukti :
(1) Akan ditunjukkan $(F + H) \times_1 J \subset (F \times_1 J) + (H \times_1 J)$

$(F + H) \times_1 J$
$= \{\langle s, \mu_F(s) + \mu_H(s) - \mu_F(s)\mu_H(s), \nu_F(s)\nu_H(s)\rangle | s \in S\} \times_1 J$
$= \{\langle (s,q), \mu_F(s)\mu_J(q) + \mu_H(s)\mu_J(q) - \mu_F(s)\mu_H(s)\mu_J(q), \nu_F(s)\nu_H(s)\nu_J(q)\rangle | s \in S, q \in Q\}$

$(F \times_1 J) + (H \times_1 J)$
$= \{\langle (s,q), \mu_F(s)\mu_J(q), \nu_F(s)\nu_J(q)\rangle | s \in S, q \in Q\} + \{\langle (s,q), \mu_H(s)\mu_J(q), \nu_H(s)\nu_J(q)\rangle | s \in S, q \in Q\}$
$= \{\langle (s,q), \mu_F(s)\mu_J(q) + \mu_H(s)\mu_J(q) - \mu_F(s)\mu_H(s)\mu_J{}^2(q), \nu_F(s)\nu_H(s)\nu_J{}^2(q)\rangle | s \in S, q \in Q\}$

Berdasarkan definisi 3.3 $(F + H) \times_1 J \subset (F \times_1 J) + (H \times_1 J)$, jika $\mu_{(F+H)\times_1 J} \leq \mu_{(F\times_1 J)+(H\times_1 J)}$ dan $\nu_{(F+H)\times_1 J} \geq \nu_{(F\times_1 J)+(H\times_1 J)}$.
$\mu_{(F\times_1 J)+(H\times_1 J)} - \mu_{(F+H)\times_1 J}$
$= -\mu_F(s)\mu_H(s)\mu_J{}^2(q) + \mu_F(s)\mu_H(s)\mu_J(q)$
$= \mu_F(s)\mu_H(s)\mu_J(q)\left(1 - \mu_J(q)\right) \geq 0$

$\nu_{(F+H)\times_1 J} - \nu_{(F\times_1 J)+(H\times_1 J)}$
$= \nu_F(s)\nu_H(s)\nu_J(q) - \nu_F(s)\nu_H(s)\nu_J{}^2(q)$
$= \nu_F(s)\nu_H(s)\nu_J(q)\left(1 - \nu_J(q)\right) \geq 0$
Karena $\mu_{(F+H)\times_1 J} \leq \mu_{(F\times_1 J)+(H\times_1 J)}$ dan $\nu_{(F+H)\times_1 J} \geq \nu_{(F\times_1 J)+(H\times_1 J)}$, maka $(F + H) \times_1 J \subset (F \times_1 J) + (H \times_1 J)$. ■

Akan ditunjukkan $(F + H) \times_2 J \subset (F \times_2 J) + (H \times_2 J)$
$(F + H) \times_2 J$
$= \{\langle s, \mu_F(s) + \mu_H(s) - \mu_F(s)\mu_H(s), \nu_F(s)\nu_H(s)\rangle | s \in S\} \times_1 J$
$= \{\langle (s,q), \mu_F(s) + \mu_H(s) + \mu_J(q) - \mu_F(s)\mu_H(s) - \mu_F(s)\mu_J(q) - \mu_H(s)\mu_J(q) + \mu_F(s)\mu_H(s)\mu_J(q), \nu_F(s)\nu_H(s)\nu_J(q)\rangle | s \in S, q \in Q\}$

$(F \times_2 J) + (H \times_2 J)$
$= \{\langle (s,q), \mu_F(s) + \mu_J(q) - \mu_F(s)\mu_J(q), \nu_F(s)\nu_J(q)\rangle | s \in S, q \in Q\} + \{\langle (s,q), \mu_H(s) + \mu_J(q) - \mu_H(s)\mu_J(q), \nu_H(s)\nu_J(q)\rangle | s \in S, q \in Q\}$
$= \{\langle (s,q), \left(\mu_F(s) + \mu_J(q) - \mu_F(s)\mu_J(q)\right) + \left(\mu_H(s) + \mu_J(q) - \mu_H(s)\mu_J(q)\right) - \left(\mu_F(s) + \mu_J(q) - \mu_F(s)\mu_J(q)\right)\left(\mu_H(s) + \mu_J(q) - \mu_H(s)\mu_J(q)\right), \nu_F(s)\nu_H(s)\nu_J{}^2(q)\rangle | s \in S, q \in Q\}$

