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OPERATOR KEBUTUHAN DAN OPERATOR KEMUNGKINAN PADA HIMPUNAN FUZZY INTUISIONISTIK

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Abstrak

Himpunan fuzzy intuisionistik merupakan perluasan dari himpunan fuzzy. memiliki fungsi keanggotaan dan nonkeanggotaan. Pada himpunan fuzzy intuisionistik terdapat beberapa operator terkait, Diantaranya adalah operator kebutuhan dan operator kemungkinan. Operator ini merupakan kemiripan dari logika modal intuisionistik. Logika modal intuisionistik adalah perkembangan dari logika intuisionistik dan operator intensional yang menggabungkan dua bentuk logika. Dua operator ini memiliki sifat keterkaitan satu sama lain. Secara sederhana akan dibahas sifat-sifat operator kebutuhan dan operator kemungkinan pada himpunan fuzzy intuisionistik.

Kata kunci: Himpunan Fuzzy Intuisionistik, Operator Kebutuhan, Operator Kemungkinan.

Abstract

Intuitionistic fuzzy set is extension of fuzzy set which has membership and non membership functions. In the intuitionistic fuzzy set there are several related operators, Among them are operator necessity and possibility. This operator is a resemblance of intuitionistic modal logic. Intuitionistic modal logic is the development of intuitionistic logic and operator intentional that combines two forms of logic. These two operators have an interrelated nature. In a simple way, It will discuss the properties of the operator necessity and operator possibility on intuitionistic fuzzy sets

Keywords : Intuitionistic Fuzzy Sets; Operator Necessity; Operator Possibility

1. PENDAHULUAN

Himpunan fuzzy pertama kali dikemukakan oleh Zadeh pada tahun 1965. Himpunan fuzzy adalah perkembangan dari himpunan klasik. Karena dalam himpunan klasik memiliki nilai benar atau salah secara tegas dan dalam kehidupaan nyata sangatlah tidak cocok, Sedangkan dalam himpunan fuzzy memiliki nilai samar yang dapat merepresentasikan keadaan dalam dunia nyata. Pada himpunan klasik fungsi keanggotaan suatu elemen dinyatakaan dengan nilai nol atau satu, Sedangkan himpunan fuzzy fungsi keanggotaan suatu elemen pada himpunan fuzzy dinyatakan dengan selang nilai tertutup [0,1]. (Ziemmermann, 1996). Himpunan fuzzy dalam berjalannya waktu mengalami perkembangan. Salah satunya yaitu himpunan fuzzy intuisionistik.

Himpunan fuzzy intisionistik pertama kali dikemukakan oleh Krasimir T. Atanassov pada tahun 1986. Terdapat penambahan satu komponen perluasan yang terdapat pada satu elemen, Yaitu fungsi nonkeanggotaan. Fungsi nonkeanggotaan ini dapat disebut nilai keragu-raguan. Fungsi keanggotaan dan non keanggotaan terdapat pada selang [0,1]. Pada himpunan fuzzy intuisionistik yang telah dikemukakan, Terdapat beberapa operator terkait. Diantaranya adalah operator Topologi, Level, Indentifikasi dan unary, Termasuk juga operator kebutuhan dan operator kemungkinan.

Operator kebutuhan dan operator kemungkinan dibahas oleh T. Atanassov pada tahun 1986. Operator ini merupakan kemiripan dari logika modal intuisionistik. Logika modal intuisionistik merupakan perkembangan dari logika intuisionistik dengan operator intensional yang menggabungkan dua bentuk logika. Logika modal intuisionistik dapat juga di artikan sebagai penalaran tentang modalitas, Menyimpulkan dari premis modal bahwa beberapa kesimpulan modal adalah valid (K.Simpson, 1994). Pada dasarnya untuk dapat menyimpulkan suatu permasalahan secara valid diperlukan properti atau bukti. Dalam karya Aristoteles tentang logika modalitas yang mempertimbangkan secara logis tentang kebutuhan dan kemungkinan, Oleh karena itu untuk dapat dibuktikan secara fisik maka ditambahkan bahasa kedalam bentuk dua operator ‘□’ kebutuhan dan ‘◇’ kemungkinan.

Pada mei 1983 ditemukan himpunan baru bahwa operator pada himpunan fuzzy tidak ada artinya karena berkurang menjadi nilai identitas. Hal ini menunjukkan fakta bahwa intuisionistik adalah ekstensi atau peluasan yang tepat dari himpunan fuzzy (Atanassov, 1999). Oleh sebab itu manfaat operator kebutuhan dan operator kemungkinan pada himpunan fuzzy intuisionistik adalah untuk mengetahui antara apa yang diperlukan dan apa yang tidak mungkin diperlukan. Operator ini juga dapat diartikan sebagai bukti atau properti yang diperlukan untuk membuktikan sesuatu tentang kebenaran.

2. KAJIAN TEORI

2.1 Himpunan Fuzzy

Definisi 1

Misalkan E kumpulan objek dinotasikan secara umum dengan λ, Himpunan fuzzy $\widetilde{\mathcal{K}}$ di E adalah :

$$\widetilde{\mathcal{K}} = \{(\lambda, \mu_{\widetilde{\mathcal{K}}}(\lambda)) \mid \lambda \in E\} \quad (1)$$

$\mu_{\mathcal{K}}(\lambda)$ disebut derajat keanggotaan λ di $\widetilde{\mathcal{K}}$ dengan $\mu_{\widetilde{\mathcal{K}}}(\lambda) : E \to [0,1]$. Elemen dengan derajat keanggotaan nol pada $\widetilde{\mathcal{K}}$ tidak ditulis (Ziemmermann, 1996).

2.2 Himpunan Fuzzy Intuisionistik

Definisi 2

Misalkan E himpunan semesta tak kosong. Himpunan fuzzy intuisionistik $\widetilde{\mathcal{K}}$ di E adalah:

$$\widetilde{\mathcal{K}} = \{(\lambda, \mu_{\widetilde{\mathcal{K}}}(\lambda), v_{\widetilde{\mathcal{K}}}(\lambda) \mid \lambda \in E)\} \quad (2)$$

Dengan $\mu_{\widetilde{\mathcal{K}}} : E \to [0,1]$ dan $v_{\widetilde{\mathcal{K}}} : E \to [0,1]$ adalah tingkat keanggotaan dan non keanggotaan dari E ke $\widetilde{\mathcal{K}}$, Dan memenuhi syarat: $0 \leq \mu_{\widetilde{\mathcal{K}}}(\lambda) + v_{\widetilde{\mathcal{K}}}(\lambda) \leq 1$. Untuk semua $\lambda \in E$, $\pi_{\widetilde{\mathcal{K}}}(\lambda) = 1 - \mu_{\widetilde{\mathcal{K}}}(\lambda) - v_{\widetilde{\mathcal{K}}}(\lambda)$, $\pi_{\widetilde{\mathcal{K}}}(\lambda)$ disebut tingkat ketidakpastian dari λ ke $\widetilde{\mathcal{K}}$ (Atanassov, 1999).

