

Prey predator Model with Holling Type II Functional Response and Hunting Cooperation of Predators

Davina Alma Ardelia Fitri

Program Studi Matematika, FMIPA, Universitas Negeri Surabaya

e-mail: davinaalma.20001@mhs.unesa.ac.id

Dian Savitri

Program Studi Matematika, FMIPA, Universitas Negeri Surabaya

*e-mail: diansavitri@unesa.ac.id

Abstract

This research examines the interaction of two populations with Holling type II response functions and hunting cooperation in predators. The mathematical model developed is based on natural phenomena, namely the interaction between wolves and deer. Wolves often cooperate to hunt in packs, with most of their prey consisting of physically or health weak individuals, one of which is deer. The dynamics analysis begins with determining the basic assumptions of model construction, equilibrium and stability points, and numerical simulation using PPlane. The results of the dynamics analysis show that there are three equilibrium points with stability types, namely $E_1(0,0)$, which is unstable, $E_2(k,0)$ and E_3 which is asymptotically stable under certain conditions. Numerical simulation results show the existence of double stability at equilibrium points E_2 and E_3 with prey to predator conversion parameter value $\beta = 2.03$ shows a change in stability that only occurs at point E_2 . The difference in the value of prey to predator conversion affects in the change in the system solution and has an impact on reducing the predator population.

Keywords: Hunting Cooperation, Bistable, Stability Analysis.

INTRODUCTION

The prey predator model shows the interaction between two populations, a prey population that is eaten by another population, and a predator population that hunts and eats another population (Aini and Savitri 2021). Each group of living things has its own characteristics in living in an ecosystem. These characteristics include the ability to survive, competition between predators for prey, and working together when hunting prey. In the context of hunting, cooperation is said to be the act of two or more individuals working together to achieve a common goal. Cooperative hunting in predators provides a number of benefits, such as increasing the probability of success in hunting, reducing the duration of hunting time, targeting a significant number of prey, and detecting food sources quickly (Pal et al. 2019a).

Cooperative hunting behavior exists not only in carnivores but also in some omnivores. This

phenomenon creates interesting problems in the interaction between prey and predator populations. As in the Alpen, France, there are several carnivore species, one of which is the wolf (*Canis Lupus*). The wolf (*Canis Lupus*) is known as one of the most common mammals in the world (Figueiredo et al. 2020). Wolves are large mammalian predators that prey on a variety of animals, most of the wolf's prey consist of young, old, and physically weakened individuals, referred to as inferiors (Mech 1974). The main prey of wolves is deer. Deer are usually found in various place, one of which is in mountainous areas. Deer have a body length of up to 2.7 meters and weigh more than 100 kg. they also have the ability to run fast, making it easier for them to escape from predators (Semiadi, Nugraha, and Jamal 2004). This reflects that wolves hunt prey in groups, coordinating to corner and capture their prey.

A wolf can be said to be a population if it consists of at least 2 wild wolf pairs that

successfully raise at least 2 children for two consecutive years in the experimental area (Fuller, Mech, and Cochrane 2003). Population dynamics studies regarding the interaction of living things based on these phenomena have continued to grow for decades. According to Capone, population dynamics analyzes how dynamic behavior affects the development of ecosystems (Capone et al. 2019). Dynamic behavior describes interaction patterns in the form of predation and competition. Predation relationships occur when one predatory organism preys on another (prey). Predation plays an important role in promoting the evolutionary development of life and maintaining ecological balance and biodiversity.

In 1925, Lotka introduced the prey predator model, which was later expanded by Volterra, known as the Lotka-Volterra model. The interaction process between populations in this model involves predation that forms a predation pattern known as a response function. This response function illustrates that the amount of prey consumed by predators is directly proportional to the prey population size (Ningrum, Abadi, and Astutia 2020). In 1953, Holling introduced a response function that became known as Holling type I, Holling type II, and Holling type III response functions based on their characteristics (Holling 1965).

