

COMPUTATIONAL ANALYSIS OF TOPOLOGICAL INDICES ON POWER GRAPHS OF MODULO PRIME POWER GROUPS USING PYTHON

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Abstrak

Studi ini menggunakan Python untuk menghitung dan menganalisis tiga indeks seperti indeks Zagreb pertama, Wiener, dan Gutman pada graf pangkat dari grup modulo pangkat bilangan prima. Memacu pada rumus-rumus yang telah dikembangkan oleh penelitian sebelumnya. Dengan menggunakan pustaka Python seperti NetworkX, Matplotlib, dan Tkinter, proses perhitungan menjadi lebih efisien serta memungkinkan visualisasi terhadap variasi indeks yang didasarkan pada perubahan nilai bilangan prima p dan eksponen k . Hasil menunjukkan bahwa nilai ketiga indeks meningkat seiring bertambahnya nilai p dan k , yang mencerminkan peningkatan kompleksitas struktur graf. Pada nilai p dan k yang besar, visualisasi graf terlalu kompleks yang menyebabkan visualisasi graf menjadi kurang jelas. Pendekatan ini terbukti efektif dalam mendukung eksplorasi struktur aljabar secara visual dan kuantitatif.

Kata Kunci: Zagreb, Wiener, Gutman, Modulo prima, Python.

Abstract

This study uses Python to calculate and analyze three indices such as the first Zagreb, Wiener, and Gutman indices on the rank graph of the group modulo the power of a prime number. It relies on formulas that have been developed by previous research. By using Python libraries such as NetworkX, Matplotlib, and Tkinter, the calculation process becomes more efficient and allows visualization of index variations based on changes in the values of prime p and exponent k . The results show that the values of the three indices increase as the values of p and k increase, reflecting the increasing complexity of the graph structure. At large values of p and k , the graph visualization is too complex which causes the graph visualization to be less clear. This approach proves to be effective in supporting visual and quantitative exploration of algebraic structures.

Keywords: Zagreb, Wiener, Gutman, Modulo prime, Python.

INTRODUCTION

Rank graphs are a type of graph developed by groups, where a group has many representations in graphs. In mathematics, group theory and graph theory are often combined, and the image of this combination can show a group. A picture that contains certain information, if interpreted logically

and correctly, is called a graph. Graph theory is very useful in mathematics, especially in algebraic structures, where graphs are used to represent a group. In recent years, graphs are used to visualize algebraic structures such as groups or rings. One of the interesting groups to study is the group modulo integer with order in the form of powers of prime numbers, because it has unique structural properties

in algebra. This research is based on the research of Evi et al. (2023), who investigated topological indices such as First Zagreb, Wiener, and Gutman on the power graphs of various groups. The analysis was performed using the Python programming language with the NetworkX, math, itertools, and Matplotlib libraries. Calculating and visualizing the indexes directly uses a Tkinter-based interactive interface. This method supports effective analysis and research in graph theory and computational algebra.

TEORY REVIEW

In recent years, graphs are used to visualize algebraic structures such as groups or rings. Some graphs that represent groups include the coprime graph of the dihedral group and the quaternion group on the dihedral group. Ma introduced coprime graphs, while Mansoori introduced non-coprime graphs, which also study modulo integers and dihedral groups. In 2013, Kalarev defined the directed power graph of a semigroup, and many recent studies have dealt with the power graph of a group. The research of Asmarani et al. discussed the power graph of the dihedral group with $n = p^m$, where p is a prime number and k is a natural number.

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The coprime graph is a representation of the group G , where the vertices consist of all elements of the group. Two vertices $a, b \in G$ are adjacent if their order is not relatively prime, or $\gcd(|a|, |b|) \neq 1$. A study by Abawajy et al. in 2013 addressed the power graph of a group, where the vertices consist of all elements of the group, and two distinct vertices $a, b \in G$, are adjacent if one is a power of the other, i.e. $a = b^x$ or $b = a^y$ for some $x, y \in \mathbb{N}$. In 2022,

power graphs were created for the dihedral group and the integer group modulo (\mathbb{Z}_n) with prime order.

To understand the complexity and nature of the relationships between elements in the power graphs of such groups, topological indices such as First Zagreb, Wiener, and Gutman are used as quantitative measures. The topological index, first introduced by Wiener in 1947, is a numerical value used to describe molecular graphs and has been widely used in chemistry.