2.3 Operasi pada Himpunan Fuzzy Intuisionistik

Definisi 3

Misalkan $\widetilde{\mathcal{K}}$ dan $\tilde{\mathcal{L}}$ dua himpunan fuzzy intuisionistik di E.

$$\widetilde{\mathcal{K}} \cap \tilde{\mathcal{L}} = \{\langle \lambda, \min(\mu_{\widetilde{\mathcal{K}}}(\lambda), \mu_{\tilde{\mathcal{L}}}(\lambda)), \max(v_{\widetilde{\mathcal{K}}}(\lambda), v_{\tilde{\mathcal{L}}}(\lambda)\rangle \mid \lambda \in E\} \quad (3)$$

$$\widetilde{\mathcal{K}} \cup \tilde{\mathcal{L}} = \{\langle \lambda, \max(\mu_{\widetilde{\mathcal{K}}}(\lambda), \mu_{\tilde{\mathcal{L}}}(\lambda)), \min(v_{\widetilde{\mathcal{K}}}(\lambda), v_{\tilde{\mathcal{L}}}(\lambda)\rangle \mid \lambda \in E\} \quad (4)$$

$$\widetilde{\mathcal{K}} + \tilde{\mathcal{L}} = \{\langle \lambda, \mu_{\widetilde{\mathcal{K}}}(\lambda) + \mu_{\tilde{\mathcal{L}}}(\lambda) - \mu_{\widetilde{\mathcal{K}}}(\lambda).\mu_{\tilde{\mathcal{L}}}(\lambda), v_{\widetilde{\mathcal{K}}}(\lambda).v_{\tilde{\mathcal{L}}}(\lambda)\rangle \mid \lambda \in E\} \quad (5)$$

$$\widetilde{\mathcal{K}}.\tilde{\mathcal{L}} = \{\langle \lambda, \mu_{\widetilde{\mathcal{K}}}(\lambda).\mu_{\tilde{\mathcal{L}}}(\lambda), v_{\widetilde{\mathcal{K}}}(\lambda) + v_{\tilde{\mathcal{L}}}(\lambda) - v_{\widetilde{\mathcal{K}}}(\lambda).v_{\tilde{\mathcal{L}}}(\lambda)\rangle \mid \lambda \in E\} \quad (6)$$

$$\overline{\mathcal{K}} = \{(\lambda, v_{\mathcal{K}}(\lambda), \mu_{\mathcal{K}}(\lambda) \mid \lambda \in E)\} \quad (7)$$

(Atanassov, 2012)

2.6 Relasi pada Himpunan Fuzzy Intuisionistik

Definisi 4

Misalkan $\widetilde{\mathcal{K}}$ dan $\tilde{\mathcal{L}}$ dua himpunan fuzzy intuisionistik di E. $\widetilde{\mathcal{K}}$ subset $\tilde{\mathcal{L}}$ didefinisikan sebagai :

$$\widetilde{\mathcal{K}} \subset \tilde{\mathcal{L}} \text{ Jika } (\forall \lambda \in E)\big(\mu_{\mathcal{K}}(\lambda) \leq \mu_{\mathcal{L}}(\lambda) \wedge v_{\mathcal{K}}(\lambda) \geq v_{\mathcal{L}}(\lambda)\big) \quad (8)$$

(Atanassov, 2012)

Definisi 5

Misalkan E_1 dan E_2 himpunan semesta. $\mathcal{K}$ dan $\mathcal{L}$ himpunan fuzzy intuisionistik.

$$\mathcal{K} = \{(\lambda, \mu_{\mathcal{K}}(\lambda), v_{\mathcal{K}}(\lambda) \mid \lambda \in E_1)\}$$
$$\mathcal{L} = \{(\gamma, \mu_{\mathcal{L}}(\gamma), v_{\mathcal{L}}(\gamma) \mid \gamma \in E_2)\}$$

Maka dapat di definisikan sebagai :

Kartesian produk "$\times_1$"

$$(\mathcal{K} \times_1 \mathcal{L}) = \{\langle\langle \lambda, \gamma\rangle, \mu_{\mathcal{K}}(\lambda), \mu_{\mathcal{L}}(\gamma), v_{\mathcal{K}}(\lambda), v_{\mathcal{L}}(\gamma)\rangle \mid \lambda \in E_1 \,\&\, \gamma \in E_2\} \quad (9)$$

(Atanassov, 1999)

3. HASIL DAN PEMBAHASAN

3.1 Operator Kebutuhan dan Operator Kemungkinan Pada Himpunan Fuzzy Intuisionistik

Definisi 6

Setiap himpunan fuzzy intuitionistik $\mathcal{K}$, Operator kebutuhan dan kemungkinan didefinisikan sebagai :

$$\square\, \mathcal{K} = \{\langle \lambda, \mu_{\mathcal{K}}(\lambda), 1 - \mu_{\mathcal{K}}(\lambda)\rangle \mid \lambda \in E\} \quad (10)$$

$$\Diamond \mathcal{K} = \{\langle \lambda, 1 - v_{\mathcal{K}}(\lambda), v_{\mathcal{K}}(\lambda)\rangle \mid \lambda \in E\} \quad (11)$$

Proposisi 1

Jika $\mathcal{K}$ himpunan fuzzy intuitionistik di E, Maka

1) $\overline{\square\, \overline{\mathcal{K}}} = \Diamond \mathcal{K}$
2) $\overline{\Diamond \overline{\mathcal{K}}} = \square\, \mathcal{K}$
3) $\square\, \mathcal{K} \subset \mathcal{K} \subset \Diamond \mathcal{K}$
4) $\square\square\, \mathcal{K} = \square\, \mathcal{K}$
5) $\square\, \Diamond \mathcal{K} = \Diamond \mathcal{K}$
6) $\Diamond\, \square\, \mathcal{K} = \square\, \mathcal{K}$
7) $\Diamond\Diamond \mathcal{K} = \Diamond \mathcal{K}$

Bukti :

1) $\overline{\square\, \overline{\mathcal{K}}} = \overline{\square\, \{\langle \lambda, v_{\mathcal{K}}(\lambda), \mu_{\mathcal{K}}(\lambda)\rangle \mid \lambda \in E\}}$
   $= \overline{\{\langle \lambda, v_{\mathcal{K}}(\lambda), 1 - v_{\mathcal{K}}(\lambda)\rangle \mid \lambda \in E\}}$
   $= \{\langle \lambda, 1 - v_{\mathcal{K}}(\lambda), v_{\mathcal{K}}(\lambda)\rangle \mid \lambda \in E\}$
   $= \Diamond \mathcal{K}$

2) $\overline{\Diamond \overline{\mathcal{K}}} = \overline{\Diamond \{\langle \lambda, v_{\mathcal{K}}(\lambda), \mu_{\mathcal{K}}(\lambda)\rangle \mid \lambda \in E\}}$
   $= \overline{\{\langle \lambda, 1 - \mu_{\mathcal{K}}(\lambda), \mu_{\mathcal{K}}(\lambda)\rangle \mid \lambda \in E\}}$
   $= \{\langle \lambda, \mu_{\mathcal{K}}(\lambda), 1 - \mu_{\mathcal{K}}(\lambda)\rangle \mid \lambda \in E\}$
   $= \square\, \mathcal{K}$

3) $\square\, \mathcal{K} \subset \mathcal{K} \subset \Diamond \mathcal{K}$

   Menurut definisi

   $\widetilde{\mathcal{K}} \subset \tilde{\mathcal{L}}$ Jika $(\forall \lambda \in E)(\mu_{\mathcal{K}}(\lambda) \leq \mu_{\mathcal{L}}(\lambda) \wedge v_{\mathcal{K}}(\lambda) \geq v_{\mathcal{L}}(\lambda))$

   Misalkan $\square\, \mathcal{K}$ adalah $\mathcal{K}$ dan $\mathcal{K}$ adalah $\mathcal{L}$, Maka :

   $$\mu_{\mathcal{K}}(\lambda) \leq \mu_{\mathcal{L}}(\lambda)$$

$$1-\mu_{\mathcal{K}}(\lambda) \geq v_{\mathcal{K}}(\lambda)$$

Maka $\square\, \mathcal{K} \subset \mathcal{K}$

Misalkan $\mathcal{K}$ adalah $\mathcal{K}$ dan $\lozenge\mathcal{K}$ adalah $\mathcal{L}$, Maka :

$$\mu_{\mathcal{K}}(\lambda) \leq 1-v_{\mathcal{K}}(\lambda)$$
$$v_{\mathcal{K}}(\lambda) \geq v_{\mathcal{K}}(\lambda)$$

Maka $\mathcal{K} \subset \lozenge\mathcal{K}$

Terbukti bahwa $\square\, \mathcal{K} \subset \mathcal{K} \subset \lozenge\mathcal{K}$