Berdasarkan definisi 3.3 $(F + H) \times_2 J \subset (F \times_2 J) + (H \times_2 J)$, jika $\mu_{(F+H)\times_2 J} \leq \mu_{(F\times_2 J)+(H\times_2 J)}$ dan $\nu_{(F+H)\times_2 J} \geq \nu_{(F\times_2 J)+(H\times_2 J)}$.
$\mu_{(F\times_2 J)+(H\times_2 J)} - \mu_{(F+H)\times_2 J}$
$= \mu_J(q) - \mu_F(s)\mu_J(q) - \mu_H(s)\mu_J(q) + \mu_F(s)\mu_H(s)\mu_J(q) - \mu_J{}^2(q) + \mu_F(s)\mu_J{}^2(q) + \mu_H(s)\mu_J{}^2(q) - \mu_F(s)\mu_H(s)\mu_J{}^2(q)$
$= \mu_J(q)\left(1 - \mu_F(s)\right)\left(1 - \mu_H(s)\right)\left(1 - \mu_J(q)\right) \geq 0$

$\nu_{(F+H)\times_2 J} - \nu_{(F\times_2 J)+(H\times_2 J)}$
$= \nu_F(s)\nu_H(s)\nu_J(q) - \nu_F(s)\nu_H(s)\nu_J{}^2(q)$
$= \nu_F(s)\nu_H(s)\nu_J(q)\left(1 - \nu_J(q)\right) \geq 0$
Karena $\mu_{(F+H)\times_2 J} \leq \mu_{(F\times_2 J)+(H\times_2 J)}$ dan $\nu_{(F+H)\times_2 J} \geq \nu_{(F\times_2 J)+(H\times_2 J)}$, maka $(F + H) \times_2 J \subset (F \times_2 J) + (H \times_2 J)$. ■

Akan ditunjukkan $(F + H) \times_3 J \subset (F \times_3 J) + (H \times_3 J)$
$(F + H) \times_3 J$
$= \{\langle s, \mu_F(s) + \mu_H(s) - \mu_F(s)\mu_H(s), \nu_F(s)\nu_H(s)\rangle | s \in S\} \times_3 J$
$= \{\langle (s,q), \left(\mu_F(s) + \mu_H(s) - \mu_F(s)\mu_H(s)\right)\mu_J(q), \nu_F(s)\nu_H(s) + \nu_J(q) - \nu_F(s)\nu_H(s)\nu_J(q)\rangle | s \in S, q \in Q\}$

$(F \times_3 J) + (H \times_3 J)$
$= \{\langle (s,q), \mu_F(s)\mu_J(q), \nu_F(s) + \nu_J(q) - \nu_F(s)\nu_J(q)\rangle | s \in S, q \in Q\} +$

$\{\langle (s,q), \mu_H(s)\mu_J(q), \nu_H(s)+\nu_J(q)-\nu_H(s)\nu_J(q)\rangle | s\in S, q\in Q\}$

$= \Big\{\langle (s,q), \mu_F(s)\mu_H(q)+\mu_H(s)\mu_J(q)-\mu_F(s){\mu_H}^2(s)\mu_J(q), \big(\nu_F(s)+\nu_J(q)-\nu_F(s)\nu_J(q)\big)\big(\nu_H(s)+\nu_J(q)-\nu_H(s)\nu_J(q)\big)\rangle | s\in S, q\in Q\Big\}$

Berdasarkan definisi 3.3 $(F+H)\times_3 J \subset (F\times_3 J)+(H\times_3 J)$, jika $\mu_{(F+H)\times_3 J} \leq \mu_{(F\times_3 J)+(H\times_3 J)}$ dan $\nu_{(F+H)\times_3 J} \geq \nu_{(F\times_3 J)+(H\times_3 J)}$.