In 2017, Alves and Hilke considered hunting cooperation in the predator population. They defined the Holling type I response function as with parameterizing hunting cooperation in predators and parameterizing the attack rate per predator and prey (Teixeira Alves and Hilker 2017). Another study also considered the effect of fear on prey using the Leslie-Gower prey predator model and considered harvesting, hunting cooperation (Naji 2023). Some researches discussed prey predator interaction models involving fear of prey and hunting cooperation with Holling type I response function (Fakhry and Naji 2023); (Liu et al. 2021). Supported by Salwa, Safinadin Indira, Shakira Lintang Alea (2023) research examining the Lotka-Volterra prey predator interaction model with hunting cooperation in predators using Holling type I response functions and competition in prey. The results of stability analysis and

numerical simulations show dual stability (bistable system) at the interior point and extinction point of predator population. This model was also developed by (Belew and Melese 2022) with hunting cooperation used Holling type II functional responses and the effect of fear on prey. The similar model also investigated with cooperation and the mate-finding allee effect using Holling type II response functions (Paul, Mondal, and Ghosh 2024).

Based on this background, the author is interested in examining mathematically the interaction between wolves and deer uses the Lotka-Volterra model that considers cooperation in hunting on predators with Holling type II response functions. This study also simplifies some of the assumptions of the model (Belew and Melese 2022) and (Paul, Mondal, and Ghosh 2024) by assuming the prey grows logistically without considering the fear effect and the predator grows without allee effect. This research is conducted by reviewing literature studies, constructing prey-predator models, determining equilibrium points, conducting stability analysis, and making numerical simulations to confirm the results of the analysis that has been done.

THEORY REVIEW

Prey Predator Model with Hunting Cooperation

In ecology, the prey predator model plays an important role in population dynamics. One interesting interaction is the hunting cooperation that occurs between predators. The hunting cooperation model is one of the topics that continues to grow and modify according to natural conditions.

Prey predator models that consider hunting cooperation have been discussed by Teixeira Alves and Hilker (2017), as follows:

$$\begin{aligned}\frac{dx}{dt} &= rx \left(1 - \frac{1}{k}\right) - (p + \alpha y)xy, \\ \frac{dy}{dt} &= c(p + \alpha y)xy - \mu y.\end{aligned}\quad (1)$$

The model (1) represent the prey population growth rate in x and the predator population in y . The model adds a hunting cooperation term to the predator with parameter α . The parameters c, p, r , and μ represent the predator conversion rate,

predator attack rate, prey population growth rate and mortality predator.

Pal et al. (2019) discussed the prey predator model in the presence of hunting cooperation in predators and the effect of fear on prey, as follow:

$$\begin{aligned}\frac{dx}{dt} &= \frac{rx}{1+ey} - (p+\alpha y)xy - bx^2 - dx, \\ \frac{dy}{dt} &= c(p+\alpha y)xy - \mu y.\end{aligned}\quad (2)$$

The model (2) considers that during hunting cooperation in predators, predators cooperate to cause fear in prey. Fear of prey represented by $f(x) = \frac{rx}{1+ey}$ in the predator population can cause the growth rate of prey to decrease.

Prey Predator Model with Hunting Cooperation with Response Function

Du, Niu, and Wei (2022) discussed a prey predator model in the presence of hunting cooperation and group defence in prey with Holling type I response function. Holling type I response function assumes that the predation rate increases linearly proportional to the prey population increase. Belew and Melese (2022) using Holling type II response function on predation rate and hunting cooperation in predators, as follow:

$$\begin{aligned}\frac{dx}{dt} &= \frac{rx}{1+ey} \left(1 - \frac{x}{k}\right) - \frac{(p+\alpha y)xy}{1+m(p+\alpha y)x} - H_1 Q_1 N, \\ \frac{dy}{dt} &= \frac{c(p+\alpha y)xy}{1+m(p+\alpha y)x} - \mu y - H_2 Q_2 N.\end{aligned}\quad (3)$$

In the model (3), hunting cooperation $(p+\alpha y)$ is restricted by the predator response function m , which is the time taken by the predator to search prey and consume. Holling type II response function occurs in predators with active characteristics in searching for prey and predators need time to consume the prey.