4) $\square\square\, \mathcal{K} = \square\, \mathcal{K}$
$= \square\square \{\langle \lambda, \mu_{\mathcal{K}}(\lambda), v_{\mathcal{K}}(\lambda)\rangle \mid \lambda \in E\}$
$= \square \{\langle \lambda, \mu_{\mathcal{K}}(\lambda), 1-\mu_{\mathcal{K}}(\lambda)\rangle \mid \lambda \in E\}$
$= \{\langle \lambda, \mu_{\mathcal{K}}(\lambda), 1-\mu_{\mathcal{K}}(\lambda)\rangle \mid \lambda \in E\}$
$= \square\, \mathcal{K}$

5) $\square \lozenge\mathcal{K} = \lozenge\mathcal{K}$
$= \square \lozenge\{\langle \lambda, \mu_{\mathcal{K}}(\lambda), v_{\mathcal{K}}(\lambda)\rangle \mid \lambda \in E\}$
$= \square \{\langle \lambda, 1-v_{\mathcal{K}}(\lambda), v_{\mathcal{K}}(\lambda)\rangle \mid \lambda \in E\}$
$=\{\langle \lambda, 1-v_{\mathcal{K}}(\lambda), v_{\mathcal{K}}(\lambda)\rangle \mid \lambda \in E\}$
$= \lozenge\mathcal{K}$

6) $\lozenge \square\, \mathcal{K} = \square\, \mathcal{K}$
$= \lozenge \square \{\langle \lambda, \mu_{\mathcal{K}}(\lambda), v_{\mathcal{K}}(\lambda)\rangle \mid \lambda \in E\}$
$= \lozenge\{\langle \lambda, \mu_{\mathcal{K}}(\lambda), 1-\mu_{\mathcal{K}}(\lambda)\rangle \mid \lambda \in E\}$
$= \{\langle \lambda, \mu_{\mathcal{K}}(\lambda), 1-\mu_{\mathcal{K}}(\lambda)\rangle \mid \lambda \in E\}$
$= \square\, \mathcal{K}$

7) $\lozenge\lozenge\mathcal{K} = \lozenge\lozenge\{\langle \lambda, \mu_{\mathcal{K}}(\lambda), v_{\mathcal{K}}(\lambda)\rangle \mid \lambda \in E\}$
$= \lozenge\{\langle \lambda, 1-v_{\mathcal{K}}(\lambda), v_{\mathcal{K}}(\lambda)\rangle \mid \lambda \in E\}$
$= \{\langle \lambda, 1-v_{\mathcal{K}}(\lambda), v_{\mathcal{K}}(\lambda)\rangle \mid \lambda \in E\}$
$= \lozenge\mathcal{K}$

Teorema 1

Jika $\mathcal{K}$ adalah himpunan fuzzy intuisionistik di E, maka

1. $\square\, (\mathcal{K} \cap \mathcal{L}) = \square\, \mathcal{K} \cap \square\, \mathcal{L}$
2. $\square\, (\mathcal{K} \cup \mathcal{L}) = \square\, \mathcal{K} \cup \square\, \mathcal{L}$
3. $\overline{\square\, (\bar{\mathcal{K}} + \bar{\mathcal{L}})} = \lozenge\mathcal{K}.\lozenge\mathcal{L}$
4. $\overline{\square\, (\bar{\mathcal{K}}.\bar{\mathcal{L}})} = \lozenge\mathcal{K} + \lozenge\mathcal{L}$
5. $\lozenge(\mathcal{K} \cap \mathcal{L}) = \lozenge\mathcal{K} \cap \lozenge\mathcal{L}$
6. $\lozenge(\mathcal{K} \cup \mathcal{L}) = \lozenge\mathcal{K} \cup \lozenge\mathcal{L}$
7. $\overline{\lozenge(\bar{\mathcal{K}} + \bar{\mathcal{L}})} = \square\, \mathcal{K}.\square\, \mathcal{L}$
8. $\overline{\lozenge(\bar{\mathcal{K}}.\bar{\mathcal{L}})} = \square\, \mathcal{K} + \square\, \mathcal{L}$

Bukti

1) $\square\, (\mathcal{K} \cap \mathcal{L})$
$= \square \{\langle \lambda, \min(\mu_{\mathcal{K}}(\lambda), \mu_{\mathcal{L}}(\lambda)), \max (v_{\mathcal{K}}(\lambda), v_{\mathcal{L}}(\lambda))\rangle \mid \lambda \in E\}$
$= \{\langle \lambda, \min(\mu_{\mathcal{K}}(\lambda), \mu_{\mathcal{L}}(\lambda)), \max(1-\mu_{\mathcal{K}}(\lambda), 1-\mu_{\mathcal{L}}(\lambda))\rangle \mid \lambda \in E\}$
$= \{\langle \lambda, \mu_{\mathcal{K}}(\lambda), 1-\mu_{\mathcal{K}}(\lambda)\rangle \mid \lambda \in E\} \cap \{\langle \lambda, \mu_{\mathcal{L}}(\lambda), 1-\mu_{\mathcal{K}}(\lambda)\rangle \mid \lambda \in E\}$
$= \square\, \mathcal{K} \cap \square\, \mathcal{L}$

2) $\square\, (\mathcal{K} \cup \mathcal{L})$
$= \square \{\langle \lambda, \max(\mu_{\mathcal{K}}(\lambda), \mu_{\mathcal{L}}(\lambda)), \min (v_{\mathcal{K}}(\lambda), v_{\mathcal{L}}(\lambda))\rangle \mid \lambda \in E\}$
$= \{\langle \lambda, \max(\mu_{\mathcal{K}}(\lambda), \mu_{\mathcal{L}}(\lambda)), \min(1-\mu_{\mathcal{K}}(\lambda), 1-\mu_{\mathcal{L}}(\lambda))\rangle \mid \lambda \in E\}$
$= \{\langle \lambda, \mu_{\mathcal{K}}(\lambda), 1-\mu_{\mathcal{K}}(\lambda)\rangle \mid \lambda \in E\} \cup \{\langle \lambda, \mu_{\mathcal{L}}(\lambda), 1-\mu_{\mathcal{K}}(\lambda)\rangle \mid \lambda \in E\}$
$= \square\, \mathcal{K} \cup \square\, \mathcal{L}$

3) $\overline{\square\, (\bar{\mathcal{K}} + \bar{\mathcal{L}})}$
$= \overline{\square \{\langle \lambda, (v_{\mathcal{K}}(\lambda) + v_{\mathcal{L}}(\lambda) - v_{\mathcal{K}}(\lambda).v_{\mathcal{L}}(\lambda), \mu_{\mathcal{K}}(\lambda).\mu_{\mathcal{L}}(\lambda)\rangle \mid \lambda \in E\}}$
$= \overline{\{\langle \lambda, (v_{\mathcal{K}}(\lambda) + v_{\mathcal{L}}(\lambda) - v_{\mathcal{K}}(\lambda).v_{\mathcal{L}}(\lambda), (1-v_{\mathcal{K}}(\lambda)).(1-v_{\mathcal{L}}(\lambda))\rangle \mid \lambda \in E\}}$
$= \{\langle \lambda, (1-v_{\mathcal{K}}(\lambda)).(1-v_{\mathcal{L}}(\lambda)), v_{\mathcal{K}}(\lambda) + v_{\mathcal{L}}(\lambda) - v_{\mathcal{K}}(\lambda).v_{\mathcal{L}}(\lambda) \mid \lambda \in E\}$
$= \square\, \mathcal{K} + \square\, \mathcal{L}$