Theorem 1.1 (Evi et al.(2023)). Let $G = \mathbb{Z}_{p^k}$ cyclic group of order p^k , with p prime numbers and $k \in \mathbb{N}$. Let $\Gamma(G)$ is the power graph of group G . Then the first Zagreb index of the graph $\Gamma(G)$ is given by:

$$M_1(\Gamma(G)) = \sum_{v \in V(\Gamma(G))} \deg(v)^2 \quad (1)$$

Theorem 1.2 (Evi et al.(2023)). Let $G = \mathbb{Z}_{p^k}$ cyclic group of order p^k , with p prime numbers and $k \in \mathbb{N}$. Let $\Gamma(G)$ is the power graph of group G . Then the Wiener index of the graph $\Gamma(G)$ is given by:

$$W(\Gamma(G)) = \sum_{\substack{u, v \in V(\Gamma(G)) \\ u \neq v}} d(u, v) \quad (2)$$

Theorem 1.3 (Evi et al.(2023)). Let $G = \mathbb{Z}_{p^k}$ cyclic group of order p^k , with p prime numbers and $k \in \mathbb{N}$. Let $\Gamma(G)$ is the power graph of group G . Then the Gutman index of the graph $\Gamma(G)$ is given by:

$$\begin{aligned} & Gut(\Gamma(G)) \\ &= \sum_{\{u, v\} \in V(\Gamma(G))} d(u, v) \cdot \deg(u) \cdot \deg(v) \end{aligned} \quad (3)$$

METHODS

To optimize the analysis of topological indices such as the first Zagreb index, Wiener index, and Gutman index on power group graphs modulo integers, existing theoretical models can be used. The theoretical approach developed by previous researchers, which refers to the power group modulo p , where p is a prime number and the exponent k , $k \in \mathbb{N}$, provides formulas that allow the calculation of such indices directly without the need to explore complex graphs. This method reduces the amount of time required to calculate the index for a given prime value of p with k . The model utilizes the structural characteristics of groups and is communicated in the form of compact arithmetic operations.

Python is the programming language used to implement these theoretical formulas in this study. Relevant topological indices, such as the first Zagreb index, Wiener index, and Gutman index, are calculated directly using the formulas that have been derived according to the existing theorems applied. The algorithm used accepts input in the form of prime power p^k , verifies the prime number p and ensures $p \geq 2$ and generates the corresponding graph for the power group modulo p using the NetworkX library. The NumPy library is used to perform numerical operations, and the Matplotlib library is used to create data visualizations. Calculations and code implementation were performed using the Spyder platform. Furthermore, line plot was used to see how the index variation relates to the group order.

RESULTS AND DISCUSSION

Algorithm 1 (Main Group Modulo Prime Powers): This algorithm aims to construct the power graph of the multiplication group modulo p^k , where p is a prime number and $k \geq 1$. Next, the algorithm verifies the validity of the input by ensuring that p is a prime number greater than 2 by calling the ISPRIME(P) function (see Algorithm 5) and k is a positive integer. The algorithm forms the set of group elements \mathbb{Z}_{p^k} after successful validation (see Algorithm 2). Creates a graph based on the element lifting relation and calculates the three indices that show the structure and complexity of the connections (see Algorithm 4). The algorithm generates an error output if the input is invalid.

Algorithm 1 Main Group Modulo Prime Powers

```

1: Input:  $p, k$  (for grup modulo pangkat prima)
2: Verify  $p$  and  $k$  are integers
3: Check if  $p$  is prime using ISPRIME( $p$ ) (Algorithm 5)
4: Check if  $p > 2$ 
5: Check if  $k \geq 1$ 
6: if validation passes then
7:   Generate modulo grup elements using Algorithm (modulo grup)
8:   Construct Power Graph  $G$  (Algorithm 3)
9:   Calculate first Zagreb index (Algorithm 4)
10:  Calculate Wiener index using (Algorithm 4)
11:  Calculate Gutman index using (Algorithm 4)
12:  (Optional) Visualize  $G$ 
13:  return  $G$ , indices

```

```

14: else
15:   Report error: "Invalid inputs for  $p$  and  $k$ "
16: Output: First Zagreb, Wiener and Gutman index values or error message

```

Algorithm 2 (Modulo Group): This algorithm is used to construct the set of elements of the multiplication group modulo p^k , where p is a prime number and k is a positive integer. First, the algorithm calculates the modulus value of p^k and then checks all integers from 1 to p^{k-1} . Any number x that is prime relative to p^k i.e., $\gcd(x, p^k) = 1$ is included in the list. The result is a list of elements of the group \mathbb{Z}_{p^k} . The set of numbers that have the ability to form a group under the multiplication operation modulo p^k .