Pal et al. (2019b) discussed Holling type II response function on predation rate, hunting cooperation in predators and fear effect on prey, as follow in model (4):

$$\begin{aligned}\frac{dx}{dt} &= \frac{rx}{1+ey} \left(1 - \frac{x}{k}\right) - \frac{(p+\alpha y)xy}{1+m(p+\alpha y)x} - dx - x^2, \\ \frac{dy}{dt} &= \frac{c(p+\alpha y)xy}{1+m(p+\alpha y)x} - \mu y.\end{aligned}\quad (4)$$

Then Paul, Mondal, and Ghosh (2024) also discussed a prey predator model in the presence of hunting cooperation in predators with Holling type II response function, as follow:

$$\begin{aligned}\frac{dx}{dt} &= rx \left(1 - \frac{x}{k}\right) - \frac{(p+\alpha y)xy}{1+m(p+\alpha y)x} - dx - x^2, \\ \frac{dy}{dt} &= \frac{c(p+\alpha y)xy}{1+m(p+\alpha y)x} - \mu y - \mu_1 y^2.\end{aligned}\quad (5)$$

Where the model (5) assumes predator populations follow the Allee effect of mate finding and the predator population engage in intra-specific competition.

RESEARCH METHODS

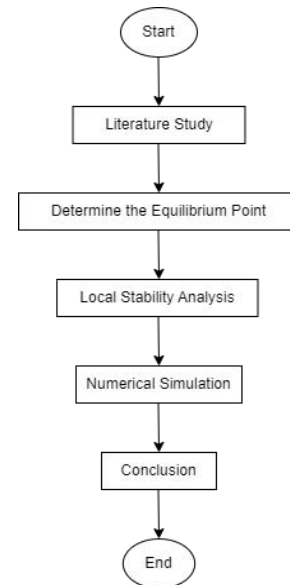


Figure 1. Research Process

Based on Figure 1, the first stage is a literature study by collecting sources from journals and articles related to previous research. The second stage is constructing a prey-predator model, extending the previous research study model. Furthermore, the third stage is dynamic analysis. Dynamic analysis includes determining the solution of the system in the form of an equilibrium point, the existence condition of the equilibrium point, analyzing the local stability of each equilibrium using eigenvalues, and numerical simulation to synchronize the analysis results using PPlane software.

RESULTS AND DISCUSSION

To construct the mathematical model, we set some assumptions:

1. The growth rate of the prey population follows a logistic equation.

2. The growth rate of the prey population decreases due to predation by wolves.
3. The growth rate of the predator population increases due to predation of deer.
4. The predation interaction uses hunting cooperation between predators to catch prey with Holling type II functional response, is the formed as $\frac{(p+\alpha y)xy}{1+m(p+\alpha y)x}$.
5. Hunting cooperation between predator populations is of the form $(p + \alpha y)$ with α parameter of hunting cooperation.
6. The predator population has a natural death rate when there is no predation.

The model was modified into:

$$\begin{aligned} \frac{dx}{dt} &= rx\left(1 - \frac{x}{k}\right) - \frac{(p + \alpha y)xy}{1 + m(p + \alpha y)x}, \\ \frac{dy}{dt} &= \frac{\beta(p + \alpha y)xy}{1 + m(p + \alpha y)x} - \mu y. \end{aligned} \quad (6)$$

In this system (6), $x(t)$ is the prey population and $y(t)$ is the predator population. Parameters are prey growth rate, carrying capacity, predator attack, hunting cooperation, handling time, natural death, prey to predator conversion. All parameters in this system are positive.

EQUILIBRIUM POINT

The equilibrium point in a model (6) is obtained by solving the system of equations that makes $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$. System (6) has different points of equilibrium E_1, E_2, E_3 .