4) $\overline{\square\, (\bar{\mathcal{K}}.\bar{\mathcal{L}})}$
$= \overline{\square \{\langle \lambda, (v_{\mathcal{K}}(\lambda).v_{\mathcal{L}}(\lambda), \mu_{\mathcal{K}}(\lambda) + \mu_{\mathcal{L}}(\lambda) - \mu_{\mathcal{K}}(\lambda).\mu_{\mathcal{L}}(\lambda)\rangle \mid \lambda \in E\}}$
$= \overline{\{\langle \lambda, (v_{\mathcal{K}}(\lambda) + v_{\mathcal{L}}(\lambda), (1-v_{\mathcal{K}}(\lambda)) + (1-v_{\mathcal{L}}(\lambda)) - (1-v_{\mathcal{K}}(\lambda)).(1-v_{\mathcal{L}}(\lambda))\rangle \mid \lambda \in E\}}$
$= \{\langle \lambda, (1-v_{\mathcal{K}}(\lambda)).(1-v_{\mathcal{L}}(\lambda)) - (1-v_{\mathcal{K}}(\lambda)).(1-v_{\mathcal{L}}(\lambda)), v_{\mathcal{K}}(\lambda).v_{\mathcal{L}}(\lambda) \mid \lambda \in E\}$
$= \lozenge\mathcal{K} + \lozenge\mathcal{L}$

5) $\lozenge(\mathcal{K} \cap \mathcal{L})$
$= \lozenge\{\langle \lambda, \min(\mu_{\mathcal{K}}(\lambda), \mu_{\mathcal{L}}(\lambda)), \max (v_{\mathcal{K}}(\lambda), v_{\mathcal{L}}(\lambda))\rangle \mid \lambda \in E\}$
$= \{\langle \lambda, \min(1-v_{\mathcal{K}}(\lambda), 1-v_{\mathcal{L}}(\lambda)), \max(v_{\mathcal{K}}(\lambda), v_{\mathcal{L}}(\lambda))\rangle \mid \lambda \in E\}$
$= \{\langle \lambda, 1-v_{\mathcal{K}}(\lambda), v_{\mathcal{K}}(\lambda)\rangle \mid \lambda \in E\} \cap \{\langle \lambda, 1-v_{\mathcal{L}}(\lambda), v_{\mathcal{L}}(\lambda)\rangle \mid \lambda \in E\}$
$= \lozenge\mathcal{K} \cap \lozenge\mathcal{L}$

6) $\lozenge(\mathcal{K} \cup \mathcal{L})$
$= \square \{\langle \lambda, \max(\mu_{\mathcal{K}}(\lambda), \mu_{\mathcal{L}}(\lambda)), \min (v_{\mathcal{K}}(\lambda), v_{\mathcal{L}}(\lambda))\rangle \mid \lambda \in E\}$
$= \{\langle \lambda, \max(1-v_{\mathcal{K}}(\lambda), 1-v_{\mathcal{L}}(\lambda)), \min(v_{\mathcal{K}}(\lambda), v_{\mathcal{L}}(\lambda))\rangle \mid \lambda \in E\}$
$= \{\langle \lambda, 1-v_{\mathcal{K}}(\lambda), v_{\mathcal{K}}(\lambda)\rangle \mid \lambda \in E\} \cup \{\langle \lambda, 1-v_{\mathcal{L}}(\lambda), v_{\mathcal{L}}(\lambda)\rangle \mid \lambda \in E\}$
$= \lozenge\mathcal{K} \cup \lozenge\mathcal{L}$

7) $\overline{\lozenge(\mathcal{K} + \bar{\mathcal{L}})} = \square\, \mathcal{K}.\square\, \mathcal{L}$
$= \overline{\lozenge\{\langle \lambda, (v_{\mathcal{K}}(\lambda) + v_{\mathcal{L}}(\lambda) - v_{\mathcal{K}}(\lambda).v_{\mathcal{L}}(\lambda), \mu_{\mathcal{K}}(\lambda).\mu_{\mathcal{L}}(\lambda)\rangle \mid \lambda \in E\}}$
$= \overline{\{\langle \lambda, (1-\mu_{\mathcal{K}}(\lambda) + (1-\mu_{\mathcal{L}}(\lambda)) - (1-\mu_{\mathcal{K}}(\lambda).(1-\mu_{\mathcal{L}}(\lambda)), \mu_{\mathcal{K}}(\lambda).\mu_{\mathcal{L}}(\lambda)\rangle \mid \lambda \in E\}}$
$= \{\langle \lambda, \mu_{\mathcal{K}}(\lambda).\mu_{\mathcal{L}}(\lambda), (1-\mu_{\mathcal{K}}(\lambda)) + (1-\mu_{\mathcal{L}}(\lambda)) - (1-\mu_{\mathcal{K}}(\lambda)).(1-\mu_{\mathcal{L}}(\lambda))\rangle \mid \lambda \in E\}$
$= \square\, \mathcal{K}.\square\, \mathcal{L}$

8) $\overline{\lozenge(\bar{\mathcal{K}}.\bar{\mathcal{L}})} = \square\, \mathcal{K} + \square\, \mathcal{L}$

$= \overline{\lozenge\{\langle\lambda, (v_{\mathcal{K}}(\lambda).v_{\mathcal{L}}(\lambda), \mu_{\mathcal{K}}(\lambda) + \mu_{\mathcal{L}}(\lambda) - \mu_{\mathcal{K}}(\lambda).\mu_{\mathcal{L}}(\lambda))\rangle | \lambda \,\epsilon\, E\}}$

$= \overline{\{\langle\lambda, (1-\mu_{\mathcal{K}}(\lambda)).(1-\mu_{\mathcal{L}}(\lambda)), \mu_{\mathcal{K}}(\lambda) + \mu_{\mathcal{L}}(\lambda) - \mu_{\mathcal{K}}(\lambda).\mu_{\mathcal{L}}(\lambda)\rangle | \lambda \,\epsilon\, E\}}$

$= \{\langle\lambda, \mu_{\mathcal{K}}(\lambda) + \mu_{\mathcal{L}}(\lambda) - \mathcal{K}(\lambda).\mu_{\mathcal{L}}(\lambda), (1-\mu_{\mathcal{K}}(\lambda))(1-\mu_{\mathcal{L}}(\lambda))\rangle | \lambda \,\epsilon\, E\}$

$= \square\, \mathcal{K} + \square\, \mathcal{L}$

Definisi 7

Misalkan $\mathcal{K}$ dan $\mathcal{L}$ dua himpunan fuzzy intuisionistik di E , Didefinisikan relasi operator kebutuhan dan kemungkinan sebagai berikut :

$$\mathcal{K} \subset_{\square} \mathcal{L} \text{ jika dan hanya jika } (\forall\lambda \in E)\big(\mu_{\mathcal{K}}(\lambda) \leq \mu_{\mathcal{L}}(\lambda)\big) \quad (12)$$

$$\mathcal{K} \subset_{\lozenge} \mathcal{L} \text{ jika dan hanya jika } (\forall\lambda \in E)\big(v_{\mathcal{K}}(\lambda) \geq v_{\mathcal{L}}(\lambda)\big) \quad (13)$$

Teorema 2

Untuk setiap dua himpunan fuzzy intuisionistik $\mathcal{K}$ dan $\mathcal{L}$. Maka :

1) $\mathcal{K} \subset_{\square} \mathcal{L}$ jika dan hanya jika $\square\, \mathcal{K} \subset \square\, \mathcal{L}$
2) $\mathcal{K} \subset_{\lozenge} \mathcal{L}$ jika dan hanya jika $\lozenge\mathcal{K} \subset \lozenge\mathcal{L}$
3) $\mathcal{K} \subset_{\square} \mathcal{L}$ dan $\mathcal{K} \subset_{\lozenge} \mathcal{L}$ jika dan hanya jika $\mathcal{K} \subset \mathcal{L}$

Bukti

1) $\mathcal{K} \subset_{\square} \mathcal{L}$ jika dan hanya jika $\square\, \mathcal{K} \subset \square\, \mathcal{L}$

Akan dibuktika ke kanan :

Misalkan $\mathcal{K} \subset_{\square} \mathcal{L}$. Akan dibuktikan bahwa $\square\, \mathcal{K} \subset \square\, \mathcal{L}$