Algorithm 2 Modulo Group

```

1: Input:  $p, k$  (for grup modulo pangkat prima)
2:  $\text{mod} = p^k$ 
3:  $\text{results} = \text{empty list}$ 
4: for  $x$  from 1 to  $\text{mod} - 1$  do
5:   if  $\gcd(x, \text{mod}) == 1$  then
6:     add  $x$  to result
7:   end if
8: end for
9: return result
10: Output: elements of the multiplication group modulo  $p^k$ 

```

Algorithm 3 (Power Graph): This algorithm generates the rank graph of the multiplication group modulo p^k . Where the vertices are elements of \mathbb{Z}_{p^k} , i.e. integers that are prime relative to p^k . The algorithm adds an edge between every pair of elements (u, v) if one of them is a power product of the other in modulo p^k . The end result is an undirected graph that shows the connected structure within the group based on the lifting relation.

Algorithm 3 power graph

```

1: Input: prime  $p$ , integer  $k \geq 1$ 
2:  $\text{mod} = p^k$ 
3:  $\text{elements} = \text{modulo\_group}(p, k)$ 
4: create empty graph  $G$ 
5: add nodes from elements to  $G$ 
6: for each pair  $(u, v)$  in elements  $\times$  elements
7:   if  $u \neq v$  and ( $\text{is\_power}(u, v, \text{mod})$  or  $\text{is\_power}(v, u, \text{mod})$ ) then
8:     add edge  $(u, v)$  to  $G$ 
9:   end if
10: end for
11: return  $G$ 
12: Output: Power Graph  $G$  with nodes of multiplication group elements modulo  $p^k$  and edge connecting the nodes

```

Algorithm 4(Compute Indices): This algorithm aims to compute three topological indices: the first Zagreb index, Wiener index, and Gutman index, of the power graph formed based on the multiplication group modulo p^k (see Algorithm 2). Where p is a prime number and k is a positive integer. After generating the elements of the group \mathbb{Z}_{p^k} , the algorithm uses the lifting relation between the elements to construct the power graph. Next, the algorithm calculates three indices namely the first Zagreb index as the sum of squared degrees of all vertices, Wiener index as the sum of shortest distances between all pairs of unordered vertices and Gutman index as the sum of the product of degrees and distances between all pairs of vertices. These three indices provide important structural information about the graph and the connectedness characteristics of the group elements.

Algorithm 4 Compute Indices

```

1: Input:  $p, k$ , prime  $p$  and  $k$  integer  $\geq 1$ 
2: Generate elements of modulo group  $\mathbb{Z}_{p^k}$  (Algoritma 2)
3: Construct power graph  $G$  (Algoritma 3)
4: Calculate First Zagreb Index:
5:    $\text{first\_zagreb\_index} \leftarrow \sum (\deg(v))^2$  for all  $v \in V(G)$ 
6: Calculate Wiener Index:
7:    $\text{wiener\_index} \leftarrow \sum d(u, v)$  for all unordered pairs  $(u, v)$  in  $G$ 
8: Calculate Gutman Index :
9:    $\text{gutman\_index} \leftarrow \sum \deg(u) \times \deg(v) \times d(u, v)$  for all unordered pairs  $(u, v)$  in  $G$ 
10: Output:  $\text{first\_zagreb\_index}$ ,  $\text{wiener\_index}$ ,  $\text{gutman\_index}$ 

```

Algorithm 5(Is Prime): This algorithm is used to determine whether an integer n is prime. The algorithm immediately returns a false value if n is less than 2, as prime numbers start from 2. Then, the algorithm checks the divisors from 2 to $\sqrt{n} + 1$. If a divisor is found that completely divides n , then n is not prime and the algorithm returns a false value. If no divisor is found, then n is prime and the algorithm returns true. This algorithm is effective because it only checks factors up to \sqrt{n} .