1. $E_1(0,0)$ declared the extinction of prey and predator populations.
2. $E_2(k,0)$ declared the predator population to be extinct.
3. $E_3(x^*, y^*)$ the interior equilibrium point that both predator and prey populations exist.

LOCAL STABILITY ANALYSIS

The general form of the Jacobian matrix (Boyce, DiPrima, and Meade 2017) of the system (6) is given by:

$$J(x, y) = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \quad (7)$$

Where $\frac{dx}{dt} = f_1(x, y)$ and $\frac{dy}{dt} = f_2(x, y)$. The matrix elements of $J(x, y)$ is

$$\begin{aligned} m_{11} &= r\left(1 - \frac{x}{k}\right) - \frac{rx}{k} - \frac{(p + \alpha y)y}{1 + m(p + \alpha y)x} \\ &\quad + \frac{(p + \alpha y)^2 x y m}{(1 + m(p + \alpha y)x)^2}, \\ m_{12} &= \frac{\alpha xy}{1 + m(p + \alpha y)x} - \frac{(p + \alpha y)y}{1 + m(p + \alpha y)x} \\ &\quad + \frac{(p + \alpha y)x^2 y m \alpha}{(1 + m(p + \alpha y)x)^2}, \\ m_{21} &= \frac{\beta(p + \alpha y)y}{1 + m(p + \alpha y)x} - \frac{\beta(p + \alpha y)^2 x y m}{(1 + m(p + \alpha y)x)^2}, \\ m_{22} &= \frac{\beta xy}{1 + m(p + \alpha y)x} + \frac{\beta(p + \alpha y)y}{1 + m(p + \alpha y)x} \\ &\quad - \frac{\beta(p + \alpha y)x^2 y m \alpha}{(1 + m(p + \alpha y)x)^2}. \end{aligned}$$

Theorem 1. *The Equilibrium point $E_1 = (0, 0)$ is unstable (saddle point)*

Proof. The Jacobian matrix in equation (7) at $E_1 = (0, 0)$ is a follows

$$J_{E_1} = \begin{bmatrix} r & 0 \\ 0 & \mu \end{bmatrix}. \quad (8)$$

The eigenvalues Jacobian matrix at E_1 are.

$$\lambda_1 = r \text{ and } \lambda_2 = -\mu.$$

It's evident that $r > 0$, so as $\lambda_1 > 0$ and $-\mu < 0$ so as $\lambda_2 < 0$. Therefore E_1 is unstable (saddle point) (Boyce, DiPrima, and Meade 2017).

Theorem 2. *The Equilibrium point $E_2 = (k, 0)$ is asymptotically stable (node) if the following condition are satisfied $\frac{\beta pk}{kmp+1} < \mu$.*

Proof. At the point $E_2 = (k, 0)$ the Jacobian matrix in equation (7) becomes

$$J_{E_2} = \begin{bmatrix} -r & \frac{-pk}{kmp+1} \\ 0 & \frac{\beta pk}{kmp+1} - \mu \end{bmatrix}. \quad (4)$$

The eigenvalues of the Jacobian matrix at E_2 are.

$$\lambda_1 = -r \text{ and } \lambda_2 = \frac{\beta pk}{kmp+1} - \mu.$$

Therefore, both eigenvalues of the Jacobian matrix at E_2 have negative genuine parts $-r < 0$ so as $\lambda_1 < 0$, and to conditions is satisfied if $\lambda_1 < 0$

$$\frac{\beta pk}{kmp+1} < \mu.$$

In that $\lambda_1 < 0$ and $\lambda_2 < 0$, then the equilibrium point E_2 is asymptotically stable (node) with conditions

$$\frac{\beta pk}{kmp+1} < \mu.$$

Theorem 3. *The Equilibrium point $E_3 = (x^*, y^*)$ is asymptotically stable (node) if the condition:*