Menurut definisi

a) $\mu_{\mathcal{K}}(\lambda) \leq \mu_{\mathcal{L}}(\lambda)$ untuk setiap $\lambda \in E$
Karena $\mathcal{K} \subset_{\square} \mathcal{L}$, maka $\mu_{\mathcal{K}}(\lambda) \leq \mu_{\mathcal{L}}(\lambda)$ untuk setiap $\lambda \in E$

b) $1 - \mu_{\mathcal{K}}(\lambda) \geq 1 - \mu_{\mathcal{L}}(\lambda)$ untuk setiap $\lambda \in E$
Karena $\mu_{\mathcal{K}}(\lambda) \leq \mu_{\mathcal{L}}(\lambda)$. Ini menunjukkan bahwa $1 - \mu_{\mathcal{K}}(\lambda) \geq 1 - \mu_{\mathcal{L}}(\lambda)$ untuk setiap $\lambda \in E$

Maka dapat dibuktikan $\square\, \mathcal{K} \subset \square\, \mathcal{L}$

Akan dibuktikan ke kiri :

Misalkan $\square\, \mathcal{K} \subset \square\, \mathcal{L}$. Akan dibuktikan bahwa $\mathcal{K} \subset_{\square} \mathcal{L}$. Dengan menujukkan $\mu_{\mathcal{K}}(\lambda) \leq \mu_{\mathcal{L}}(\lambda)$ untuk setiap $\lambda \in E$.

a) Karena $\square\, \mathcal{K} \subset \square\, \mathcal{L}$. Maka $\mu_{\mathcal{K}}(\lambda) \leq \mu_{\mathcal{L}}(\lambda)$ untuk setiap $\lambda \in E$.

b) Karena $\mu_{\mathcal{K}}(\lambda) \leq \mu_{\mathcal{L}}(\lambda)$ untuk setiap $\lambda \in E$ maka $\mathcal{K} \subset_{\square} \mathcal{L}$

Karena $\mathcal{K}$ dan $\mathcal{L}$ adalah sebarang himpunan fuzzy intuisionistik. maka berlaku :

$\mathcal{K} \subset_{\square} \mathcal{L}$ jika dan hanya jika $\square\, \mathcal{K} \subset \square\, \mathcal{L}$

2) $\mathcal{K} \subset_{\lozenge} \mathcal{L}$ jika dan hanya jika $\lozenge\mathcal{K} \subset \lozenge\mathcal{L}$

Akan dibuktikan ke kanan :

Misalkan $\mathcal{K} \subset_{\lozenge} \mathcal{L}$ akan dibuktikan bahwa $\lozenge\mathcal{K} \subset \lozenge\mathcal{L}$

Menurut definisi

a) $v_{\mathcal{K}}(\lambda) \geq v_{\mathcal{L}}(\lambda)$ untuk setiap $\lambda \in E$
Karena $\mathcal{K} \subset_{\lozenge} \mathcal{L}$, maka $v_{\mathcal{K}}(\lambda) \geq v_{\mathcal{L}}(\lambda)$ untuk setiap $\lambda \in E$.

b) $1 - v_{\mathcal{K}}(\lambda) \leq 1 - v_{\mathcal{L}}(\lambda)$ untuk setiap $\lambda \in E$
Karena $v_{\mathcal{K}}(\lambda) \geq v_{\mathcal{L}}(\lambda)$. Ini menunjukkan bahwa $1 - v_{\mathcal{K}}(\lambda) \leq 1 - v_{\mathcal{L}}(\lambda)$ untuk setiap $\lambda \in E$

maka dapat dibuktikan $\lozenge\mathcal{K} \subset \lozenge\mathcal{L}$

Akan dibuktikan ke kiri :

Misalkan $\lozenge\mathcal{K} \subset \lozenge\mathcal{L}$. Akan dibuktikan bahwa $\mathcal{K} \subset_{\lozenge} \mathcal{L}$. Dengan menunjukkan $v_{\mathcal{K}}(\lambda) \geq v_{\mathcal{L}}(\lambda)$ untuk setiap $\lambda \in E$

a) Karena $\lozenge\mathcal{K} \subset \lozenge\mathcal{L}$. Maka $v_{\mathcal{K}}(\lambda) \geq v_{\mathcal{L}}(\lambda)$ untuk setiap $\lambda \in E$.

b) Karena $v_{\mathcal{K}}(\lambda) \geq v_{\mathcal{L}}(\lambda)$ untuk setiap $\lambda \in E$ maka $\mathcal{K} \subset_{\lozenge} \mathcal{L}$.

Karena $\mathcal{K}$ dan $\mathcal{L}$ adalah sebarang himpunan fuzzy intuisionistik. maka berlaku :

$\mathcal{K} \subset_{\lozenge} \mathcal{L}$ jika dan hanya jika $\lozenge\mathcal{K} \subset \lozenge\mathcal{L}$

3) $\mathcal{K} \subset_{\square} \mathcal{L}$ dan $\mathcal{K} \subset_{\lozenge} \mathcal{L}$ jika dan hanya jika $\mathcal{K} \subset \mathcal{L}$

Berdasarkan definisi 11 dan 12

$$\mathcal{K} \subset_{\square} \mathcal{L} \text{ jika dan hanya jika } (\forall\lambda \in E)\big(\mu_{\mathcal{K}}(\lambda) \leq \mu_{\mathcal{L}}(\lambda)\big)$$

$$\mathcal{K} \subset_{\lozenge} \mathcal{L} \text{ jika dan hanya jika } (\forall\lambda \in E)\big(v_{\mathcal{K}}(\lambda) \geq v_{\mathcal{L}}(\lambda)\big)$$

Maka dapat dikatakan bahwa

$\mathcal{K} \subset_{\square} \mathcal{L}$ dan $\mathcal{K} \subset_{\lozenge} \mathcal{L}$ jika dan hanya jika $(\forall\lambda \in E)\big(\mu_{\mathcal{K}}(\lambda) \leq \mu_{\mathcal{L}}(\lambda) \& v_{\mathcal{K}}(\lambda) \geq v_{\mathcal{L}}(\lambda)\big)$ dapat dikatakan juga sebagai $\mathcal{K} \subset \mathcal{L}$

Disisi lain, jika $\mathcal{K} \subset \mathcal{L}$ jika dan hanya jika $(\forall\lambda \in E)\big(\mu_{\mathcal{K}}(\lambda) \leq \mu_{\mathcal{L}}(\lambda) \& v_{\mathcal{K}}(\lambda) \geq v_{\mathcal{L}}(\lambda)\big)$ yaitu $\mathcal{K} \subset_{\square} \mathcal{L}$ dan $\mathcal{K} \subset_{\lozenge} \mathcal{L}$

Definisi 8

Dua himpunan fuzzy intuisionistik $\mathcal{K}$ dan $\mathcal{L}$, didefinisikan relasi sebagai berikut :

$$\mathcal{K} \sqsubset \mathcal{L} \; jika \; (\forall\lambda \in E)(\pi_{\mathcal{K}}(\lambda) \leq \pi_{\mathcal{L}}(\lambda)) \quad (14)$$

Proposisi 2

Untuk setiap dua himpunan fuzzy intuisionistik $\mathcal{K}$ dan $\mathcal{L}$ berlaku :

1) Jika $\mathcal{K} \subset_{\square} \mathcal{L}$ dan $\mathcal{K} \sqsubset \mathcal{L}$, maka $\mathcal{K} \subset \mathcal{L}$
2) Jika $\mathcal{K} \subset_{\lozenge} \mathcal{L}$ dan $\mathcal{K} \sqsubset \mathcal{L}$, maka $\mathcal{K} \subset \mathcal{L}$

Bukti

1) Jika $\mathcal{K} \subset_{\square} \mathcal{L}$ dan $\mathcal{K} \sqsubset \mathcal{L}$, maka $\mathcal{K} \subset \mathcal{L}$