$\mu_{(F\times_3 J)+(H\times_3 J)} - \mu_{(F+H)\times_3 J}$
$= \mu_F(s)\mu_H(s)\mu_J(q) - \mu_F(s){\mu_H}^2(s)\mu_J(q)$
$= \mu_F(s)\mu_H(s)\mu_J(q)\big(1-\mu_H(s)\big) \geq 0$

$\nu_{(F+H)\times_3 J} - \nu_{(F\times_3 J)+(H\times_3 J)}$
$= \nu_J(q) - \nu_F(s)\nu_J(q) - \nu_H(s)\nu_J(q) + \nu_F(s)\nu_H(s)\nu_J(q) - {\nu_J}^2(q) + \nu_F(s){\nu_J}^2(q) + \nu_H(s){\nu_J}^2(q) - \nu_F(s)\nu_H(s){\nu_J}^2(q)$
$= \nu_J(q)\big(1-\nu_F(s)\big)\big(1-\nu_H(s)\big)\big(1-\nu_J(q)\big) \geq 0$

Karena $\mu_{(F+H)\times_3 J} \leq \mu_{(F\times_3 J)+(H\times_3 J)}$ dan $\nu_{(F+H)\times_3 J} \geq \nu_{(F\times_3 J)+(H\times_3 J)}$, maka $(F+H)\times_3 J \subset (F\times_3 J)+(H\times_3 J)$. ■

Akan ditunjukkan $(F+H)\times_6 J \subset (F\times_6 J)+(H\times_6 J)$

$(F+H)\times_6 J$
$= \{\langle s, \mu_F(s)+\mu_H(s)-\mu_F(s)\mu_H(s), \nu_F(s)\nu_H(s)\rangle | s\in S\} \times_6 J$
$= \Big\{\langle (s,q), \frac{\mu_F(s)+\mu_H(s)-\mu_F(s)\mu_H(s)+\mu_J(q)}{2}, \frac{\nu_F(s)\nu_H(s)+\nu_J(q)}{2}\rangle | s\in S, q\in Q\Big\}$

$(F\times_6 J)+(H\times_6 J)$
$= \Big\{\langle (s,q), \frac{\mu_F(s)+\mu_J(q)}{2}, \frac{\nu_F(s)+\nu_J(q)}{2}\rangle | s\in S, q\in Q\Big\} + \Big\{\langle (s,q), \frac{\mu_H(s)+\mu_J(q)}{2}, \frac{\nu_H(s)+\nu_J(q)}{2}\rangle | s\in S, q\in Q\Big\}$
$= \Big\{\langle (s,q), \frac{\mu_F(s)+\mu_H(s)+2\mu_J(q)}{2} - \frac{\mu_F(s)\mu_H(s)+\mu_F(s)\mu_J(q)+\mu_H(s)\mu_J(q)+{\mu_J}^2(q)}{4}, \frac{\nu_F(s)\nu_H(s)+\nu_F(s)\nu_J(q)+\nu_H(s)\nu_J(q)+{\nu_J}^2(q)}{4}\rangle | s\in S, q\in Q\Big\}$

Berdasarkan definisi 3.3 $(F+H)\times_6 J \subset (F\times_6 J)+(H\times_6 J)$, jika $\mu_{(F+H)\times_6 J} \leq \mu_{(F\times_6 J)+(H\times_6 J)}$ dan $\nu_{(F+H)\times_6 J} \geq \nu_{(F\times_6 J)+(H\times_6 J)}$.

$\mu_{(F\times_6 J)+(H\times_6 J)} - \mu_{(F+H)\times_6 J}$
$= \frac{2\mu_J(q)+\mu_F(s)\mu_H(s)-\mu_F(s)\mu_J(q)-\mu_H(s)\mu_J(q)-{\mu_J}^2(q)}{4}$
$= \frac{1}{4}\Big(\mu_F(s)\mu_H(s)+\mu_J(q)\Big(2-\big(\mu_F(s)+\mu_H(s)+\mu_J(q)\big)\Big)\Big) \geq 0$

$\nu_{(F+H)\times_6 J} - \nu_{(F\times_6 J)+(H\times_6 J)}$
$= \frac{\nu_F(s)\nu_H(s)+2\nu_J(q)-\nu_F(s)\nu_J(q)-\nu_H(s)\nu_J(q)-{\nu_J}^2(q)}{4}$
$= \frac{1}{4}\Big(\nu_F(s)\nu_H(s)+\nu_J(q)\Big(2-\big(\nu_F(s)+\nu_H(s)+\nu_J(q)\big)\Big)\Big) \geq 0$

Karena $\mu_{(F+H)\times_6 J} \leq \mu_{(F\times_6 J)+(H\times_6 J)}$ dan $\nu_{(F+H)\times_6 J} \geq \nu_{(F\times_6 J)+(H\times_6 J)}$, maka $(F+H)\times_6 J \subset (F\times_6 J)+(H\times_6 J)$. ■

Jadi, $(F+H)\times J \subset (F\times J)+(H\times J)$ untuk $\times \in \{\times_1, \times_2, \times_3, \times_6\}$. ■

Bukti (2)-(6) analog dengan (1).