$$(i) \quad (m_{11} + m_{22}) < 0,$$

$$(ii) \quad m_{11}^2 + m_{22}^2 + 4m_{12}m_{21} > 2m_{11}m_{22}.$$

Proof. We evaluate the Jacobian matrix at $E_3 = (x^*, y^*)$ to obtained:

$$J_{E_3} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}.$$

The respective components are given by:

$$\begin{aligned} m_{11} &= r(1 - \frac{x^*}{k}) - \frac{rx^*}{k} - \frac{(p + \alpha y^*)y^*}{1 + m(p + \alpha y^*)x^*} \\ &\quad + \frac{(p + \alpha y^*)^2 x^* y^* m}{(1 + m(p + \alpha y^*)x^*)^2}, \\ m_{12} &= \frac{\alpha x^* y^*}{1 + m(p + \alpha y^*)x^*} - \frac{(p + \alpha y^*)y^*}{1 + m(p + \alpha y^*)x^*} \\ &\quad + \frac{(p + \alpha y^*)x^2 y^* m \alpha}{(1 + m(p + \alpha y^*)x^*)^2}, \\ m_{21} &= \frac{\beta(p + \alpha y^*)y^*}{1 + m(p + \alpha y^*)x^*} - \frac{\beta(p + \alpha y^*)^2 x^* y m}{(1 + m(p + \alpha y^*)x^*)^2}, \\ m_{22} &= \frac{\beta x^* y^*}{1 + m(p + \alpha y^*)x^*} + \frac{\beta(p + \alpha y^*)y^*}{1 + m(p + \alpha y^*)x^*} \\ &\quad - \frac{\beta(p + \alpha y^*)x^* y^* m \alpha}{(1 + m(p + \alpha y^*)x^*)}. \end{aligned}$$

The characteristic equation of J_{E_3} is given by:

$$\lambda^2 + \delta\lambda + \gamma = 0, \quad (9)$$

From characteristic equation of (9), the eigenvalue of J_{E_3} are given by:

$$\lambda_{1,2} = \frac{(\delta) \pm \sqrt{\delta^2 - 4(\gamma)}}{2},$$

if $\delta = (m_{11} + m_{22}) < 0$ and $\gamma = (m_{11} + m_{22})^2 - 4(m_{11}m_{22} - m_{12}m_{21}) > 0$, then the equilibrium point E_3 is asymptotically stable with conditions $(m_{11} + m_{22}) < 0$, and $m_{11}^2 + m_{22}^2 + 4m_{12}m_{21} > 2m_{11}m_{22}$.

NUMERICAL SIMULATION

This section describes the simulation of a system (6) based on parameter values. The selection of parameter values is based on the references and assumptions in Table 1 below.

Table 1. Parameter Values

Parameter	Description	Value	Reference
r	Prey Growth Rate	9.03	(Salwa et al., 2023)
k	Carrying Capacity	0.3	Assumption
p	Predator Attack Hunting	0.5	(Pal et al. 2019a)
α	Cooperation	0.5	(Du et al., 2022)
m	Handling Time	0.1	Assumption
μ	Natural death Prey to Predator	0.3	Assumption
β	Conversion	1	Assumption

In this article, the prey to predator conversion parameter (β) is varied to determine the changes in the stability of some equilibrium points. Numerical simulations were conducted by setting different values of parameter β ($\beta = 1, \beta = 1.2, \beta = 2.05, \beta = 15, 11$). In this simulation, four different initial values conditions are also given.

1. The first simulation with prey to predator conversion parameter when $\beta = 1$.

Based on the parameter values used in Table 1 with $\beta = 1$ there are two equilibrium points, namely $E_1 = (0,0)$, $E_2 = (0.3,0)$. Stability analysis with these parameter values, the eigenvalues of each equilibrium point are obtained as follows:

- $E_1 = (0,0)$, with $\lambda_1 = -0.3 < 0$ or $\lambda_2 = 9.03 > 0$, equilibrium point E_1 is unstable.
- Equilibrium point $E_2 = (0.3,0)$ is asymptotically stable with $\lambda_1 = -9.03 < 0$ or $\lambda_2 = -0.152 < 0$.