Berdasarkan definisi

$\mathcal{K} \subset_{\square} \mathcal{L}$ jika $(\forall\lambda \in E)\big(\mu_{\mathcal{K}}(\lambda) \leq \mu_{\mathcal{L}}(\lambda)\big)$
$\mathcal{K} \sqsubset \mathcal{L}$ jika $(\forall\lambda \in E)(\pi_{\mathcal{K}}(\lambda) \leq \pi_{\mathcal{L}}(\lambda))$
Akan dibuktikan jika $\mathcal{K} \subset_{\square} \mathcal{L}$ dan $\mathcal{K} \sqsubset \mathcal{L}$, Maka $\mathcal{K} \subset \mathcal{L}$
$\mathcal{K} \subset \mathcal{L}$ jika $(\forall \lambda \epsilon E)\big(\mu_{\mathcal{K}}(\lambda) \leq \mu_{\mathcal{L}}(\lambda) \wedge v_{\mathcal{K}}(\lambda) \geq v_{\mathcal{L}}(\lambda)\big)$

Analisis pendahuluan
$= \pi_{\mathcal{K}}(\lambda) \leq \pi_{\mathcal{L}}(\lambda)$ dengan $\pi(\lambda) = 1 - \mu(\lambda) - v(\lambda)$
$= 1 - \mu_{\mathcal{K}}(\lambda) - v_{\mathcal{K}}(\lambda) \leq 1 - \mu_{\mathcal{L}}(\lambda) - v_{\mathcal{L}}(\lambda)$
Karena $\mu_{\mathcal{K}}(\lambda) \leq \mu_{\mathcal{L}}(\lambda)$
$=1 - v_{\mathcal{K}}(\lambda) \leq 1 - v_{\mathcal{L}}(\lambda)$
$=-v_{\mathcal{K}}(\lambda) \leq -v_{\mathcal{L}}(\lambda)$
$=v_{\mathcal{K}}(\lambda) \geq v_{\mathcal{L}}(\lambda)$
Maka terbukti jika $\mathcal{K} \subset_{\square} \mathcal{L}$ dan $\mathcal{K} \sqsubset \mathcal{L}$, maka $\mathcal{K} \subset \mathcal{L}$

2) Jika $\mathcal{K} \subset_{\Diamond} \mathcal{L}$ dan $\mathcal{K} \sqsubset \mathcal{L}$, Maka $\mathcal{K} \subset \mathcal{L}$
Berdasarkan definisi
$\mathcal{K} \subset_{\Diamond} \mathcal{L}$ jika $(\forall\lambda \in E)\big(v_{\mathcal{K}}(\lambda) \geq v_{\mathcal{L}}(\lambda)\big)$
$\mathcal{K} \sqsubset \mathcal{L}$ jika $(\forall\lambda \in E)(\pi_{\mathcal{K}}(\lambda) \leq \pi_{\mathcal{L}}(\lambda))$ maka
$\mathcal{L} \sqsubset \mathcal{K}$ jika $(\forall\lambda \in E)(\pi_{\mathcal{L}}(\lambda) \leq \pi_{\mathcal{K}}(\lambda))$
Akan dibuktikan jika $\mathcal{K} \subset_{\Diamond} \mathcal{L}$ dan $\mathcal{L} \sqsubset \mathcal{K}$, Maka $\mathcal{K} \subset \mathcal{L}$
$\mathcal{K} \subset \mathcal{L}$ jika $(\forall \lambda \epsilon E)\big(\mu_{\mathcal{K}}(\lambda) \leq \mu_{\mathcal{L}}(\lambda) \wedge v_{\mathcal{K}}(\lambda) \geq v_{\mathcal{L}}(\lambda)\big)$

Analisis pendahuluan
$= \pi_{\mathcal{L}}(\lambda) \leq \pi_{\mathcal{K}}(\lambda)$ dengan $\pi(\lambda) = 1 - \mu(\lambda) - v(\lambda)$
$= 1 - \mu_{\mathcal{L}}(\lambda) - v_{\mathcal{L}}(\lambda) \leq 1 - \mu_{\mathcal{K}}(\lambda) - v_{\mathcal{K}}(\lambda)$
Karena $v_{\mathcal{K}}(\lambda) \geq v_{\mathcal{L}}(\lambda)$
$= 1 - \mu_{\mathcal{L}}(\lambda) \leq 1 - \mu_{\mathcal{K}}(\lambda)$
$= -\mu_{\mathcal{L}}(\lambda) \leq -\mu_{\mathcal{K}}(\lambda)$
$= \mu_{\mathcal{K}}(\lambda) \leq \mu_{\mathcal{L}}(\lambda)$
Maka terbukti jika $\mathcal{K} \subset_{\Diamond} \mathcal{L}$ dan $\mathcal{L} \sqsubset \mathcal{K}$, maka $\mathcal{K} \subset \mathcal{L}$

Proposisi 3
Jika $\mathcal{K}$ adalah IFS di E_1 dan $\mathcal{L}$ adalah IFS di E_2.
1) $\square\,(\mathcal{K} \times_1 \mathcal{L}) \subset \square\,\mathcal{K} \times_1 \square\,\mathcal{L}$
2) $\Diamond(\mathcal{K} \times_1 \mathcal{L}) \supset \Diamond\mathcal{K} \times_1 \Diamond\mathcal{L}$
3) $\square\,(\mathcal{K} \times_2 \mathcal{L}) = \square\,\mathcal{K} \times_2 \square\,\mathcal{L}$
4) $\Diamond(\mathcal{K} \times_2 \mathcal{L}) = \Diamond\mathcal{K} \times_2 \Diamond\mathcal{L}$
5) $\square\,(\mathcal{K} \times_3 \mathcal{L}) = \square\,\mathcal{K} \times_3 \square\,\mathcal{L}$
6) $\Diamond(\mathcal{K} \times_3 \mathcal{L}) = \Diamond\mathcal{K} \times_3 \Diamond\mathcal{L}$
7) $\square\,(\mathcal{K} \times_4 \mathcal{L}) = \square\,\mathcal{K} \times_4 \square\,\mathcal{L}$
8) $\Diamond(\mathcal{K} \times_4 \mathcal{L}) = \Diamond\mathcal{K} \times_4 \Diamond\mathcal{L}$
9) $\square\,(\mathcal{K} \times_5 \mathcal{L}) = \square\,\mathcal{K} \times_5 \square\,\mathcal{L}$
10) $\Diamond(\mathcal{K} \times_5 \mathcal{L}) = \Diamond\mathcal{K} \times_5 \Diamond\mathcal{L}$

Bukti
1) $\square\,(\mathcal{K} \times_1 \mathcal{L}) \subset \square\,\mathcal{K} \times_1 \square\,\mathcal{L}$
$= \square\,\{\langle\langle\lambda,\gamma\rangle\mu_{\mathcal{K}}(\lambda).\ \mu_{\mathcal{L}}(\gamma), v_{\mathcal{K}}(\lambda).v_{\mathcal{L}}(\gamma)\rangle, \mid \lambda \epsilon E_1, \gamma \epsilon E_2\} \subset \square\,\mathcal{K} \times_1 \square\,\mathcal{L}$
$= \{\langle\langle\lambda,\gamma\rangle,\mu_{\mathcal{K}}(\lambda).\mu_{\mathcal{L}}(\gamma), (1-\mu_{\mathcal{K}}(\lambda)).(1-\mu_{\mathcal{L}}(\gamma))\rangle \mid \lambda \epsilon E_1, \gamma \epsilon E_2\} \subset \{\langle\langle\lambda,\gamma\rangle,\mu_{\mathcal{K}}(\lambda).\mu_{\mathcal{L}}(\gamma), (1-\mu_{\mathcal{K}}(\lambda)).(1-\mu_{\mathcal{L}}(\gamma))\rangle | \lambda \epsilon E_1, \gamma \epsilon E_2\}$
Berdasarkan definisi 4. Terbukti bahwa :
$\square\,(\mathcal{K} \times_1 \mathcal{L}) \subset \square\,\mathcal{K} \times_1 \square\,\mathcal{L}$