Algorithm 5 Is Prime

```

1: Input: Integer  $n$ 
2: if  $n < 2$  then
3:   return false
4: end if
5: for  $i$  from 2 to  $\sqrt{n} + 1$ 
6:   if  $n \% i == 0$  then
7:     return false

```

```

8:   end if
9: end for
10: return true

```

Calculation of Topological Indices

When the user has run the syntax and wants to try calculating the index, it will look like this:

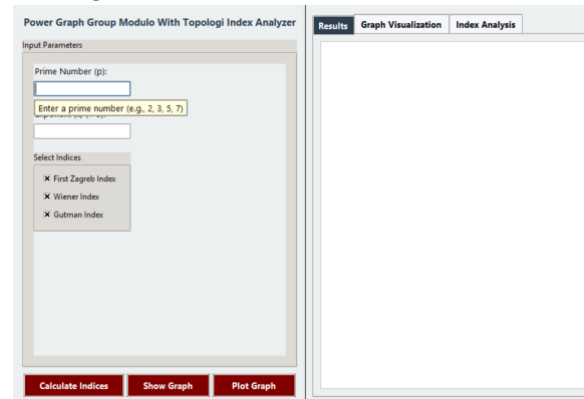


Figure 1. Initial View of the Application

The first step to do that is that the user is asked to enter a prime number p that is greater than equal to 2, for example 2,3,5,7 and so on. Next, enter the value of $k \geq 1$, where k is a member of N and also the user is asked to select the index that they want to calculate. For example, we entered the value of $p = 2$, the value of $k = 3$ and selected the three indices to be calculated. After the values are entered, the user can continue to click the index calculation button. The results of the index calculation can be seen in the result menu. Then the result display will be like this:

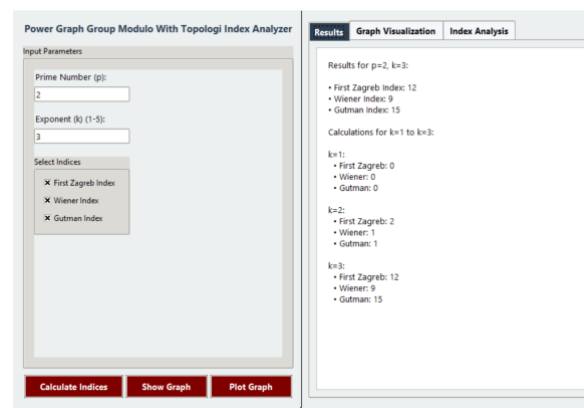


Figure 2. Calculation Result First Zagreb, Wiener and Gutman Indices

For the additional index calculation, the prime number value p used is $\{2,3,5,7,11\}$, with different restrictions on the value of k . The purpose of limiting both the prime number and the value of k prevents

excessive index calculation. The index calculation is summarized in the following table:

Table 1. Computed Values of the First Zagreb, Wiener and Gutman Indices

p	k	First Zagreb Index	Wiener Index	Gutman Index
2	1	0	0	0
2	2	2	1	1
2	3	12	9	15
3	1	2	1	1
3	2	116	17	304
3	3	4.034	173	36.253
5	2	5.896	210	58.716
7	2	48.254	1.025	1.064.863
11	2	1.021.376	6.769	58.200.994

The results show the calculation of the first zagreb index, wiener index and gutman index for different primes p and k values. The results show an increase in the values of the first zagreb index, wiener index and gutman index for each increasing p value and k value. These results will affect the shape of the graph visualization.

Generate the Graph Modulo Prime Powers

When the user has run the syntax and wants to try visualizing the index, the first step to do that is that the user is asked to enter a prime number p greater than 2. Next the user selects the value of k , where k is a member of N and selects the index that he wants to visualize. For example, we enter the value of $p = 2$, the value of $k = 3$. After the value is entered, the user can continue to click the show graph button. The index visualization results can be seen in the graph visualization menu. Then the index analysis display will be like this:

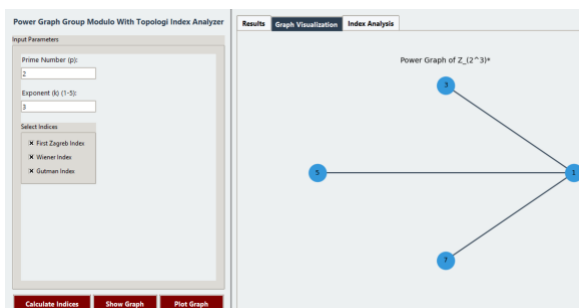


Figure 3. Visualization Graph For \mathbb{Z}_p^k , $p = 2$ and $k = 3$

Creating Comparison Charts for the Three Indices

First Zagreb Index Line Graph

When the user has run the syntax and wants to try index analysis for the first Zagreb index, the first step

to do that is that the user is asked to enter a prime number p greater than 2. Next the user enters the value of $k \geq 1$, where k is a member of N and selects the index to analyze, such as the first Zagreb index. After the value is entered, the user can continue to click the index analysis button on the right. The index analysis results can be seen in the index analysis menu. Then the index analysis display will be like this:

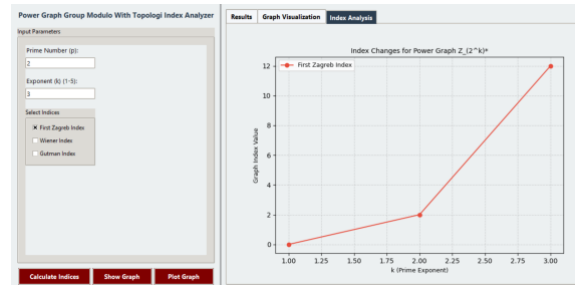


Figure 4. First Zagreb Index Line Graph For \mathbb{Z}_p^k , $p = 2$ and $k = 3$

Wiener Index Line Graph

When the user has run the syntax and wants to try index analysis for the Wiener index, the first step to do that is that the user is asked to enter a prime number p greater than 2. Next the user enters the value of $k \geq 1$, where k is a member of N and selects the index to analyze, such as the Wiener index. After the value is entered, the user can continue to click the index analysis button on the right. The index analysis results can be seen in the index analysis menu. Then the index analysis display will be like this:

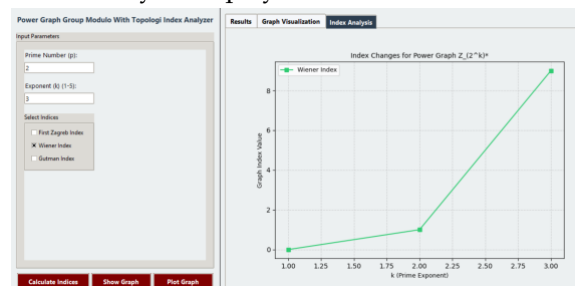


Figure 5. Wiener Index Line Graph For \mathbb{Z}_p^k , $p = 2$ and $k = 3$

Gutman Index Line Graph

When the user has run the syntax and wants to try index analysis for the Gutman index, the first step to do that is that the user is asked to enter a prime number p greater than 2. Next the user enters the value of $k \geq 1$, where k is a member of N and selects the index to analyze, such as the Gutman index. After the value is entered, the user can continue to click the

index analysis button on the right. The index analysis results can be seen in the index analysis menu. Then the index analysis display will be like this:

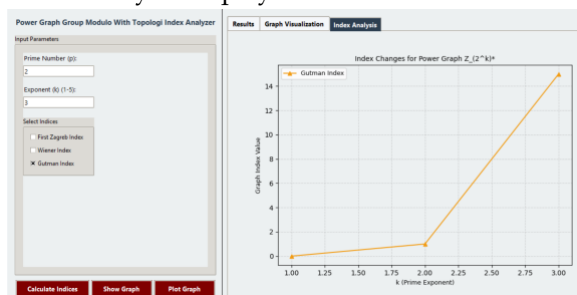


Figure 6. Gutman Index Line Graph For \mathbb{Z}_{p^k} , $p = 2$ and $k = 3$

Based on the three index graphs above, the horizontal axis shows the exponent k , while the vertical axis shows the calculated index value. The three graphs above show that the index increases as the value of p and the value of k increases but at different rates. The Gutman index will increase the steepest compared to the other indexes, indicating that the relationship between vertices in the graph is more complex for larger k . Overall, this visualization indicates that for larger values of k , the graph structure becomes more complex, especially in the distribution of vertex degrees and the relationships between vertices.

CONCLUSION

Using Python, this research develops a computational approach to compute and analyze the first Zagreb, Wiener, and Gutman indices on group power graphs modulo the power of prime numbers. The calculation process becomes more efficient, and visualization and analysis of the index variation with the help of libraries such as NetworkX, Matplotlib, and Tkinter. The results show that the index value increases as the value of p and the value of k increase, reflecting the complexity of the graph structure. However, for large primes p and exponents k , the visualization becomes less clear. Overall, this approach is effective for exploring algebraic structures visually and quantitatively.

SUGGESION

This research provides several suggestions for further development. First, it is necessary to develop a more efficient method to handle the values of and in terms of algorithms and large visualization so that

the graph remains clear despite its high complexity. Second, from the perspective of theory, in order to find more diverse topological index patterns, the research can be extended to other algebraic groups, such as dihedral or non-abelian groups. In addition, the development of general formulas for Zagreb, Wiener, and Gutman indices for a particular class.

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