The numerical simulation based on Table 1 with $\beta = 1$ is illustrated as follows:

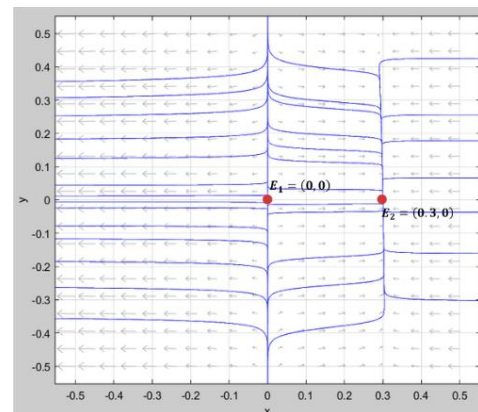


Figure 2. Phase portrait with $\beta = 1$

Based on the assumption for the parameter value $\beta = 1$. In figure 1, two existent points are obtained and there is one stable existent point, namely $E_2 = (0.3, 0)$ which is consistent with the analytical results.

2. The second simulation with prey to predator conversion parameter when $\beta = 1.2$

Based on the parameter values used in Table 1 with $\beta = 1.2$ there four equilibrium points, namely $E_1 = (0, 0)$, $E_2 = (0.3, 0)$, $E_3 = (0.13, 2.69)$, $E_4 = (0.27, 0.87)$. Stability analysis with these parameter values, the eigenvalues of each equilibrium point are obtained as follows:

- $E_1 = (0, 0)$, with $\lambda_1 = -0.3 < 0$ or $\lambda_2 = 9.03 > 0$, equilibrium point E_1 is unstable.
- Equilibrium point $E_2 = (0.3, 0)$ is asymptotically stable with $\lambda_1 = -9.03 < 0$ or $\lambda_2 = -0.123 < 0$.
- Equilibrium point $E_3 = (0.13, 2.69)$ is asymptotically stable with $\lambda_1 = -3.382 < 0$ or $\lambda_2 = -0.462 < 0$.
- $E_4 = (0.27, 0.87)$, with $\lambda_1 = -8.168 < 0$ or $\lambda_2 = 0.095 > 0$, equilibrium point E_4 is unstable.

The numerical simulation based on Table 1 with $\beta = 1.2$ is illustrated as follows:

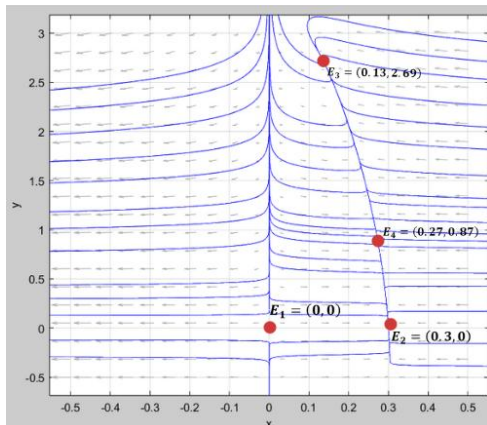


Figure 3. Phase portrait with $\beta = 1.2$

Based on the assumption for the parameter value $\beta = 1.2$. In figure 2, it is obtained that all points are on the x-axis and there are two stable existing points, namely $E_2 = (0.3, 0)$ and $E_3 = (0.13, 2.69)$ which is consistent with the analytical results. A system that has stability at two equilibrium points is called bistable. Changes in the stability of the system (6) are shown through numerical simulations by increasing the value of prey to predator conversion parameter $\beta = 1$ with $\beta = 1.2$