2) $\Diamond(\mathcal{K} \times_1 \mathcal{L}) \supset \Diamond\mathcal{K} \times_1 \Diamond\mathcal{L}$
$= \Diamond\mathcal{K} \times_1 \Diamond\mathcal{L} \subset \Diamond\{\langle\langle\lambda,\gamma\rangle,\mu_{\mathcal{K}}(\lambda).\ \mu_{\mathcal{L}}(\gamma), v_{\mathcal{K}}(\lambda).\ v_{\mathcal{L}}(\gamma)\rangle \mid \lambda \epsilon E_1, \gamma \epsilon E_2\}$
$= \{\langle\langle\lambda,\gamma\rangle,(1-v_{\mathcal{K}}(\lambda)).(1-v_{\mathcal{L}}(\gamma)), v_{\mathcal{K}}(\lambda).v_{\mathcal{L}}(\gamma)\rangle \mid \lambda \epsilon E_1 \& \gamma \epsilon E_2\} \subset \{\langle\langle\lambda,\gamma\rangle,(1-v_{\mathcal{K}}(\lambda)).(1-v_{\mathcal{L}}(\gamma)), v_{\mathcal{K}}(\lambda).v_{\mathcal{L}}(\gamma)\rangle | \lambda \epsilon E_1, \gamma \epsilon E_2\}$
Berdasarkan definisi 4. Terbukti bahwa :
$\Diamond(\mathcal{K} \times_1 \mathcal{L}) \supset \Diamond\mathcal{K} \times_1 \Diamond\mathcal{L}$

3) $\square\,(\mathcal{K} \times_2 \mathcal{L}) = \square\,\mathcal{K} \times_2 \square\,\mathcal{L}$
$= \square\,\{\langle\langle\lambda,\gamma\rangle, \mu_{\mathcal{K}}(\lambda) + \mu_{\mathcal{L}}(\gamma) - \mu_{\mathcal{K}}(\lambda).\ \mu_{\mathcal{L}}(\gamma), v_{\mathcal{K}}(\lambda).v_{\mathcal{L}}(\gamma)\rangle \mid \lambda \epsilon E_1, \gamma \epsilon E_2\}$
$= \{\langle\langle\lambda,\gamma\rangle,\mu_{\mathcal{K}}(\lambda) + \mu_{\mathcal{L}}(\gamma) - \mu_{\mathcal{K}}(\lambda).\ \mu_{\mathcal{L}}(\gamma), | (1-\mu_{\mathcal{K}}(\lambda)).(1-\mu_{\mathcal{L}}(\gamma))\rangle | \lambda \epsilon E_1, \gamma \epsilon E_2\}$
$= \square\,\mathcal{K} \times_2 \square\,\mathcal{L}$

4) $\Diamond(\mathcal{K} \times_2 \mathcal{L}) = \Diamond\mathcal{K} \times_2 \Diamond\mathcal{L}$
$= \Diamond\{\langle\langle\lambda,\gamma\rangle,\mu_{\mathcal{K}}(\lambda) + \mu_{\mathcal{L}}(\gamma) - \mu_{\mathcal{K}}(\lambda).\ \mu_{\mathcal{L}}(\gamma), v_{\mathcal{K}}(\lambda).v_{\mathcal{L}}(\gamma)\rangle \mid \lambda \epsilon E_1, \gamma \epsilon E_2\}$
$= \{\langle\langle\lambda,\gamma\rangle,(1-v_{\mathcal{K}}(\lambda)) + (1-v_{\mathcal{L}}(\gamma)) - (1-v_{\mathcal{K}}(\lambda)).(1-v_{\mathcal{L}}(\gamma)), v_{\mathcal{K}}(\lambda).v_{\mathcal{L}}(\gamma)\rangle | \lambda \epsilon E_1, \gamma \epsilon E_2\}$
$= \Diamond\mathcal{K} \times_2 \Diamond\mathcal{L}$

5) $\square\,(\mathcal{K} \times_3 \mathcal{L}) = \square\,\mathcal{K} \times_3 \square\,\mathcal{L}$
$= \square\,\{\langle\langle\lambda,\gamma\rangle, \mu_{\mathcal{K}}(\lambda).\ \mu_{\mathcal{L}}(\gamma), v_{\mathcal{K}}(\lambda) + v_{\mathcal{L}}(\gamma) - v_{\mathcal{K}}(\lambda).v_{\mathcal{L}}(\gamma)\rangle \mid \lambda \epsilon E_1, \gamma \epsilon E_2\}$
$= \{\langle\langle\lambda,\gamma\rangle,\mu_{\mathcal{K}}(\lambda).\mu_{\mathcal{L}}(\gamma), (1-\mu_{\mathcal{K}}(\lambda)) + (1-\mu_{\mathcal{L}}(\gamma)) -(1-\mu_{\mathcal{K}}(\lambda)).(1-\mu_{\mathcal{L}}(\gamma))\rangle | \lambda \epsilon E_1, \gamma \epsilon E_2\}$
$= \square\,\mathcal{K} \times_3 \square\,\mathcal{L}$

6) $\Diamond(\mathcal{K} \times_3 \mathcal{L}) = \Diamond\mathcal{K} \times_3 \Diamond\mathcal{L}$
$= \Diamond\{\langle\langle\lambda,\gamma\rangle,\mu_{\mathcal{K}}(\lambda).\ \mu_{\mathcal{L}}(\gamma), v_{\mathcal{K}}(\lambda) + v_{\mathcal{L}}(\gamma) - v_{\mathcal{K}}(\lambda).v_{\mathcal{L}}(\gamma)\rangle \mid \lambda \epsilon E_1, \gamma \epsilon E_2\}$
$= \{\langle\langle\lambda,\gamma\rangle,(1-v_{\mathcal{K}}(\lambda)).(1-v_{\mathcal{L}}(\gamma)), v_{\mathcal{K}}(\lambda) + v_{\mathcal{L}}(\gamma) -v_{\mathcal{K}}(\lambda).v_{\mathcal{L}}(\gamma)\rangle | \lambda \epsilon E_1, \gamma \epsilon E_2\}$
$= \Diamond\mathcal{K} \times_3 \Diamond\mathcal{L}$

7) $\square\,(\mathcal{K} \times_4 \mathcal{L}) = \square\,\mathcal{K} \times_4 \square\,\mathcal{L}$
$= \square\,\{\langle\langle\lambda,\gamma\rangle, \min(\mu_{\mathcal{K}}(\lambda).\ \mu_{\mathcal{L}}(\gamma)), \max(v_{\mathcal{K}}(\lambda), v_{\mathcal{L}}(\gamma) | \lambda \epsilon E_1 \& \gamma \epsilon E_2\}$
$= \{\langle\langle\lambda,\gamma\rangle,, \min(\mu_{\mathcal{K}}(\lambda).\ \mu_{\mathcal{L}}(\gamma)), \max(1-\mu_{\mathcal{K}}(\lambda)), (1-\mu_{\mathcal{L}}(\gamma))\rangle | \lambda \epsilon E_1, \gamma \epsilon E_2\}$
$= \square\,\mathcal{K} \times_4 \square\,\mathcal{L}$

8) $\Diamond(\mathcal{K} \times_4 \mathcal{L}) = \Diamond\mathcal{K} \times_4 \Diamond\mathcal{L}$
$= \Diamond\{\langle\langle\lambda,\gamma\rangle,\min(\mu_{\mathcal{K}}(\lambda)).\ \mu_{\mathcal{L}}(\gamma)), \max(v_{\mathcal{K}}(\lambda), v_{\mathcal{L}}(\gamma))\rangle \mid \lambda \epsilon E_1, \gamma \epsilon E_2\}$
$= \{\langle\langle\lambda,\gamma\rangle,\min(1-v_{\mathcal{K}}(\lambda)).(1-v_{\mathcal{L}}(\gamma)), \max(v_{\mathcal{K}}(\lambda).v_{\mathcal{L}}(\gamma))\rangle | \lambda \epsilon E_1, \gamma \epsilon E_2\}$
$= \Diamond\mathcal{K} \times_4 \Diamond\mathcal{L}$