Proposisi 3.7

Jika F himpunan fuzzy intuisionistik di S and H himpunan fuzzy intuisionistik di Q, maka

(1) $\overline{(\bar{F}\times_1 \bar{J})} = F\times_1 J$
(2) $\overline{(\bar{F}\times_2 \bar{J})} = F\times_3 J$
(3) $\overline{(\bar{F}\times_3 \bar{J})} = F\times_2 J$
(4) $\overline{(\bar{F}\times_4 \bar{J})} = F\times_5 J$
(5) $\overline{(\bar{F}\times_5 \bar{J})} = F\times_4 J$
(6) $\overline{(\bar{F}\times_6 \bar{J})} = F\times_6 J$

Bukti :

(1) $\overline{(\bar{F}\times_1 \bar{J})}$
$= \overline{\{\langle (s,q), \nu_F(s)\nu_J(q), \mu_F(s)\mu_J(q)\rangle | s\in S, q\in Q\}}$
$= \{\langle (s,q), \mu_F(s)\mu_J(q), \nu_F(s)\nu_J(q)\rangle | s\in S, q\in Q\}$
$= F\times_1 J$ ■

Bukti (2)-(6) analog dengan (1).

4. PENUTUP

Simpulan

Berdasarkan pembahasan pada penelitian ini, maka diperoleh kesimpulan sebagai berikut.

(1) Misalkan S himpunan tak kosong. Jika F, H, dan J himpunan fuzzy intuisionistik di S, maka berlaku
    (a) Hukum komutatif terhadap operasi penjumlahan $(+)$ dan perkalian $(.)$
    (b) Hukum asosiatif terhadap operasi penjumlahan $(+)$ dan perkalian $(.)$
    (c) Hukum distributif operasi $(+)$, $(.)$, dan $(@)$ terhadap operasi $(\cap)$
    (d) Hukum distributif operasi $(+)$, $(.)$, dan $(@)$ terhadap operasi $(\cup)$

(e) Hukum idempoten pada operasi $(\cap)$, $(\cup)$, dan $(@)$
(f) $\overline{(\bar{F}+\bar{J})} = F.H$
(g) $\overline{(\bar{F}.\bar{J})} = F + H$
(h) $\overline{(\bar{F}@\bar{J})} = F@H$
(i) $F \underset{1,1}{\otimes} H = F@H$

(2) Misalkan S himpunan tak kosong. F disebut himpunan fuzzy intuisionistik tautologi di S, jika $\forall s \in S$, $\mu_F(s) \geq v_F(s)$

(3) Misalkan S, Q, P himpunan tak kosong. Jika F, H, J, dan L himpunan fuzzy intuisionistik, dimana F, H di S, J di Q, dan L di P, maka berlaku
(a) Hukum asosiatif terhadap operasi $\times$, untuk $\times \in (\times_1, \times_2, \times_3, \times_4, \times_5)$
(b) Hukum distributif operasi $\times$ terhadap operasi $(\cap)$ dan $(\cup)$, untuk $\times \in (\times_1, \times_2, \times_3, \times_4, \times_5, \times_6)$
(c) Hukum distributif operasi $\times$ terhadap operasi $(@)$, untuk $\times \in (\times_1, \times_2, \times_3, \times_6)$
(d) $\overline{(\bar{F} \times_1 \bar{J})} = F \times_1 J$
(e) $\overline{(\bar{F} \times_2 \bar{J})} = F \times_3 J$
(f) $\overline{(\bar{F} \times_3 \bar{J})} = F \times_2 J$
(g) $\overline{(\bar{F} \times_4 \bar{J})} = F \times_5 J$
(h) $\overline{(\bar{F} \times_5 \bar{J})} = F \times_4 J$
(i) $\overline{(\bar{F} \times_6 \bar{J})} = F \times_6 J$

Saran

Dalam penelitian ini telah dibahas tentang operasi dan relasi pada himpunan fuzzy intuisionistik dan sifat-sifatnya. Operasi dan relasi yang digunakan adalah operasi dan relasi sederhana. Penulis menantikan penelitian lebih lanjut yang berkaitan dengan operasi dan relasi pada himpunan fuzzy intuisionistik yang belum dibahas dalam penelitian ini.

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