3. The third simulation with prey to predator conversion parameter when $\beta = 2.05$

Based on the parameter values used in Table 1 with $\beta = 2.05$ there three equilibrium points, namely $E_1 = (0, 0)$, $E_2 = (0.3, 0)$, $E_3 = (0.069, 3.28)$. Stability analysis with these parameter values, the eigenvalues of each equilibrium point are obtained as follows:

- $E_1 = (0, 0)$, with $\lambda_1 = -0.3 < 0$ or $\lambda_2 = 9.03 > 0$, equilibrium point E_1 is unstable.
- $E_2 = (0.3, 0)$ with $\lambda_1 = -9.03 < 0$ or with $\lambda_2 = 0.003 > 0$, equilibrium point E_2 is unstable.
- Equilibrium point $E_3 = (0.069, 3.28)$ is asymptotically stable with $\lambda_1 = -0.00005 + 2.1539i < 0$ or $\lambda_2 = -0.00005 - 2.1539i < 0$.

The numerical simulation based on Table 1 with $\beta = 2.05$ is illustrated as follows:

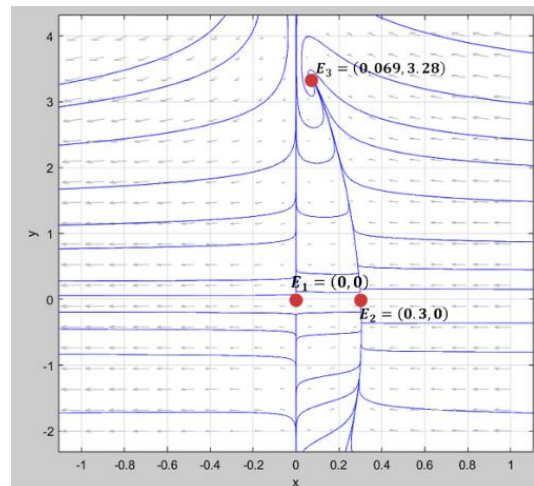


Figure 4. Phase portrait with $\beta = 2.03$

Based on the assumption of the parameter value point $\beta = 2.05$ in Figure 3, three existent points are obtained and there is one stable existent point, namely point $E_3 = (0.069, 3.28)$ which agrees with the analytical calculation results.

4. The fourth simulation with prey to predator conversion parameter when $\beta = 15.11$

Based on the parameter values used in Table 1 with $\beta = 15.11$ there three equilibrium points, namely $E_1 = (0, 0)$, $E_2 = (0.3, 0)$, $E_3 = (0.0084, 3.723)$. Stability analysis with these parameter values, the eigenvalues of each equilibrium point are obtained as follows:

- $E_1 = (0, 0)$ with $\lambda_1 = -0.3 < 0$ or $\lambda_2 = 9.03 > 0$, equilibrium point E_1 is unstable.
- $E_2 = (0.3, 0)$ with $\lambda_1 = -9.03 < 0$ or $\lambda_2 = 1.93 > 0$, equilibrium point E_2 is unstable.
- Equilibrium point $E_3 = (0.0084, 3.723)$ is asymptotically stable with $\lambda_1 = -0.878 + 1.544i < 0$ or $\lambda_2 = -0.878 - 1.544i < 0$.

The numerical simulation based on Table 1 with $\beta = 15.11$ is illustrated as follows:

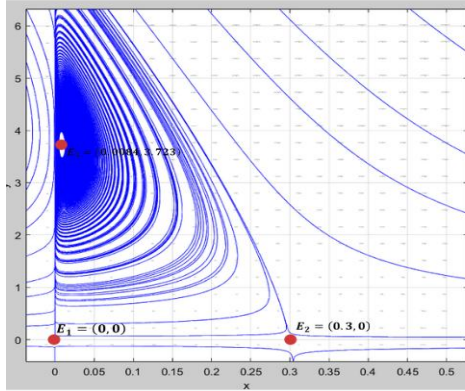


Figure 5. Phase portrait with $\beta = 15.11$

Based on the assumption of the parameter value point $\beta = 15.11$ in Figure 4, three existent points are obtained and there is one stable existent point $E_3 = (0.0084, 3.723)$ which agrees with the analytical calculation results. The eigenvalues in $E_3 = (0.0084, 3.723)$ are complex values in the form of a stable spiral.