9) $\square(\mathcal{K} \times_5 \mathcal{L}) = \square\mathcal{K} \times_5 \square\mathcal{L}$
$= \square\{\langle\langle\lambda,\gamma\rangle, \max(\mu_{\mathcal{K}}(\lambda).\ \mu_{\mathcal{L}}(\gamma)), \min(v_{\mathcal{K}}(\lambda), v_{\mathcal{L}}(\gamma)|\lambda \in E_1, \gamma \in E_2\}$
$= \{\langle\langle\lambda,\gamma\rangle,, \max(\mu_{\mathcal{K}}(\lambda).\ \mu_{\mathcal{L}}(\gamma)), \min(1-\mu_{\mathcal{K}}(\lambda)), (1-\mu_{\mathcal{L}}(\gamma))\rangle|\lambda \in E_1, \gamma \in E_2\}$
$= \square\mathcal{K} \times_5 \square\mathcal{L}$

10) $\Diamond(\mathcal{K} \times_5 \mathcal{L}) = \Diamond\mathcal{K} \times_5 \Diamond\mathcal{L}$
$= \Diamond\{\langle\langle\lambda,\gamma\rangle, \max(\mu_{\mathcal{K}}(\lambda)).\ \mu_{\mathcal{L}}(\gamma)), \min(v_{\mathcal{K}}(\lambda), v_{\mathcal{L}}(\gamma))\rangle \mid \lambda \in E_1, \gamma \in E_2\}$
$= \{\langle\langle\lambda,\gamma\rangle, \max(1-v_{\mathcal{K}}(\lambda)).(1-v_{\mathcal{L}}(\gamma)), \min(v_{\mathcal{K}}(\lambda). v_{\mathcal{L}}(\gamma))\rangle|\lambda \in E_1, \gamma \in E_2\}$
$= \Diamond\mathcal{K} \times_5 \Diamond\mathcal{L}$

4. PENUTUP

Simpulan

1. Misalkan E himpunan semesta tak kosong. $\mathcal{K}$ adalah himpunan fuzzy intuisionistik di E. Maka berlaku :
   a. $\overline{\square\overline{\mathcal{K}}} = \Diamond\mathcal{K}$
   b. $\overline{\Diamond\overline{\mathcal{K}}} = \square\mathcal{K}$
   c. $\square\mathcal{K} \subset \mathcal{K} \subset \Diamond\mathcal{K}$
   d. $\square\square\mathcal{K} = \square\mathcal{K}$
   e. $\square\Diamond\mathcal{K} = \Diamond\mathcal{K}$
   f. $\Diamond\square\mathcal{K} = \square\mathcal{K}$
   g. $\Diamond\Diamond\mathcal{K} = \Diamond\mathcal{K}$
   h. $\square(\mathcal{K} \cap \mathcal{L}) = \square\mathcal{K} \cap \square\mathcal{L}$
   i. $\square(\mathcal{K} \cup \mathcal{L}) = \square\mathcal{K} \cup \square\mathcal{L}$
   j. $\overline{\square(\overline{\mathcal{K}} + \overline{\mathcal{L}})} = \Diamond\mathcal{K}.\Diamond\mathcal{L}$
   k. $\overline{\square(\overline{\mathcal{K}}.\overline{\mathcal{L}})} = \Diamond\mathcal{K} + \Diamond\mathcal{L}$
   l. $\Diamond(\mathcal{K} \cap \mathcal{L}) = \Diamond\mathcal{K} \cap \Diamond\mathcal{L}$
   m. $\Diamond(\mathcal{K} \cup \mathcal{L}) = \Diamond\mathcal{K} \cup \Diamond\mathcal{L}$
   n. $\overline{\Diamond(\overline{\mathcal{K}} + \overline{\mathcal{L}})} = \square\mathcal{K}.\square\mathcal{L}$
   o. $\overline{\Diamond(\overline{\mathcal{K}}.\overline{\mathcal{L}})} = \square\mathcal{K} + \square\mathcal{L}$

2. Misalkan E himpunan semesta tak kosong. $\mathcal{K}$ dan $\mathcal{L}$ himpunan fuzzy intuisionistik di E. Maka berlaku relasi pada $\square\mathcal{K}$ & $\Diamond\mathcal{K}$:
   a. $\mathcal{K} \subset_{\square} \mathcal{L}$ jika dan hanya jika $\square\mathcal{K} \subset \square\mathcal{L}$
   b. $\mathcal{K} \subset_{\Diamond} \mathcal{L}$ jika dan hanya jika $\Diamond\mathcal{K} \subset \Diamond\mathcal{L}$
   c. $\mathcal{K} \subset_{\square} \mathcal{L}$ dan $\mathcal{K} \subset_{\Diamond} \mathcal{L}$ jika dan hanya jika $\mathcal{K} \subset \mathcal{L}$
   d. Jika $\mathcal{K} \subset_{\square} \mathcal{L}$ dan $\mathcal{K} \sqsubset \mathcal{L}$, maka $\mathcal{K} \subset \mathcal{L}$
   e. Jika $\mathcal{K} \subset_{\Diamond} \mathcal{L}$ dan $\mathcal{K} \sqsubset \mathcal{L}$, maka $\mathcal{K} \subset \mathcal{L}$

3. Misalkan E himpunan semesta tak kosong. $\mathcal{K}$ himpunan fuzzy intuisionistik di E_1 dan $\mathcal{L}$ himpunan fuzzy intuisionistik di E_2. Maka berlaku kartesian produk pada $\square\mathcal{K}$ & $\Diamond\mathcal{K}$:
   a. $\square(\mathcal{K} \times_1 \mathcal{L}) \subset \square\mathcal{K} \times_1 \square\mathcal{L}$
   b. $\Diamond(\mathcal{K} \times_1 \mathcal{L}) \supset \Diamond\mathcal{K} \times_1 \Diamond\mathcal{L}$
   c. $\square(\mathcal{K} \times_2 \mathcal{L}) = \square\mathcal{K} \times_2 \square\mathcal{L}$
   d. $\Diamond(\mathcal{K} \times_2 \mathcal{L}) = \Diamond\mathcal{K} \times_2 \Diamond\mathcal{L}$
   e. $\square(\mathcal{K} \times_3 \mathcal{L}) = \square\mathcal{K} \times_3 \square\mathcal{L}$
   f. $\Diamond(\mathcal{K} \times_3 \mathcal{L}) = \Diamond\mathcal{K} \times_3 \Diamond\mathcal{L}$
   g. $\square(\mathcal{K} \times_4 \mathcal{L}) = \square\mathcal{K} \times_4 \square\mathcal{L}$
   h. $\Diamond(\mathcal{K} \times_4 \mathcal{L}) = \Diamond\mathcal{K} \times_4 \Diamond\mathcal{L}$
   i. $\square(\mathcal{K} \times_5 \mathcal{L}) = \square\mathcal{K} \times_5 \square\mathcal{L}$
   j. $\Diamond(\mathcal{K} \times_5 \mathcal{L}) = \Diamond\mathcal{K} \times_5 \Diamond\mathcal{L}$

Saran

Pembahasan dalam penelitian ini merupakan pembahasan tentang operator kebutuhan dan operator kemungkinan yang disertai sifat-sifatnya. Kedua operator tersebut memiliki keterkaitan satu sama lain. Pembahasan dari penelitian ini masih sederhana dan penulis berharap agar penelitian berikutnya terkait operator kebutuhan dan operator kemungkinan dapat lebih disempurnakan serta dapat mengembangkan penelitian ini.

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