5. The Numerical Continuation of Parameter β
Numerical continuation was performed on system (6) by varying the value of β , which is the parameter for the prey to predator conversion. Continuation begins when $\beta = 1$ shows that around point E_2 is stable. Then β is moved forward and LP (Limit Point) at $\beta = 1.009$. Then β is moved forward to $\beta = 1.2$ indicating a Saddle-Node bifurcation which illustrates that when $\beta = 2.03$ the system has two stable existing points namely E_2 and E_3 . Therefore, a system that has two stable equilibrium points is called a bistable system (Umaroh and Savitri 2023). The nature of stability in this system changes when passing BP (Branch Point) at $\beta = 2.03$. The BP phenomena is called transcritical bifurcation which means that there is a change in stability from the point E_2 which was originally stable to unstable when passing the value of the prey to predator conversion parameter $\beta = 2.03$. When $\beta > 2.03$, point E_2 is unstable and when $\beta < 2.03$, point E_2 is stable. Changes in the value of β can cause changes in the stability of E_2 and bring up the existing point E_3 which is known as a forward bifurcation (Savitri and Panigoro 2020). Then β is moved forward again and a Hopf bifurcation at $\beta = 15.11$. The numerical

continuation results also match the simulation results displayed in the phase portrait. In this research, the simulation results show complete dynamics with the appearance of Saddle-Node (LP) bifurcation, Transcritical (BP) bifurcation, and Hopf bifurcation.

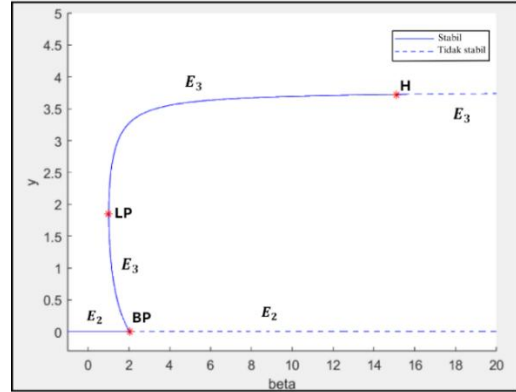


Figure 5. Bifurcation Diagram

CONCLUSION

Construction of a prey predator model with Holling type II response function and hunting cooperation of predators is.

$$\begin{aligned} \frac{dx}{dt} &= rx \left(1 - \frac{x}{k}\right) - \frac{(p+\alpha y)xy}{1+m(p+\alpha y)x'} \\ \frac{dy}{dt} &= \frac{\beta(p+\alpha y)xy}{1+m(p+\alpha y)x} - \mu y. \end{aligned}$$

The results of the dynamic analysis produce three equilibrium points, namely $E_1(0,0)$ which is always unstable saddle, $E_2(k, 0)$ and $E_3 = (x^*, y^*)$ stable under certain conditions of existence. Selection of parameter β as the prey to predator conversion that is numerically continued has influence the stability of each solution system. The result of the numerical analysis show agreement with the results of the analysis for the type of stability of each equilibrium solution, When numerical continuation on the parameter $\beta = 1$ there is only one stable point, namely $E_2(k, 0)$ meaning that the predator population has become extinct, when $\beta = 1.2$ shows a bistable system at points E_2 and E_3 meaning that the predator population is extinct and the two populations can coexist, when $\beta = 2.05$ there is only one stable point namely E_3 meaning that the two populations can coexist, and when $\beta = 15.11$ the system experiences Hopf bifurcation.

SUGGESTION

This study examines the analysis of interaction models in prey-predator using Holling type II response functions with hunting cooperation in

predators. Research can be developed with different parameters at the numerical simulation stage to show the stability of the results or changes in the system solution.

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